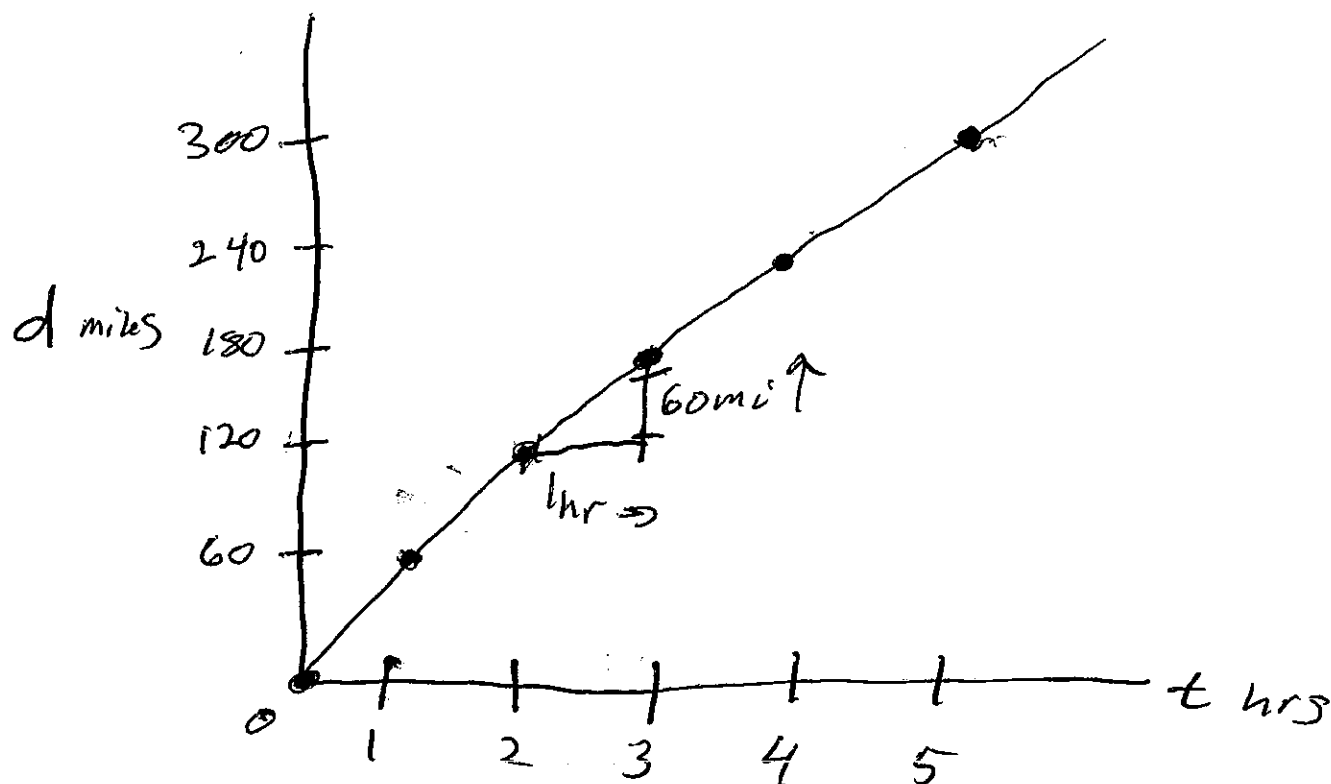


Linear equations

This section is all about lines. In math a line means something that has a constant rate of change. To get an idea of this lets think about driving to moab. Suppose you drive to moab at 60 mph along the way you check the distance you have traveled every hour and your passenger writes it down. You might get data like this:

t	distance
0 hrs	0 miles
1 hr	60 mi
2 hrs	120 mi
3 hrs	180 mi
4 hrs	240 mi
5 hrs	300 mi

