

Practice Exam I

Math 1050 001 Summer 2013
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Name and Unid:

Solutions

by writing my name I swear by the honor code

Read all of the following information before starting the exam:

- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Justify your answers algebraically whenever possible to ensure full credit. Sketch all relevant graphs and explain all relevant mathematics.
- Circle or otherwise indicate your final answers.
- You MAY NOT use a calculator or notes. You may use the formulas provided.
- This test has 11 problems and one bonus problem and is worth 100 points, plus some extra credit at the end. It is your responsibility to make sure that you have all of the pages!
- Good luck!

For this use the distance formula to find the side lengths of the triangle

$$d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

1. (5 points)

a. (3 pts) Is the triangle with vertices $(-1, 3)$, $(3, 5)$ and $(5, 1)$ a right triangle? Justify your answer.

we can find the distance between each vertex between and see if it satisfies the Pythagorean theorem.

$$(-1, 3)(3, 5) d_1 = \sqrt{(5-3)^2 + (3-(-1))^2} = \sqrt{2^2 + 4^2} = \sqrt{4+16} = \sqrt{20}$$

$$(3, 5)(5, 1) d_2 = \sqrt{(1-5)^2 + (5-3)^2} = \sqrt{4^2 + 2^2} = \sqrt{16+4} = \sqrt{20} \quad (\sqrt{20})^2 + (\sqrt{20})^2 = (\sqrt{40})^2$$

$$(-1, 3)(5, 1) d_3 = \sqrt{(1-3)^2 + (5-(-1))^2} = \sqrt{(-2)^2 + 6^2} = \sqrt{4+36} = \sqrt{40} \quad 20+20=40$$

so we have a right triangle

b. (2 pts) What is the mid point between the points $(3, 5)$ and $(5, 1)$?

using the mid point formula $(x_m, y_m) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2} \right)$

$$\Rightarrow (x_m, y_m) = \left(\frac{3+5}{2}, \frac{5+1}{2} \right) = \left(\frac{8}{2}, \frac{6}{2} \right) = \boxed{(4, 3)}$$

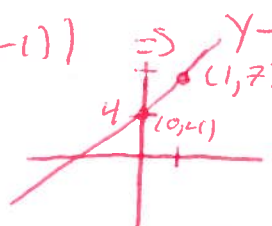
2. (10 points) Find the equation of the line through the points $(-1, 1)$ and $(1, 7)$ and sketch it.

we can use point slope form $y - y_1 = m(x - x_1)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-1}{1-(-1)} = \frac{6}{2} = 3 \quad \text{choose } (x_1, y_1) = (-1, 1)$$

$$\text{then } y - 1 = 3(x - (-1)) \Rightarrow y - 1 = 3x + 3$$

$$\Rightarrow \boxed{y = 3x + 4}$$



I'll give more room for the sketch. sorry...

3. (5 points) Find the equation of the line perpendicular to the line $y = \frac{2}{3}x + 1$ going through the point $(0, 2)$

$m_{\perp} = -\frac{1}{m_1}$ so the slope of the line we are looking

for is $-\frac{1}{2/3} = -\frac{3}{2}$ we want it to go through

$$(0, 2) \quad y - y_1 = m(x - x_1) \Rightarrow y - 2 = -\frac{3}{2}(x - 0)$$

$$\Rightarrow y - 2 = -\frac{3}{2}x \Rightarrow$$

$$\boxed{y = -\frac{3}{2}x + 2}$$

4. (10 points) Find the domain of the function $f(x) = \frac{x^2}{\sqrt{x-2}}$

① make sure denominator is not zero

$$\sqrt{x-2} = 0 \Rightarrow x-2=0 \Rightarrow x=2 \text{ so } x \neq 2$$

② make sure $x-2$ is not negative

$$x-2 \geq 0 \Rightarrow x \geq 2$$

Putting ① and ② together
says $x > 2$ or $(2, \infty)$

5. (10 points) Classify the following functions as odd, even or neither. Justify your answer

a. (5 pts) $f(x) = x^2 + x^4$

$$f(-x) = (-x)^2 + (-x)^4 = (-1)^2 x^2 + (-1)^4 x^4 = x^2 + x^4 = f(x)$$

Since $f(-x) = f(x)$ this function is even.

