

Extra Credit 1

Math 1050 001 Summer 2013

For this extra credit assignment you will analyze some data from a physics experiment and use logarithms to interoperate the results.

The Experiment.

We are going to determine the acceleration due to the earth's gravity by dropping a rock or some other heavy object from different heights. Imagine you go find a rock, tape measure and ladder that is 5 meters tall. Starting at 0.5 m you drop the rock and time how long it takes to hit the ground next you go to 1 m drop the rock and time the fall again. You do the same thing in 0.5 m intervals until you get all they way to 5 m each time measuring how long it took for your rock to hit the ground. If you did this you would get data that will look something like:

Height (meters)	Time (seconds)
0.5	0.32
1	0.45
1.5	0.54
2	0.61
2.5	0.72
3	0.78
3.5	0.87
4	0.91
4.5	0.95
5	1.0

Now you are very clever and you know that the function which gives the distance a falling object has traveled is given by $h(t) = \frac{1}{2} g t^2$. You realize that you could figure out the gravitational acceleration on the earth (g) by plugging a height and time from the data table and solving. But you want an accurate answer and you know your timing with the stopwatch is not perfect. You decide that to reduce the error you want to calculate g using all of the data. We will do this in two ways.

Method 1.

For each of the values in the table calculate the gravitational constant g by solving the equation $h(t) = \frac{1}{2}gt^2$ for g and plugging in the corresponding time and heights. Then take the average of all the different values you get for g . The table below will be helpful.

Height (meters)	Time(Seconds)	$g \text{ (m/s}^2\text{)}$
0.5	0.32	
1	0.46	
1.5	0.54	
2	0.61	
2.5	0.72	
3	0.78	
3.5	0.87	
4	0.91	
4.5	0.95	
5	1.0	
		Average:

Method 2.

For this method we will do what is called a linearization of the equation. Lets examine our equation

$$h = \frac{1}{2}gt^2$$

Lets take the logarithm of both sides

$$\ln(h) = \ln\left(\frac{1}{2}gt^2\right)$$

Using the properties of logarithms we can expand the right hand side out to get the equation:

$$\ln(h) = 2\ln(t) + \ln\left(\frac{1}{2}g\right)$$

Here is the cool part we can think of the $\ln(h)$ as a y value and $\ln(t)$ as an x value noting that $\ln\left(\frac{1}{2}g\right)$ is just a constant value we notice that this equation looks a lot like

$$y = 2x + b$$

The equation of a line with slope 2 and y intercept b . Note that y corresponds to $\ln(h)$, x corresponds to $\ln(t)$ and b corresponds to $\ln(\frac{1}{2}g)$. What we could then do is take $\ln(h)$ for each height we have and $\ln(t)$ for each time we have and plot them on the x-y axis as ordered pairs $(x, y) = (\ln(t), \ln(h))$. These points should then fall on a line with slope 2 and y intercept $b = \ln(\frac{1}{2}g)$. Because of error in the measurements however the points will not fall perfectly on a line. What you will need to do is draw the best fit line that you can. Then you can look at the y intercept $b = \ln(1/2g)$ and solve for g . That is $g = 2e^b$. For an extra 5 points you can use software like excel, matlab or maple to find the real best fit line but include the print out of your work.

You might find the following table useful to organize the numbers.

Height (meters)	Time (seconds)	$\ln(t)$	$\ln(h)$	$(x,y)=(\ln(t),\ln(h))$
0.5	0.32			
1	0.46			
1.5	0.54			
2	0.61			
2.5	0.72			
3	0.78			
3.5	0.87			
4	0.91			
4.5	0.95			
5	1.0			

What I want you to do:

1. Fill out all of the tables using a calculator or computer (5pts)
2. Find the average value for g in Method 1. (5pts)
3. Reproduce the linearization from method 2 justifying the steps using properties of logarithms (5pts)
4. Plot the ordered pairs $(\ln(t), \ln(h))$ and draw a line that best captures the data. (5pts)
5. From your best fit line find the y intercept and calculate g from it.) (5pts) You can get an additional 5 points if you use software or special formulas to calculate the actual best fit line.

