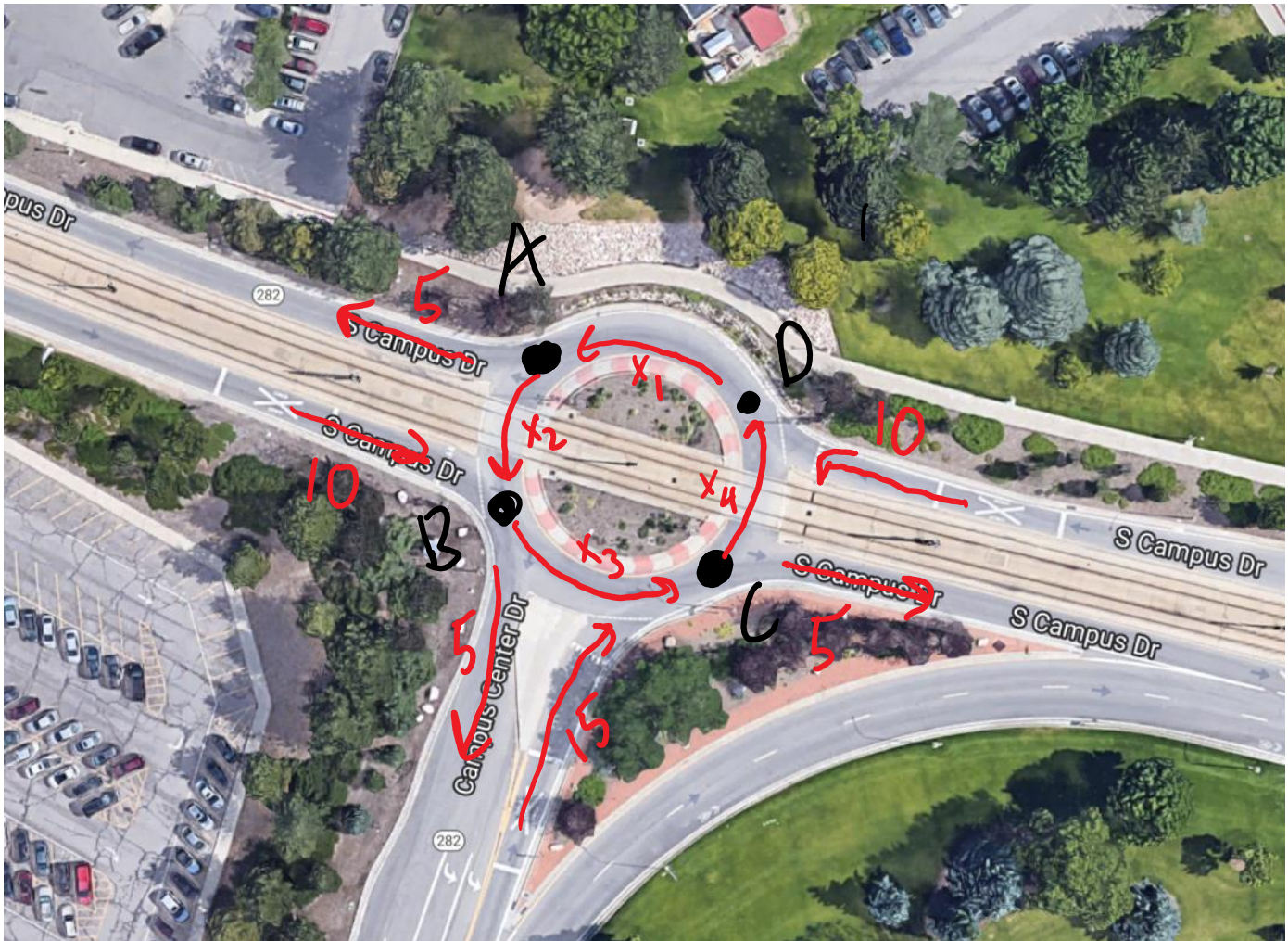


Section 1.3 Applications of Linear Systems

Traffic flow cars in = cars out



Section 1.3 Applications of Linear Systems

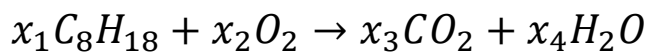
In the last example we were balancing input and output of traffic. What goes in must come out, this concept is pervasive in science and linear equations often result. We often make the same considerations when thinking about chemical reactions.

Gasoline

Clear liquid; distinctive odor.
Irritating to eyes/skin/
respiratory tract.
Also causes: dizziness,
drunkenness, unconscious-
ness. Absorbed through
skin. Chronic: dermatitis.
Possible cancer hazard.
Flammable. Can form explosive
mixtures in air.