b. (5 pts) $f(x) = x^3 - 1$

$$f(-x) = (-x)^3 - 1 = (-1)^3 x^3 - 1 = -x^3 - 1$$

$$-x^3 - 1 \neq f(x) \text{ and } -1x^3 - 1 \neq -f(x)$$

Since $-f(x) = -(x^3 - 1) = -x^3 + 1$ so this function is neither.

6. (5 points) Let $f(x) = x^2 + x + 1$ and $g(x) = x + 3$ find $f(g(x))$.

$$f(g(x)) = (g(x))^2 + g(x) + 1$$

replace every x in $f(x)$ with $g(x)$

$$= (x+3)^2 + x+3 + 1$$

$g(x) = x+3$ so we can plug that in

$$\begin{array}{c} \text{Foil} \\ = x^2 + 6x + 9 + x + 3 + 1 \end{array}$$

$$= \boxed{x^2 + 7x + 13}$$

7. (15 points)

a. (5 pts) For the parent function $f(x) = |x|$ find the equation that results from a reflection over the x-axis a shift to the right 3 and shift down 2.

Reflection do $-f(x) = -|x|$

Shift right 3 $-f(x+3) = -|x+3|$

Shift down 2 $-f(x+3) - 2 = -|x+3| - 2$

so our equation is

$$h(x) = -|x+3| - 2$$

b. (10 pts) Sketch the graph of $f(x) = -2(x-2)^2 + 8$ Include the x and y intercepts as points on your graph. Label the vertex. What is the maximum value of $f(x)$

x intercepts set $y=0$ solve y intercepts plug in 0 for x

$$0 = -2(x-2)^2 + 8$$

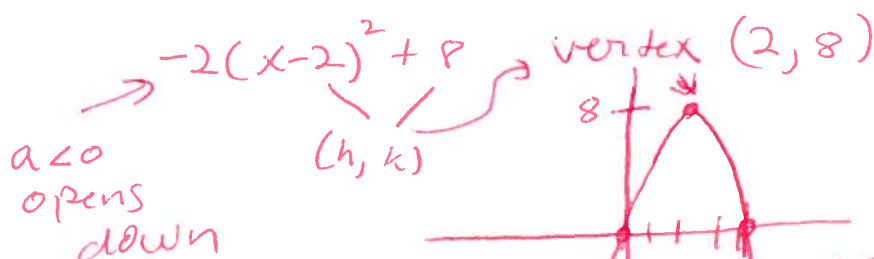
$$-8 = -2(x-2)^2$$

$$\sqrt{4} = \sqrt{(x-2)^2}$$

$$\pm 2 = x - 2$$

$$x = 2 \pm 2 \Rightarrow x = 4, 0$$

$$f(0) = -2(0-2)^2 + 8 = -8 + 8 = 0$$



See end pages for more...

8. (10 points) Write the quadratic equation $f(x) = x^2 - 6x + 8$ in standard form.

By completing the square:

$$x^2 - 6x + 8 \quad b = -6$$

$$\frac{b}{2} = -3$$

$$x^2 - 6x + 9 - 9 + 8 \quad \left(\frac{b}{2}\right)^2 = (-3)^2 = 9$$

$$f(x) = (x-3)^2 - 1$$

Using the fact that the

vertex is $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

$$a = 1 \quad b = -6 \Rightarrow \frac{-b}{2a} = \frac{-(-6)}{2(1)} = 3$$

$$f(3) = 3^2 - 6(3) + 8 = 9 - 18 + 8 = -1$$

$$\text{vertex } (3, -1) \Rightarrow h = 3 \quad k = -1$$

and $a = 1$ so

$$f(x) = a(x-h)^2 + k = (x-3)^2 - 1$$

Use rational root theorem



9. (20 points) For the polynomial $f(x) = 2x^4 + 5x^3 - 5x^2 - 5x + 3$

a. (5 pts) List all of the possible rational zeros of $f(x)$

$$a_n = 2 \mid \pm 2, \pm 1 \leftarrow q \text{ values}$$

$$a_0 = 3 \mid \pm 3, \pm 1 \leftarrow p \text{ values}$$

possible roots are of the form $\frac{p}{q}$

$$\text{Possible roots} \mid \pm \frac{3}{2}, \pm \frac{1}{2}, \pm \frac{3}{1}, \pm \frac{1}{1}$$

