

Section 2.2

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Polynomial functions of Higher Degree.

What if we wanted to understand the behavior of the graph of something like,

$$f(x) = x^4 - 5x^2 + 4$$

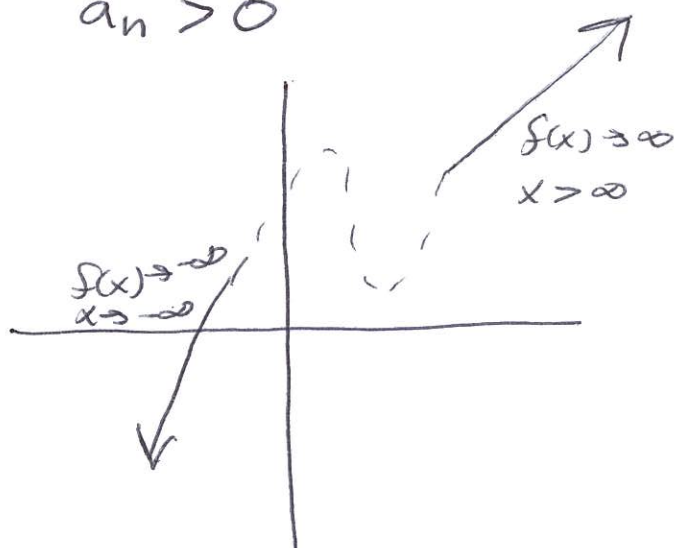
It would be nice to have some rules we could employ to tell us about the graph. Luckily there are!

~~Some~~ Leading coefficient test

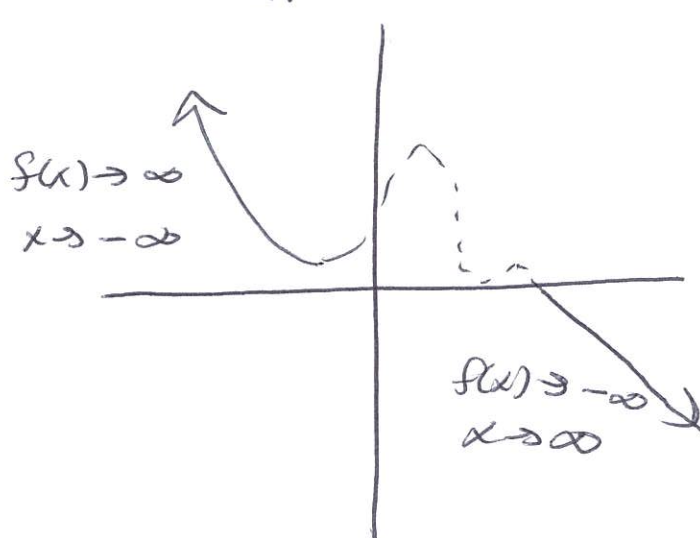
for a polynomial: $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$

