

Practice Exam II

Math 1050 001 Summer 2013
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Name and Unid:

Solution S

by writing my name I swear by the honor code

Read all of the following information before starting the exam:

- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Justify your answers algebraically whenever possible to ensure full credit. Sketch all relevant graphs and explain all relevant mathematics.
- Circle or otherwise indicate your final answers.
- You MAY NOT use a calculator or notes. You may use the formulas provided.
- This test has 10 problems and one bonus problem and is worth 100 points, plus some extra credit at the end. It is your responsibility to make sure that you have all of the pages!
- Good luck!

1. (10 points)

a. (5 pts) Solve $3^{x+2} = 27^{4x}$

$$\begin{aligned}
 3^{x+2} &= (3^3)^{4x} \\
 3^{x+2} &= 3^{12x} \\
 (1-1) \text{ property } \quad x+2 &= 12x \\
 -x &\quad -x
 \end{aligned}
 \rightarrow 2 = 11x$$

$$x = \frac{2}{11}$$

b. (5 pts) Calculate $\log_9\left(\frac{1}{27}\right)$

$$\begin{aligned}
 \log_9\left(\frac{1}{27}\right) &= \log_9(27^{-1}) = -\log_9(27) \\
 &= -\log_9(3^3) = -\log_9((\sqrt{9}))^3 = -\log_9((9^{1/2})^3) \\
 &= -\log_9(9^{3/2}) = -\frac{3}{2}
 \end{aligned}$$

2. (10 points)

a. (5 pts) Solve $\log_3(x^2 + 2x + 1) - \log_3(x^2 - 1) + \log_3(x - 1) = 1$

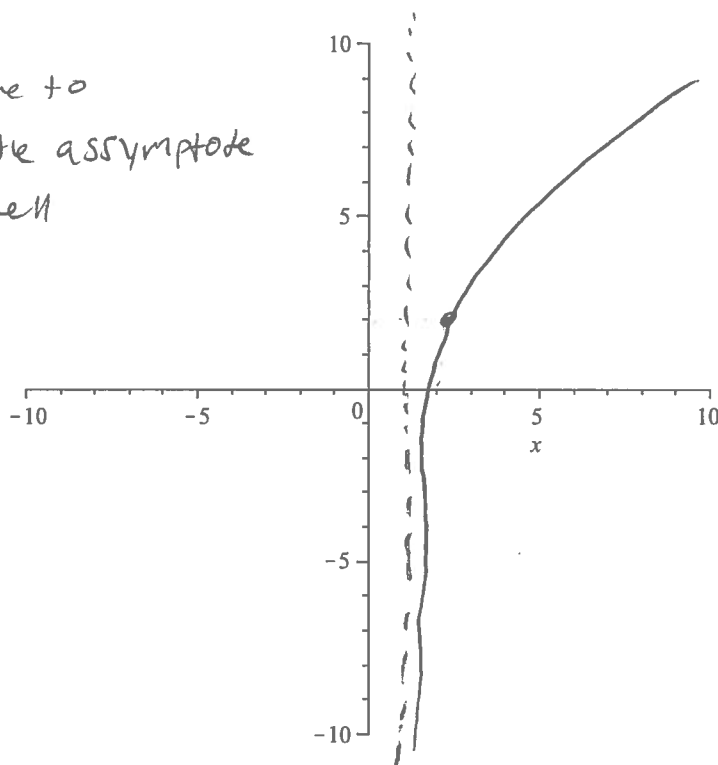
$$\begin{aligned}
 \Rightarrow \log_3\left(\frac{x^2 + 2x + 1}{x^2 - 1}\right) + \log_3(x - 1) &= 1 \\
 \Rightarrow \log_3\left(\frac{(x^2 + 2x + 1)(x - 1)}{x^2 - 1}\right) &= 1 \\
 \Rightarrow \log_3\left(\frac{(x+1)^2 \cancel{(x-1)}}{\cancel{(x+1)} \cancel{(x-1)}}\right) &= 1 \\
 \Rightarrow \log_3(x+1) &= 1 \\
 \Rightarrow 3^{\log_3(x+1)} &= 3^1
 \end{aligned}$$

$$\begin{aligned}
 \Rightarrow x+1 &= 3 \\
 \Rightarrow x &= 2
 \end{aligned}$$

check to see if 2 is in the domain of original equation and here it is.