b. (5 pts) How many possible numbers of positive roots are there?

Descartes rule of signs!

$$f(x) = 2x^4 + \underbrace{5x^3}_{1} - \underbrace{5x^2}_{2} - 5x + 3$$

2 sign changes so either 2 positive roots
or $2 - 2 = 0$ positive roots 2 or 0

c. (5 pts) How many possible numbers negative roots are there?

check sign changes for $f(-x)$

$$\begin{aligned} f(-x) &= 2(-x)^4 + 5(-x)^3 - 5(-x)^2 - 5(-x) + 3 \\ &= \underbrace{2x^4}_{1} - \underbrace{5x^3}_{2} - 5x^2 + 5x + 3 \end{aligned}$$

2 sign changes so either 2 or $2 - 2 = 0$ negative roots 2 or 0

d. (5 pts) Find the zeros of $f(x)$ and write it as a product of its linear factors.

Try roots from the list

$$f(1) = 2(1)^4 + 5(1)^3 - 5(1)^2 - 5(1) + 3 = 0 \quad \checkmark \quad \begin{array}{l} 1 \text{ is a root} \\ \text{so } (x-1) \text{ is a factor} \end{array}$$

$$f(-1) = 2(-1)^4 + 5(-1)^3 - 5(-1)^2 - 5(-1) + 3 = 0 \quad \begin{array}{l} -1 \text{ is a root} \\ \text{so } (x - (-1)) = x + 1 \\ \text{is a factor} \end{array}$$

$$f(-3) = 2(-3)^4 + 5(-3)^3 - 5(-3)^2 - 5(-3) + 3 = 0$$

so $(x - (-3)) = (x + 3)$ is a factor

$$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^4 + 5\left(\frac{1}{2}\right)^3 - 5\left(\frac{1}{2}\right)^2 - 5\left(\frac{1}{2}\right) + 3 = 0 \quad \begin{array}{l} \text{so } (x - \frac{1}{2}) \text{ is} \\ \text{a factor} \end{array}$$

$$\text{so } \boxed{f(x) = (x-1)(x+1)(x+3)(x-\frac{1}{2})}$$

← don't worry #'s on the exam will be easier.

11. (10 points)

a. (5 pts) Given $f(x) = \sqrt{x+2} - 1$ find $f^{-1}(x)$

① Replace $f(x)$ with Y $\rightarrow X = \sqrt{Y+2} - 1$ $\rightarrow f^{-1}(x) = (x+1)^2 - 2$
 $Y = \sqrt{x+2} - 1$
 $X+1 = \sqrt{Y+2}$
 ② Swap Y and x
 $x = \sqrt{Y+2} - 1$
 $(x+1)^2 = (\sqrt{Y+2})^2$
 $(x+1)^2 = Y+2$
 $(x+1)^2 - 2 = Y = x$
 ③ Solve for y

b. (5 pts) If $f(x) = 3x + 1$ and $f^{-1}(x) = \frac{x-1}{3}$ verify that that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$

$$f(f^{-1}(x)) = 3\left(\frac{x-1}{3}\right) + 1 = x - 1 + 1 = x \quad \checkmark$$

$$f^{-1}(f(x)) = \frac{f(x)-1}{3} = \frac{3x+1-1}{3} = \frac{3x}{3} = x \quad \checkmark$$

12. (5 points) (Bonus all or nothing) The equation for the height of a falling object is given by $h(t) = -\frac{1}{2}gt^2 + v_0t + h_0$ where t is the time in seconds v_0 is the initial velocity of the object and h_0 the initial height. Suppose you drop your math book off of the library which has a height of 50m how long before it hits the ground? Take the acceleration of gravity $g = 10\text{m/s}^2$.