If n is odd

$a_n > 0$



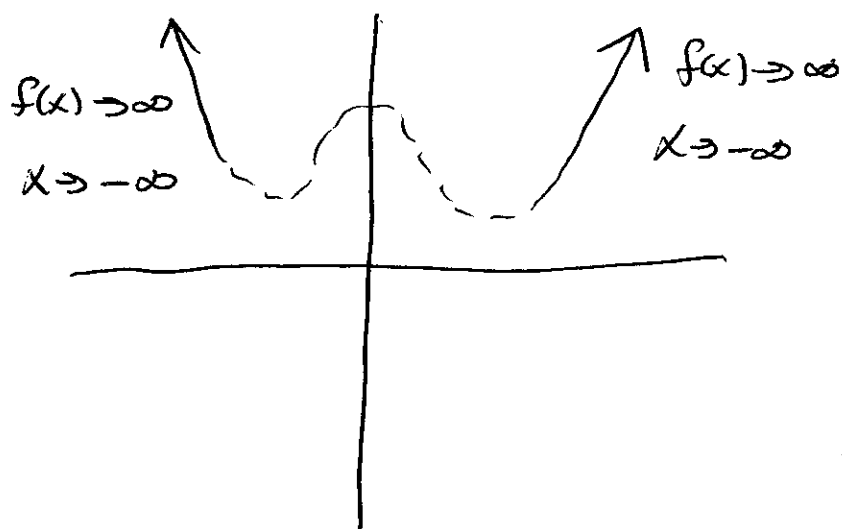
$a_n < 0$



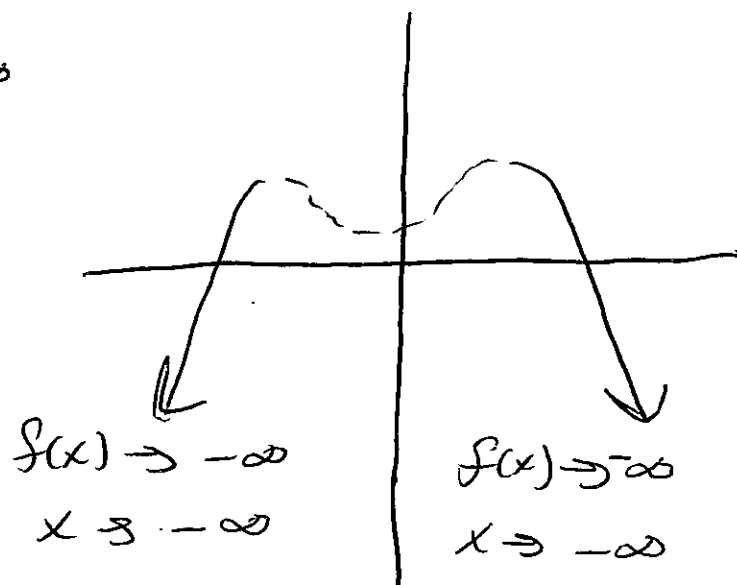
If n is even

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$$a_n > 0$$



$$a_n < 0$$



To understand this let's consider
the polynomials

$$4x^3 + 2x^2 + 1 \quad \text{and} \quad -4x^3 + 2x^2 + 1$$

Suppose we take x really big like 100,000
Plugging these in gives us

$$4(100,000)^3 + 2(100,000)^2 + 1$$

↓

This number is the
largest and eclipses the others

$$-4(100,000)^3 + 2(100,000)^2 + 1$$

↓

This number
is the largest and negative
The other positives don't
compare,

so for

$$4x^3 - 2x^2 + 1$$

and

$$-4x^3 + 2x^2 + 1$$

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When x is very large the only term that matters is the term with the highest power of x . The coefficient in front determines whether or not the graph falls or rises.

HW Example like # 17 HW 2

$$\text{Given } P(x) = 6x^{10} - 2x^7 + 6x + 10$$

$$P(x) \rightarrow \text{---} \quad \text{If } x \rightarrow -\infty$$

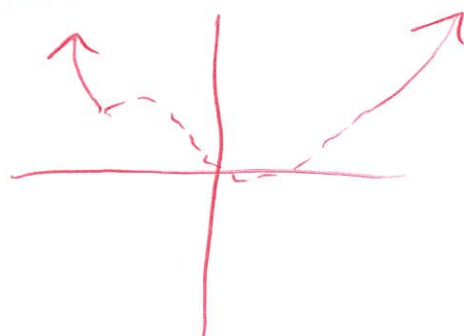
$$P(x) \rightarrow \text{---} \quad \text{If } x \rightarrow +\infty$$

Soln

The leading coefficient is 6 and the degree of $P(x)$ is 10 which is even and $6 > 0$ so by the leading coefficient test

$$P(x) \xrightarrow{\text{infinity}} +\infty \quad \text{as } x \rightarrow \infty$$

$$P(x) \xrightarrow{\text{infinity}} +\infty \quad \text{as } x \rightarrow -\infty$$



HW Example like # 18 HW 2

for the function $y = x^3 + x^2 - 42x$

its y intercept is —

its x intercepts are — — —

when $x \rightarrow \infty$ $y \rightarrow$ —

when $x \rightarrow -\infty$ $y \rightarrow$ —

Soln

To find the y intercepts set $x = 0$

$$y = 0^3 + 0^2 - 42(0) = \boxed{0}$$

for the x intercepts set $y = 0$ solve for x

$$0 = x^3 + x^2 - 42x$$

$$0 = x(x^2 + x - 42)$$

$$0 = x(x+7)(x-6)$$

$$0 = x \Rightarrow x = 0$$

$$0 = x+7 \Rightarrow x = -7$$

$$0 = x-6 \Rightarrow x = 6$$

so the x intercepts are $\boxed{0, 6, -7}$

Now we apply the leading coefficient test

$$y = \underset{\uparrow}{1}x^3 + x^2 - 42x$$

$$\text{leading coefficient} = 1 > 0$$

The degree is 3 odd

so by the leading coefficient test

$$\text{as } x \rightarrow \infty \quad y \rightarrow \infty$$

$$x \rightarrow -\infty \quad y \rightarrow -\infty$$

Real Zeros of Polynomials

If $x=a$ is a solution to $f(x)=0$

- ① $x=a$ is called a zero of f
- ② $(x-a)$ is a factor of $f(x)$
- ③ $(a,0)$ is an x intercept