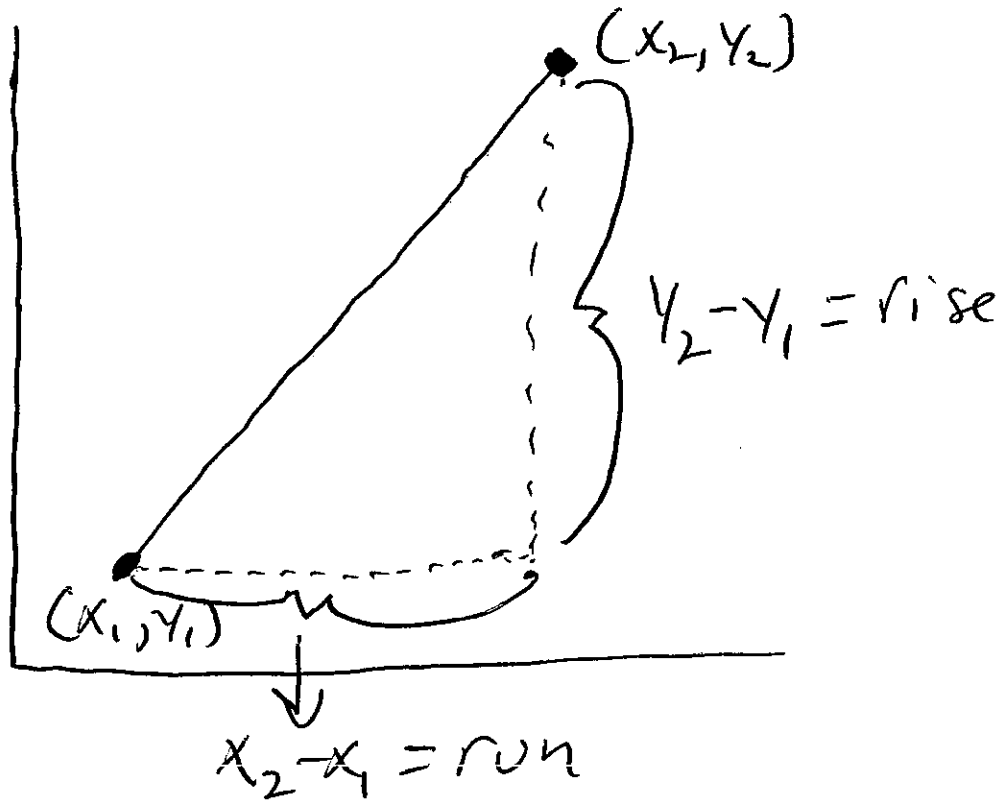
We notice that every hour we go 60 more miles that is the rate of change is 60 miles for every hour lets plot the data.



we notice that we get a line the steepness of the line we call the slope which is the rate of change in t 's case $\frac{60 \text{ miles}}{1 \text{ hr}} = \frac{60}{1} = 60$ that is on the plot when we move forward 1 hr to stay on the line we move up 60 the slope can be thought of as the rise over the run

$$\frac{\text{rise} = 60}{\text{run} = 1} \quad \text{slope} = \frac{60}{1} = 60.$$