$h_0 = 50\text{ m}$ $v_0 = 0 \leftarrow$ Since at the moment it was dropped it was not moving

So $h(t) = -\frac{1}{2}(10)t^2 + 50 = -5t^2 + 50$

When it hits the ground $h(t) = 0$ so

$$0 = -5t^2 + 50 \Rightarrow 5t^2 = 50 \Rightarrow t^2 = \frac{50}{5}$$

$$\Rightarrow t^2 = 10 \Rightarrow t = \sqrt{10} \text{ seconds.}$$

10. (15 points) For the function $f(x) = \frac{2x^2+6x+4}{x^2+x-6}$

a. (5 pts) Find all of the vertical and horizontal asymptotes

Step 1 Factor everything if you can $f(x) = \frac{2x^2+6x+4}{x^2+x-6} = \frac{2(x^2+3x+2)}{x^2+x-6}$
 $= \frac{2(x+1)(x+2)}{(x-2)(x+3)}$ Recall the V.A.'s are where the bottom is

Zero $(x-2)(x+3) = 0 \Rightarrow \boxed{\text{V.A.'s } x=2 \text{ } x=-3}$

H.A.'s have degree of the top is the same as the bottom so we have a horizontal asymptote at

$\boxed{y = \frac{2}{1} = 2}$ the ratio of the leading coefficients.