CAS No. 8006-61-9



GASOLINE & FIRE NEVER MIX

HELP US SPREAD THE WORD. Most parents today say they handle gasoline more safely than their own parents did. Parents know that gasoline is extremely flammable and its vapors can travel and ignite. And they know that kids learn by example.

That's why most parents don't use gas to start fires. Help us make sure every parent knows.

MOST PARENTS DON'T MIX GAS AND FIRE. BUT EVERY TIME I SEE A CHILD SERIOUSLY BURNED, I REMEMBER THERE ARE STILL PEOPLE TO REACH.

FIREFIGHTER TOM HUFFORD
OKLAHOMA BURN CAMP ORGANIZER



STOPGASFIRE.org

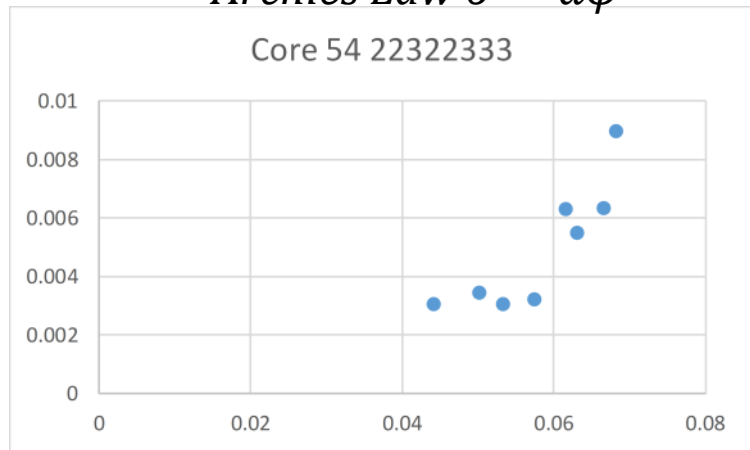
Apparently this is necessary in 2019.....
Also, don't eat tide pods.

Section 1.3 Applications of Linear Systems

Physical Modeling

Brine Volume Fraction ϕ	Conductivity σ
0.068309	0.00897
0.066612	0.006353
0.05029	0.003431
0.04422	0.003045
0.057476	0.003223
0.063163	0.005508
0.0534	0.003045
0.061583	0.006294

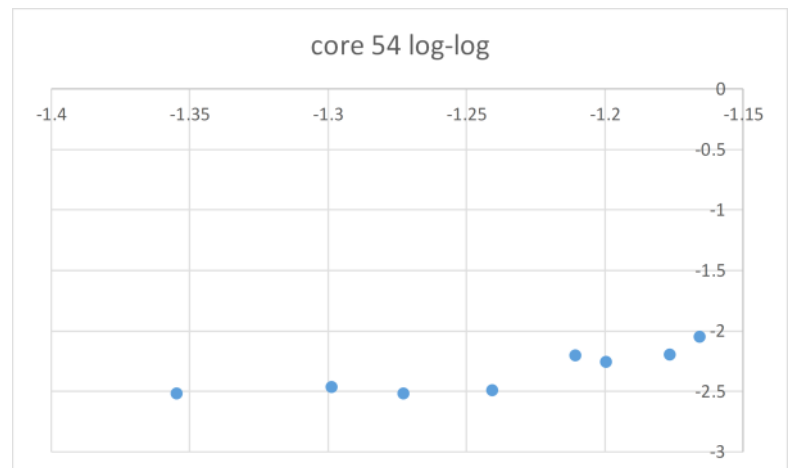
$$\text{Archies Law } \sigma = a\phi^m$$



We can change this to a linear equation!



$\ln(\phi)$	$\ln(\sigma)$
-1.16552	-2.04721
-1.17645	-2.19704
-1.29852	-2.46455
-1.35438	-2.51647
-1.24052	-2.49179
-1.19954	-2.25902
-1.27246	-2.51637
-1.21054	-2.20106



Let's choose the first and fourth data points since they cover a good x range: in this case our linear system is:

Section 1.3 Applications of Linear Systems

Our augmented matrix is then

$\ln(\phi)$	$\ln(\sigma)$
-1.16552	-2.04721
-1.17645	-2.19704
-1.29852	-2.46455
-1.35438	-2.51647
-1.24052	-2.49179
-1.19954	-2.25902
-1.27246	-2.51637
-1.21054	-2.20106

$$A = \left[\begin{array}{cc|c} -1.16552 & 1 & -2.04721 \\ -1.35438 & 1 & -2.19704 \end{array} \right]$$