- b. (5 pts) Sketch the graph of $\log_2(x-1) + 2$ ← UP 2

make sure to
shift the asymptote
as well



3. (10 points) Suppose you invest \$1000 into an account with an interest rate of $r = 5\%$ compounded monthly how long before you double your money. You can leave your answer in logarithmic form

Relevant equation

$$A = P \left(1 + \frac{r}{n} \right)^{nt}$$

Interest rate $\rightarrow r$
 Initial Investment $\rightarrow P$
 number of times per year interest is compounded $\rightarrow n$
 number of years $\rightarrow t$

To double our money we would have \$2000

so

$$2000 = 1000 \left(1 + \frac{0.05}{12} \right)^{12t} \quad \leftarrow \text{Solve for } t$$

$$2 = \left(1 + \frac{0.05}{12} \right)^{12t}$$

$$\ln(2) = \ln \left(\left(1 + \frac{0.05}{12} \right)^{12t} \right)$$

$$\ln(2) = 12t \ln \left(1 + \frac{0.05}{12} \right)$$

$$t = \frac{\ln(2)}{12 \ln \left(1 + \frac{0.05}{12} \right)}$$

4. (5 points) A certain bacteria doubles its population every 10 minutes. Using the $P = P_0 e^{rt}$ population model find the growth rate r

growth rate
time
Initial pop
2.71828

If it doubles in 10 min suppose we start with 1 bacteria ($P_0 = 1$) then after $t = 10$ min we have $P = 2$ so: $2 = e^{r(10)}$

$$2 = e^{r(10)}$$

$$\Rightarrow \ln(2) = \ln(e^{r(10)})$$

$$\Rightarrow \ln(2) = r(10)$$

$\leftarrow \ln = \log_e$ so inverse property

$$\Rightarrow \boxed{\frac{\ln(2)}{10} = r}$$

5. (5 points) Indicate whether the following systems of equations have a unique solution, no solution or infinitely many solutions but do not solve.

a. (3 pts)

Compare the lines

$$x + 4y = 1$$

$$2x + 8y = 2$$

$$x + 4y = 1$$

$$2x + 8y = 2$$

$$4y = 1 - x$$

$$8y = -2x + 2$$

$$y = \frac{1}{4} - \frac{1}{4}x$$

$$y = \frac{-2x}{8} + \frac{2}{8}$$

$$y = -\frac{1}{4}x + \frac{1}{4}$$

$$y = -\frac{1}{4}x + \frac{1}{4}$$

Since they are the same line they coincide everywhere thus there are infinitely many solutions.

b. (2 pts)

$$x + 3y = 2$$

$$2x + 3y = 2$$

$$x + 3y = 2$$

$$2x + 3y = 2$$

$$3y = -x + 2$$

$$3y = -2x + 2$$

$$y = -\frac{1}{3}x + \frac{2}{3}$$

$$y = -\frac{2}{3}x + \frac{2}{3}$$

The lines have different slopes so they must cross at some point

Unique solution

6. (10 points) Solve the following system of equations any way you like:

$$x + y + z = 6$$

$$3x + 2y + z = 10$$

$$2x + 4y - 2z = 4$$

Using elimination

$$\begin{array}{rcl} x + y + z & = & 6 \\ 3x + 2y + z & = & 10 \leftarrow \\ 2x + 4y - 2z & = & 4 \end{array} \quad \begin{array}{l} -3E_1 + E_2 = \\ -3x - 3y - 3z = -18 \\ + 3x + 2y + z = 10 \\ \hline 0x - y - 2z = -8 \end{array}$$

↓

$$\begin{array}{rcl} x + y + z & = & 6 \\ 0x - y - 2z & = & -8 \rightarrow \\ 2x + 4y - 2z & = & 4 \leftarrow \end{array} \quad \begin{array}{l} -2E_1 + E_3 = \\ -2x - 2y - 2z = -12 \\ + 2x + 4y - 2z = 4 \\ \hline 0x + 2y - 4z = -8 \end{array}$$

↓

$$\begin{array}{rcl} x + y + z & = & 6 \\ 0x - y - 2z & = & -8 \rightarrow \\ 0x + 2y - 4z & = & -8 \leftarrow \end{array} \quad \begin{array}{l} 2E_2 + E_3 = \\ 0x - 2y - 4z = -16 \\ + 0x + 2y - 4z = -8 \\ \hline 0x + 0y - 8z = -24 \end{array}$$