b. (5 pts) Find the x and y intercepts

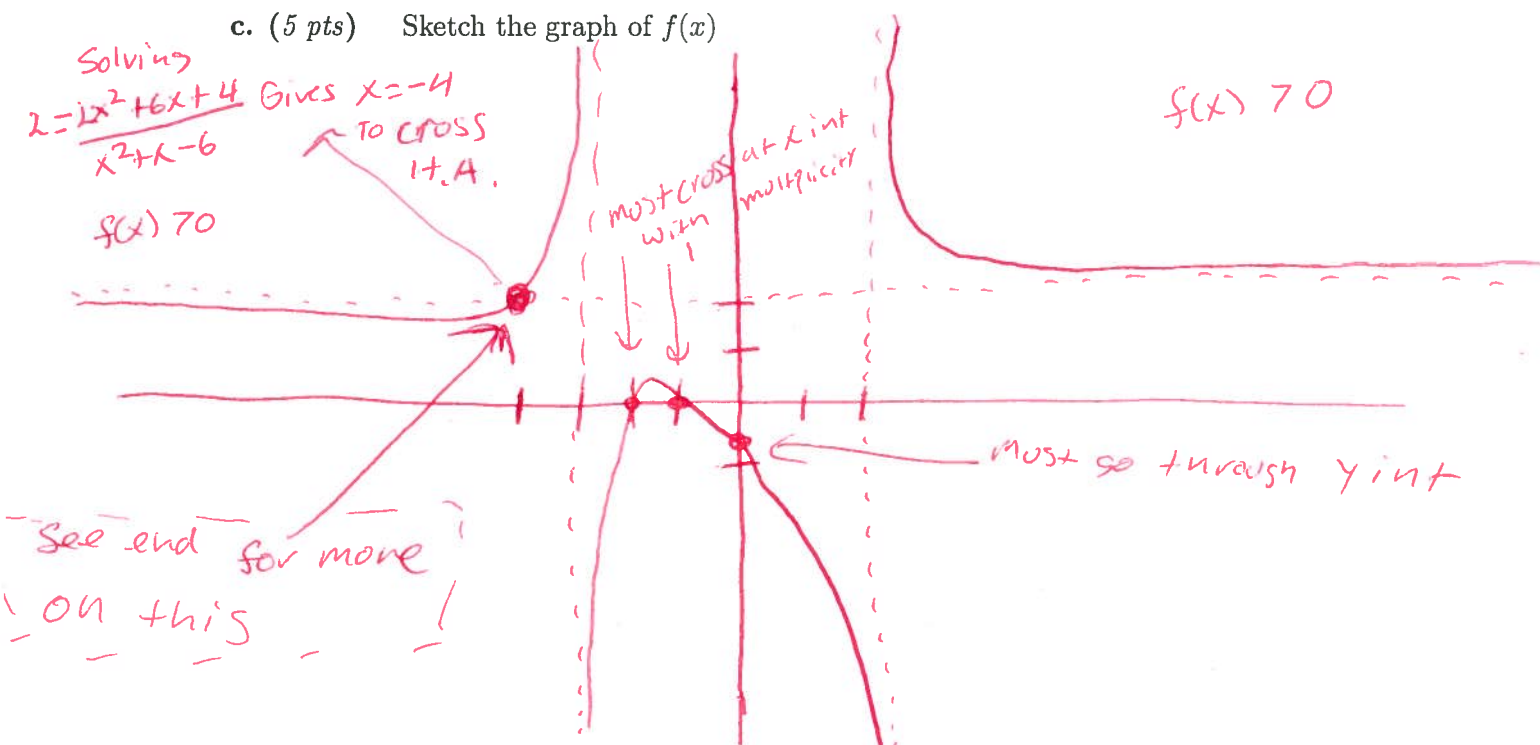
x intercepts are where the top is zero

$2(x+1)(x+2) = 0 \Rightarrow x+1=0 \text{ or } x+2=0$
 $\Rightarrow \boxed{x=-1 \text{ or } x=-2}$ xint

y intercepts plug in $x=0$

$f(0) = \frac{2(0)^2+6(0)+4}{0^2+0-6} = \frac{4}{-6} = \boxed{-\frac{2}{3}}$

c. (5 pts) Sketch the graph of $f(x)$



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#2

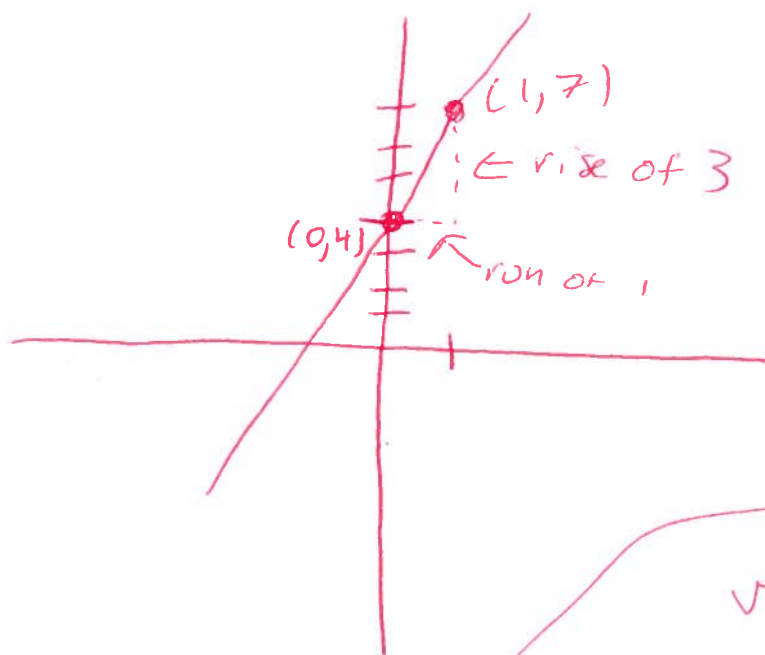
For Problem 2 we got the equation

$$y = 3x + 4$$

4 the y-intercept

3 is the slope and

$$\frac{3}{1} = \frac{\text{rise}}{\text{run}}$$



7 b)