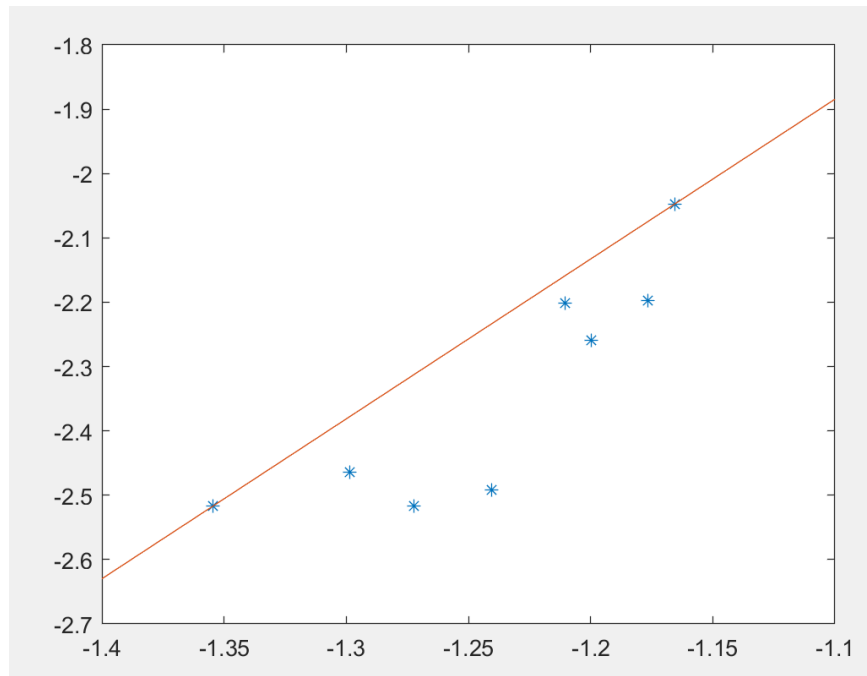
Inviting matlab to get the rref form with the following script

```
xx=[-1.16552,-1.17645,-1.29852,-1.35438,-1.24052,-1.19954,-1.27246,-1.21054];
yy=[-2.04721,-2.19704,-2.46455,-2.51647,-2.49179,-2.25902,-2.51637,-2.20106];

AA=[xx(1),1,yy(1);xx(4),1,yy(4)]
BB=rref(AA)
p=[-1.4:0.001:-1.1];
s=BB(1,3).*p+BB(2,3);
plot(xx,yy,'*')
hold on
plot(p,s)
```

$$rref(A) = \left[\begin{array}{cc|c} 1 & 0 & 2.485 \\ 0 & 1 & 0.085 \end{array} \right]$$

So the equation of our line is $y = 2.485x + 0.085$



Notice that while the line follows the trend and hits the points we asked it to it is missing the others in practice we will make use of all available data and find something called the *least squares solution* but this is a topic for another day. Are there any other two points we could pick to better capture the data?

Section 1.3 Applications of Linear Systems

Polynomial Curve fitting (When things sometimes go too far)

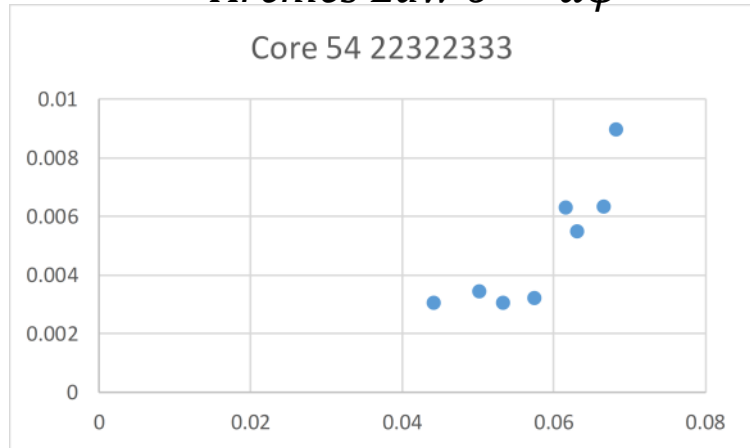
Here we have 8 data points and if I like we can find an 7th degree polynomial (8 terms total with the constant)

$$\sigma(\phi) = a\phi^7 + b\phi^6 + c\phi^5 + d\phi^4 + e\phi^3 + f\phi^2 + g\phi + h$$

Which goes through all 8 points exactly.