Slope of a line between two points



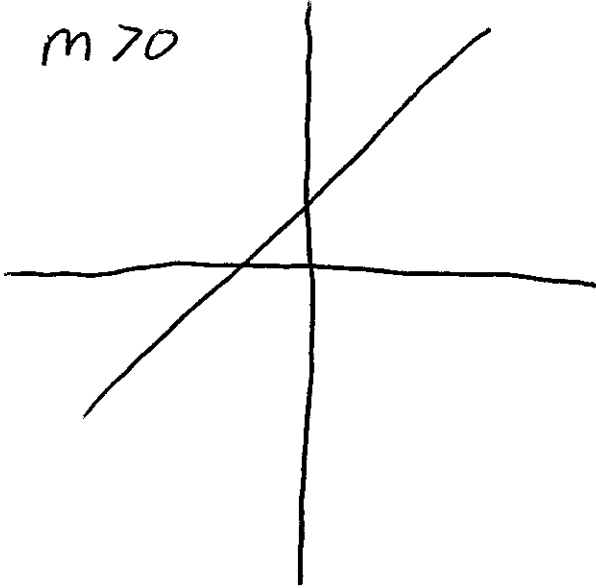
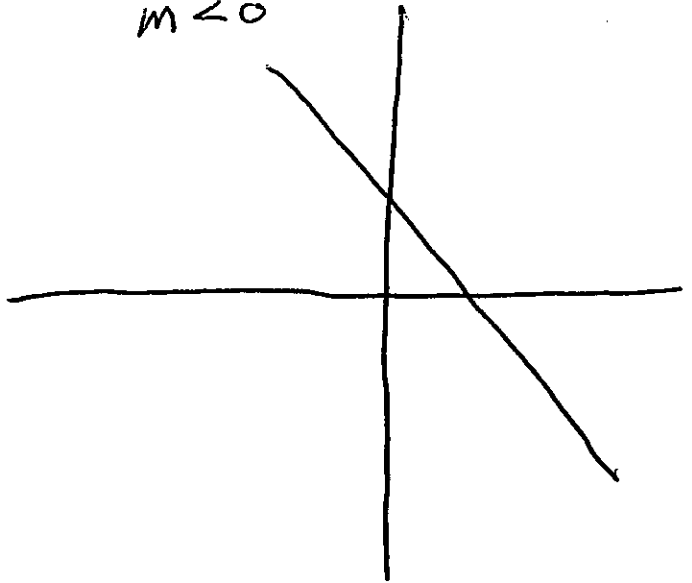
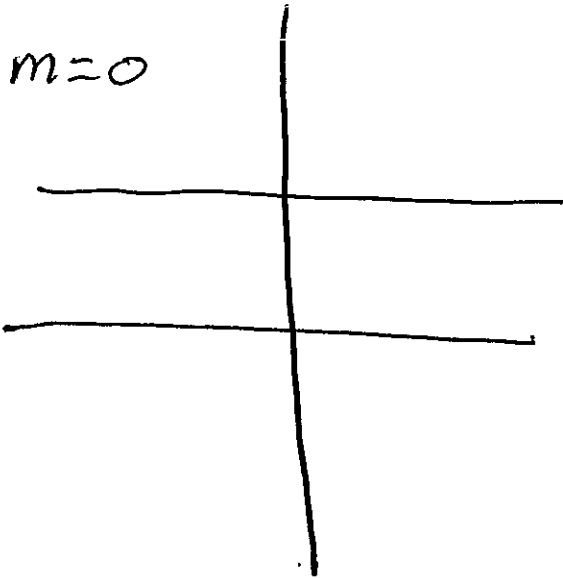
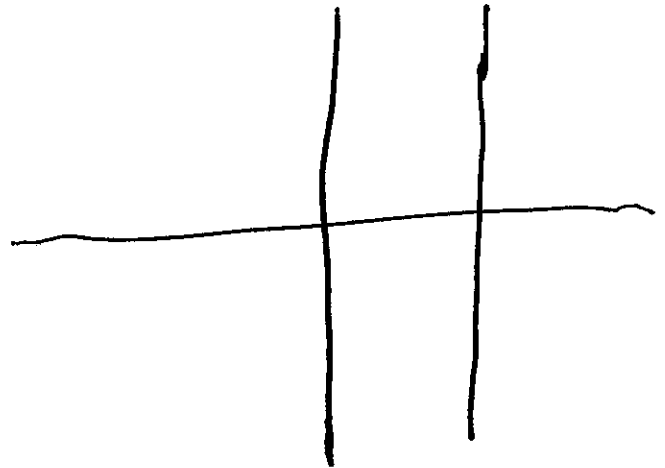
$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = m$$

HW example like #18 HW 1

Find the slope of the line through $(0, -2)$ and $(3, 10)$

$(0, -2)$	$(3, 10)$
$x_1 \ y_1$	$x_2 \ y_2$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10 - (-2)}{3 - 0} = \frac{10 + 2}{3} = \frac{12}{3} = \boxed{4}$$

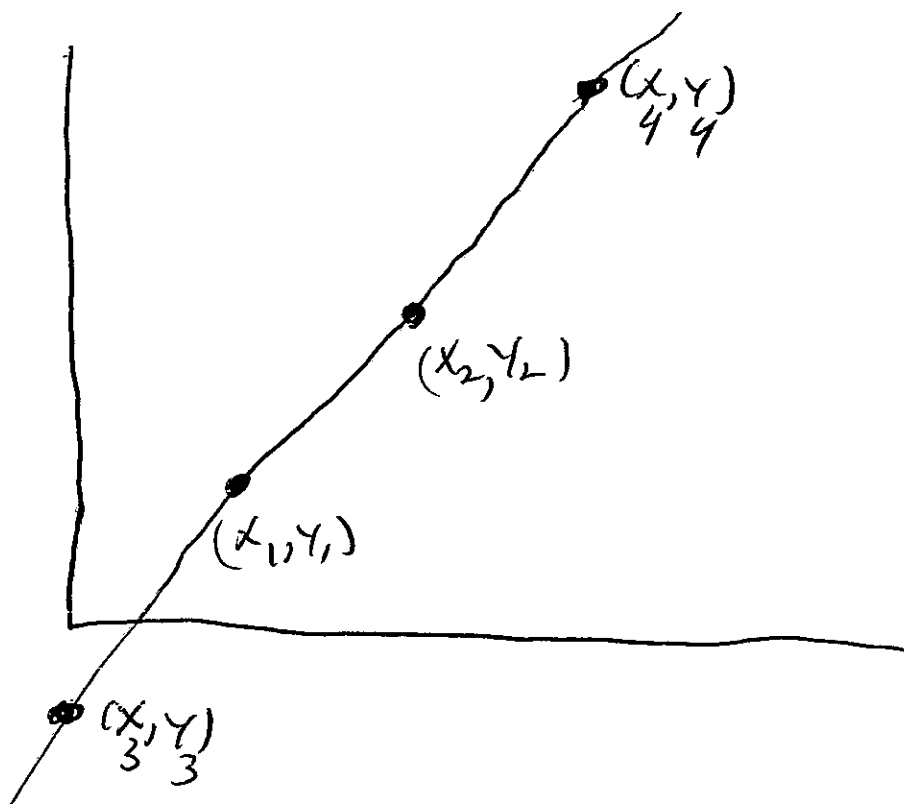
Types of slope $m > 0$  $m < 0$  $m = 0$  $m = \text{undefined}$ 