↓

$$\begin{array}{l} x + y + z = 6 \\ 0x - y - 2z = -8 \\ 0x + 0y - 8z = -24 \end{array} \quad \begin{array}{l} \text{Now Back substitute} \\ -8z = -24 \Rightarrow z = \frac{-24}{-8} = 3 \quad \boxed{z = 3} \end{array}$$

$$0x - y - 2(3) = -8$$

$$-y - 6 = -8$$

$$-y = -8 + 6 = -2$$

$$\Rightarrow \boxed{y = 2}$$

$$x + 2 + 3 = 6$$

$$x + 5 = 6$$

$$x = 6 - 5 = 1$$

$$\rightarrow \boxed{x = 1}$$

7. (15 points)

a. (5 pts)

Does the following system have a solution? Determine the answer by taking the determinant of the coefficient matrix.

$$2x + 3y = 5$$

$$4x + 2y = 6$$

coeff matrix: $\begin{bmatrix} 2 & 3 \\ 4 & 2 \end{bmatrix}$ $\det\left(\begin{bmatrix} 2 & 3 \\ 4 & 2 \end{bmatrix}\right) = 2(2) - 4(3)$

$$= 4 - 12 = \boxed{-8} \neq 0 \text{ so unique solution}$$

b. (10 pts) Solve the matrix equation by using the inverse of the coefficient matrix.

$$\begin{bmatrix} 2 & 3 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$$

$$\text{If } A = \begin{bmatrix} 2 & 3 \\ 4 & 2 \end{bmatrix} \quad A^{-1} = \frac{-1}{\det(A)} \begin{bmatrix} 2 & -3 \\ -4 & 2 \end{bmatrix} = \frac{-1}{2(2) - 4(3)} \begin{bmatrix} 2 & -3 \\ -4 & 2 \end{bmatrix}$$

$$= -\frac{1}{8} \begin{bmatrix} 2 & -3 \\ -4 & 2 \end{bmatrix} \text{ now multiply both sides}$$

$$-\frac{1}{8} \begin{bmatrix} 2 & -3 \\ -4 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{8} \begin{bmatrix} 2 & -3 \\ -4 & 2 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \frac{-1}{8} \begin{bmatrix} 2(5) + 6(-3) \\ -4(5) + 2(6) \end{bmatrix} = \frac{-1}{8} \begin{bmatrix} 10 - 18 \\ -20 + 12 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{-1}{8} \begin{bmatrix} -8 \\ -8 \end{bmatrix} = \begin{bmatrix} \frac{1}{8}(-8) \\ \frac{-1}{8}(-8) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\Rightarrow \boxed{x=1 \quad y=1}$$

8. (20 points) Let $A = \begin{bmatrix} 1 & 1 & -2 \\ 3 & -1 & 1 \\ 0 & 4 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 & -1 \\ 4 & 2 & 10 \\ 5 & 1 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & 1 & -2 \\ 4 & 2 & 1 \end{bmatrix}$

a. (5 pts) Calculate $2A - B$

$$2 \begin{bmatrix} 1 & 1 & -2 \\ 3 & -1 & 1 \\ 0 & 4 & -3 \end{bmatrix} - \begin{bmatrix} 2 & 1 & -1 \\ 4 & 2 & 10 \\ 5 & 1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 & -4 \\ 6 & -2 & 2 \\ 0 & 8 & -6 \end{bmatrix} - \begin{bmatrix} 2 & 1 & -1 \\ 4 & 2 & 10 \\ 5 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 2-2 & 2-1 & -4-(-1) \\ 6-4 & -2-2 & 2-10 \\ 0-5 & 8-1 & -6-4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & -3 \\ 2 & -4 & -8 \\ -5 & 7 & -10 \end{bmatrix}$$

b. (5 pts) Which is calculable CB or BC what is the size of the resulting matrix for the one that can be done?