Archies Law $\sigma = a\phi^m$

Brine Volume Fraction ϕ	Conductivity σ
0.068309	0.00897
0.066612	0.006353
0.05029	0.003431
0.04422	0.003045
0.057476	0.003223
0.063163	0.005508
0.0534	0.003045
0.061583	0.006294



This is done by creating a linear system where we plug in each ϕ value into our equation and setting equal to the corresponding σ value.

$$0.00897 = a(0.068309)^7 + b(0.068309)^6 + c(0.068309)^5 + d(0.068309)^4 + e(0.068309)^3 + f(0.068309)^2 + g(0.068309) + h$$

$$0.006353 = a(0.066612)^7 + b(0.066612)^6 + c(0.066612)^5 + d(0.066612)^4 + e(0.066612)^3 + f(0.066612)^2 + g(0.066612) + h$$

$$0.003432 = a(0.05029)^7 + b(0.05029)^6 + c(0.05029)^5 + d(0.05029)^4 + e(0.05029)^3 + f(0.05029)^2 + g(0.05029) + h$$

$$0.003045 = a(0.04422)^7 + b(0.04422)^6 + c(0.04422)^5 + d(0.04422)^4 + e(0.04422)^3 + f(0.04422)^2 + g(0.04422) + h$$

$$0.003223 = a(0.057476)^7 + b(0.057476)^6 + c(0.057476)^5 + d(0.057476)^4 + e(0.057476)^3 + f(0.057476)^2 + g(0.057476) + h$$

$$0.005508 = a(0.063163)^7 + b(0.063163)^6 + c(0.063163)^5 + d(0.063163)^4 + e(0.063163)^3 + f(0.063163)^2 + g(0.063163) + h$$

$$0.003045 = a(0.0534)^7 + b(0.0534)^6 + c(0.0534)^5 + d(0.0534)^4 + e(0.0534)^3 + f(0.0534)^2 + g(0.0534) + h$$

$$0.006294 = a(0.061583)^7 + b(0.061583)^6 + c(0.061583)^5 + d(0.061583)^4 + e(0.061583)^3 + f(0.061583)^2 + g(0.061583) + h$$

If we compute the powers and transfer to a matrix form we obtain:

```

0.000000006939777  0.000000101593884  0.000001487269384  0.000021772671011  0.000318737955628  0.004666119481000  0.068309000000000  1.000000000000000  0.008970000000000
0.000000005524970  0.000000083559738  0.000001263758892  0.000019113110895  0.000289067012928  0.004371854400000  0.066120000000000  1.000000000000000  0.006353000000000
0.000000000813526  0.000000016176696  0.000000321668236  0.000006396266385  0.000127187639389  0.002529084100000  0.050290000000000  1.000000000000000  0.003431000000000
0.000000000330622  0.000000007476743  0.000000169080565  0.000003823622011  0.000086468159448  0.001955408400000  0.044220000000000  1.000000000000000  0.003045000000000
0.000000002072076  0.00000036051158  0.000000627238461  0.000010913049986  0.000189871424346  0.003303490576000  0.057476000000000  1.000000000000000  0.003223000000000
0.000000004010876  0.000000063500405  0.000001005341813  0.000015916625450  0.000251992866872  0.003989564569000  0.063163000000000  1.000000000000000  0.005508000000000
0.000000001238194  0.000000023187159  0.000000434216463  0.000008131394434  0.000152273304000  0.002851560000000  0.053400000000000  1.000000000000000  0.003045000000000
0.000000003359123  0.000000054546269  0.000000885735820  0.000014382797519  0.000233551426842  0.003792465889000  0.061583000000000  1.000000000000000  0.006294000000000

```

A bit yucky... Let's have matlab do the heavy lifting!

Section 1.3 Applications of Linear Systems

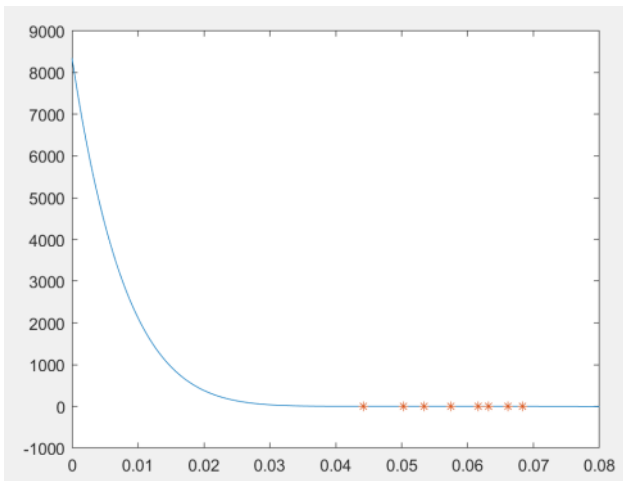
Using the rref command in matlab we obtain the following matrix and can read off the coefficients!