Point Slope form of a line.

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Consider the equation for the slope of a line

$$\frac{y_2 - y_1}{x_2 - x_1} = m$$



Suppose we choose the point (x_1, y_1) on the line and then calculate the slope for a few points on the line since they are all on the same line it must be that

$$\frac{y_4 - y_1}{x_4 - x_1} = \frac{y_3 - y_1}{x_3 - x_1} = \frac{y_2 - y_1}{x_2 - x_1} = m$$

That is to say if we choose any other point on the line to find slope we get the same answer.

This means for any (x, y) on the line (36)
 we can say

$$\frac{y - y_1}{x - x_1} = m \quad \left(\begin{array}{l} \text{Here } x, y \text{ are variables} \\ \text{and } (x_1, y_1) \text{ a point on} \\ \text{the line} \end{array} \right)$$

If we multiply both sides of the equation
 by $(x - x_1)$ we get the equation

$$\cancel{(x - x_1)} \frac{y - y_1}{\cancel{(x - x_1)}} = m(x - x_1) \Rightarrow y - y_1 = m(x - x_1)$$

This is the point/slope form of a
 line.

Now variables.

$$y - y_1 = m(x - x_1)$$

slope

Point on the line

Ex Find the equation of the line
 that goes through $(1, 2)$ with
 slope $m = 2$

soln

$$m = 2$$

$$\begin{array}{cc} (1, 2) \\ \downarrow \quad \downarrow \\ (x_1, y_1) \end{array}$$

Plugging this in to $y - y_1 = m(x - x_1)$ gives

(37)

$$\boxed{y - 2 = 2(x - 1)}$$

Slope intercept form of a line.

Let's solve

$$\begin{array}{ccc} y - y_1 & = m(x - x_1) & \text{for } y \\ + y_1 & & + y_1 \end{array}$$

$$y = m(x - x_1) + y_1$$

$$y = mx - \underbrace{mx_1 + y_1}_{\text{Note that } m, x_1, y_1 \text{ are numbers so they are like terms}}$$

Note that m, x_1, y_1 are numbers

so they are like terms

Let's call $-mx_1 + y_1 = b$

Then we have

$$\boxed{y = mx + b} \leftarrow \text{Slope intercept form.}$$

Slope

y coordinate of y intercept

To see why b is the y intercept

Let's set $x=0$ to find it.

Setting $x=0$ gives

$$y = m(0) + b = b$$

$\Rightarrow (0, b)$ is the y -intercept

HW example like # 20 HW 1

Find the equation of going through $(-6, -3)$ and has slope -4 in slope intercept form