$$f(x) = -2(x-2)^2 + 8$$

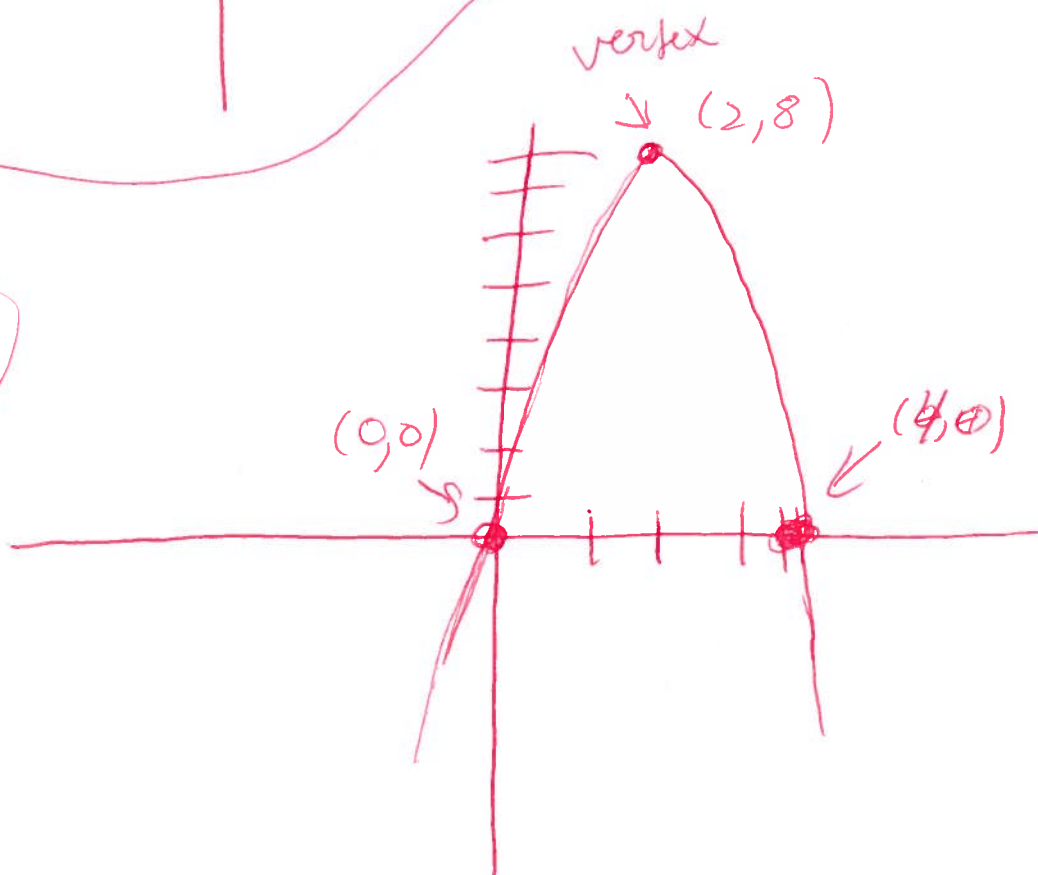
$$a(x-h)^2 + k$$

$$h=2 \quad k=8$$

vertex

$$a = -2 < 0 \text{ so}$$

it opens down



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(please do not remove this page from the test packet)

10 c)

We want to know if the Graph crosses the ~~line~~
the horizontal asymptote $y=2$ so we set
 $y=2$ and solve

$$2 = \frac{2x^2 + 6x + 4}{x^2 + x - 6}$$

$$2(x^2 + x - 6) = 2x^2 + 6x + 4$$

multiply
both sides
by $x^2 + x - 6$

$$\begin{array}{rcl} 2x^2 + 2x - 12 & = & 2x^2 + 6x + 4 \\ -2x^2 & & -2x^2 \end{array}$$

$$\begin{array}{rcl} 2x - 12 & = & 6x + 4 \\ +12 & & +12 \end{array}$$

$$\begin{array}{rcl} 2x & = & 6x + 16 \\ -6x & -6x & \\ -4x & = & 16 \\ \underline{-4} & & \underline{-4} \end{array}$$

$$\boxed{x = -4}$$

So the Graph has
y value 2 at $x = -4$
So it will cross the
Horizontal asymptote
there.