1	0	0	0	0	0	0	0	-4220701885864.06
0	1	0	0	0	0	0	0	1705264710529.54
0	0	1	0	0	0	0	0	-294243066420.417
0	0	0	1	0	0	0	0	28105948029.5212
0	0	0	0	1	0	0	0	-1604935086.03702
0	0	0	0	0	1	0	0	54783499.2300331
0	0	0	0	0	0	1	0	-1034944.27128364
0	0	0	0	0	0	0	1	8346.79928915277

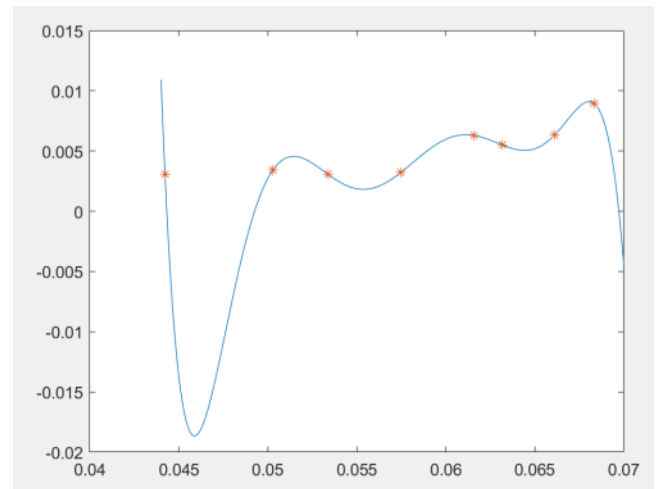
Here is the polynomial we find:

$$\begin{aligned}\sigma(\phi) &= -4.2 \times 10^{12} \phi^7 + 1.7053 \times 10^{12} \phi^6 - 2.942 \times 10^{11} \phi^5 + 2.81 \times 10^{10} \phi^4 - 1.6 \times 10^9 \phi^3 + 5.47 \times 10^7 \phi^2 \\ &\quad - 1.034 \times 10^6 \phi + 8.3468 \times 10^3\end{aligned}$$

Plots with the data, beware the dangers of overfitting!



Our polynomial on the interval [0,0.08]
Terrible everywhere but the data points!



Close up of the polynomial around the data points, hits them all! However, the function itself is complete nonsense physically.

Section 1.3 Applications of Linear Systems

Matlab code for the last example

```
x=[0.0683,0.0661,0.0503,0.0442,0.0575,0.0632,0.0534,0.0616]
y=[0.0090,0.0064,0.0034,0.0030,0.0032,0.0055,0.0030,0.0063]

A=[x(1)^7,x(1)^6,x(1)^5,x(1)^4,x(1)^3,x(1)^2,x(1),1,y(1);x(2)^7,x(2)^6,x(2)^5,x(2)^4,x(2)^3,x(2)^2,x(2),1,y(2);x(3)^7,x(3)^6,x(3)^5,x(3)^4,x(3)^3,x(3)^2,x(3),1,y(3);x(4)^7,x(4)^6,x(4)^5,x(4)^4,x(4)^3,x(4)^2,x(4),1,y(4);x(5)^7,x(5)^6,x(5)^5,x(5)^4,x(5)^3,x(5)^2,x(5),1,y(5);x(6)^7,x(6)^6,x(6)^5,x(6)^4,x(6)^3,x(6)^2,x(6),1,y(6);x(7)^7,x(7)^6,x(7)^5,x(7)^4,x(7)^3,x(7)^2,x(7),1,y(7);x(8)^7,x(8)^6,x(8)^5,x(8)^4,x(8)^3,x(8)^2,x(8),1,y(8)]
B=rref(A)
phi=[0.044:0.0001:0.07]
sigma=B(1,9).*(phi.^7)+B(2,9).*(phi.^6)+B(3,9).*(phi.^5)+B(4,9).*(phi.^4)+B(5,9).*(phi.^3)+B(6,9).*(phi.^2)+B(7,9).*(phi.^1)+B(8,9).*(phi.^0)
plot(x,y,'*')
hold on
plot(phi,sigma)
```

Note that the definition of A should be typed out as a single row... This probably makes copy and paste hard. You can find two .m files in the recourses folder on Sakai which contain the matlab code from today's examples should you like to play around with it.