Soln

we can use Point Slope form to find the equation

$$y - y_1 = m(x - x_1)$$


$(-3, -6)$

$$y - (-3) = -4(x - (-6))$$

$$\Rightarrow y + 3 = -4(x + 6)$$

$$\Rightarrow y + 3 = -4x - 24$$

$-3 \qquad -3$

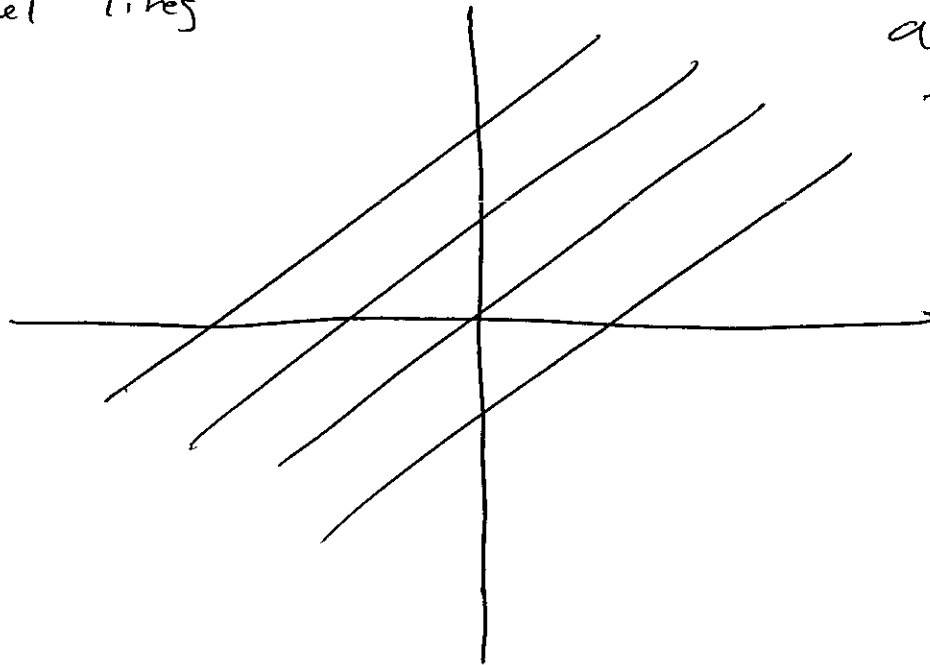
$$\Rightarrow \boxed{y = -4x - 27}$$

Parallel and perpendicular lines

★ A line parallel to a line with slope m_1 will have slope $m_2 = m_1$

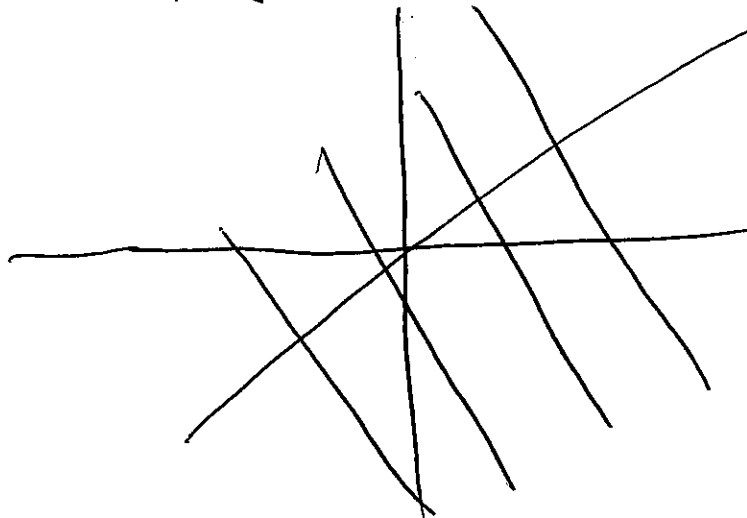
★ A line perpendicular to a line with slope m_1 will have slope $m_2 = -\frac{1}{m_1}$

parallel lines



all have
the same
slope

Perpendicular lines



Negative
reciprocal
slope.

Ex

For the line with slope $\frac{1}{2}$ and y intercept $(0,1)$ find the parallel line that goes through $(0,0)$ and the perpendicular line that goes through $(-1,1)$

Soln

The equation of the given line is

$$y = \frac{1}{2}x + 1$$

$\uparrow$
 m_1

a parallel line must have the same slope

$m_2 = m_1 = \frac{1}{2}$ we want it to go through

$(0,0)$ Using point-slope form

$$y - y_1 = m(x - x_1)$$

$\overset{0}{y} - \overset{0}{y_1} = \overset{\frac{1}{2}}{m}(x - \overset{0}{x_1})$

$$y - 0 = \frac{1}{2}(x - 0)$$

$$\Rightarrow \boxed{y = \frac{1}{2}x} \leftarrow \text{our parallel line.}$$

The perpendicular line will have slope

$$m_2 = -\frac{1}{m_1} = -\frac{1}{\frac{1}{2}} = -\frac{1}{\frac{1}{2}} = -\frac{1}{1} \cdot \frac{2}{1} = -2$$

we want it to go through the point

(4)

~~Q282~~ $(-1, 1)$ using point slope form

$$y - y_1 = m(x - x_1)$$

$\downarrow$
 -2
 $\swarrow$
 $(-1, 1)$

$$y - (1) = -2(x - 1)$$

$$y + 1 = -2(x - 1)$$

$$y + 1 = -2x + 2$$

$-1 \qquad -1$

$$y = -2x + 1$$