$$C = 2 \times 3$$

$$B = 3 \times 3$$

$C B$

$$2 \times 3 \times 3 \times 3$$

same
so can

size of new matrix

$B C$

$$3 \times 3 \times 2 \times 3$$

not same
can't multiply

CB of size 2×3

c. (10 pts) Calculate CA

$$CA = \begin{bmatrix} 0 & 1 & -2 \\ 4 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & -2 \\ 3 & -1 & 1 \\ 0 & 4 & -3 \end{bmatrix} = \begin{bmatrix} 0+3+0 & 0+(-1)+(-8) & 0+1+6 \\ 4+6+0 & 4+(-2)+4 & (-2)+1+(-3) \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -9 & 7 \\ 10 & 6 & -4 \end{bmatrix}$$

9. (10 points) Take the determinant of $A = \begin{bmatrix} 0 & 1 & -2 \\ 4 & 2 & 1 \\ 0 & 6 & -3 \end{bmatrix}$

Expand across the column with most zeros
or row

$$\begin{vmatrix} 0 & 1 & -2 \\ 4 & 2 & 1 \\ 0 & 6 & -3 \end{vmatrix} = 0 \begin{vmatrix} 2 & 1 \\ 6 & -3 \end{vmatrix} - 4 \begin{vmatrix} 1 & -2 \\ 6 & -3 \end{vmatrix} + 0 \begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix}$$

$$= -4(1(-3) - 6(-2)) = -4(-3 + 8) = -4(5)$$

$$= \boxed{-20}$$

10. (5 points)

Find the inverse of $A = \begin{bmatrix} 0 & 0 & 1 \\ 4 & 2 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

augment with Identity matrix
and reduce rows all the way

$$\left[\begin{array}{ccc|ccc} 0 & 0 & 1 & 1 & 0 & 0 \\ 4 & 2 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{-R_1 + R_2} \left[\begin{array}{ccc|ccc} 0 & 0 & 1 & 1 & 0 & 0 \\ 4 & 2 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 0 & 0 & 1 & 1 & 0 & 0 \\ 4 & 2 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{-R_1 + R_3} \left[\begin{array}{ccc|ccc} 0 & 0 & 1 & 1 & 0 & 0 \\ 4 & 2 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 0 & 0 & 1 & 1 & 0 & 0 \\ 4 & 2 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{-4R_3 + R_2} \left[\begin{array}{ccc|ccc} 0 & 0 & 1 & 1 & 0 & 0 \\ 4 & 2 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 2 & 0 & 3 & 1 & -4 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\frac{R_2}{2}} \left[\begin{array}{ccc|ccc} 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & \frac{3}{2} & \frac{1}{2} & -2 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & \frac{3}{2} & \frac{1}{2} & -2 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\text{swap } R_1, R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 0 & 1 \\ 0 & 1 & 0 & \frac{3}{2} & \frac{1}{2} & -2 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{array} \right]$$

This is the inverse

$$A^{-1} = \begin{bmatrix} -1 & 0 & 1 \\ \frac{3}{2} & \frac{1}{2} & -2 \\ 1 & 0 & 0 \end{bmatrix}$$

11. (5 points) (Bonus all or nothing) Derive as we did in class the formula for the partial sum of a geometric sequence. That is that

$$\sum_{i=0}^n r^i = \frac{1 - r^{n+1}}{1 - r}$$

Explain what happens when $n \rightarrow \infty$ and $|r| < 1$.

$$\sum_{i=0}^n r^i = 1 + r + r^2 + r^3 + r^4 + \dots + r^n$$

The trick

$$\hookrightarrow 1 + r + r^2 + r^3 + \dots + r^n - r(1 + r + r^2 + r^3 + \dots + r^n)$$

$$= 1 + \cancel{r} + \cancel{r^2} + \cancel{r^3} + \dots + \cancel{r^n} - \cancel{r} - \cancel{r^2} - \cancel{r^3} - \dots - \cancel{r^n} - r^{n+1}$$

$$= 1 - r^{n+1}$$

so

$$(1 + r + r^2 + \dots + r^n) - r(1 + r + r^2 + \dots + r^n) = 1 - r^{n+1}$$

Factoring out the sum

$$(1 + r + r^2 + \dots + r^n)(1 - r) = 1 - r^{n+1}$$

divide by $(1 - r)$

$$\boxed{(1 + r + r^2 + \dots + r^n) = \frac{1 - r^{n+1}}{1 - r} \quad \checkmark}$$

as $n \rightarrow \infty$ and $|r| < 1$ $r^{n+1} \rightarrow 0$ so

$$\boxed{\sum_{i=0}^{\infty} r^i = \frac{1}{1 - r}}$$

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