Ex

$$x^2 + 5x + 6 = 0 \quad \text{has solutions } x = -3 \quad x = -2$$

which means $(x - (-3))(x - (-2))$ are factors

$$\text{Note } (x - (-3))(x - (-2)) = (x+3)(x+2) = x^2 + 5x + 6, \quad \text{FOIL}$$

Repeated Zeros

suppose we factor a polynomial like

$$x^2 + 2x + 1 = (x+1)(x+1) = (x+1)^2 \text{ we would}$$

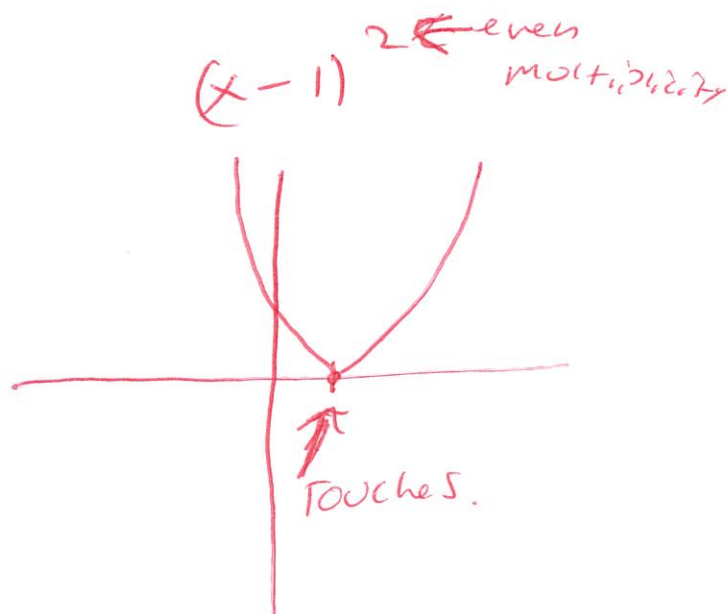
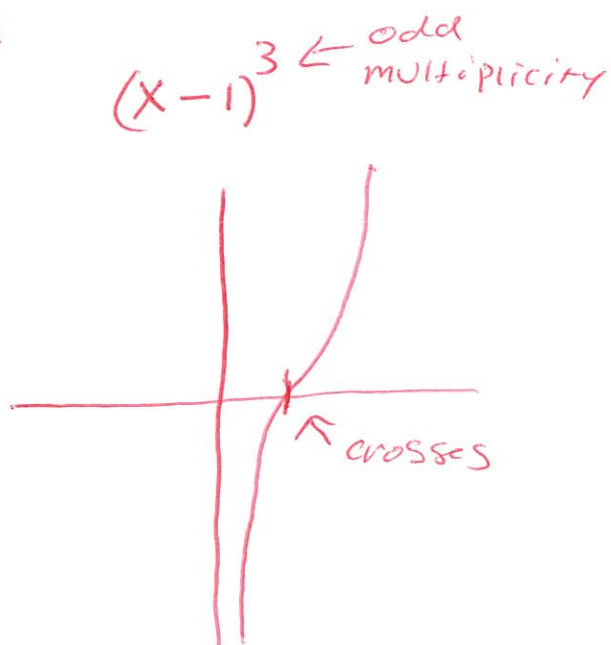
say that $x = -1$ is a repeated root with multiplicity 2.

Def

A factor $(x-a)^k$ $k > 1$ yields a repeated root $x=a$ of multiplicity k .

- ① when k is odd the Graph crosses the x -axis
- ② when k is even the Graph touches but does not cross the x -axis.

EX



Sketching the Graph of a polynomial function.

EX

Sketch the Graph of $3x^4 - 4x^3 = f(x)$

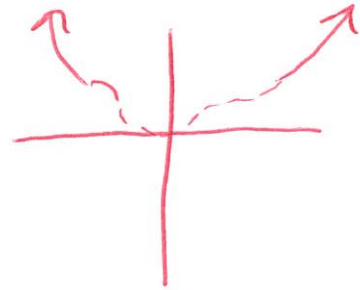
① Apply the leading coefficient test

$$L.C. = 3 > 0$$

degree = 4 even so

$$\text{as } x \rightarrow \infty \quad f(x) \rightarrow \infty$$

$$x \rightarrow -\infty \quad f(x) \rightarrow -\infty$$



② Find the zeros

$$3x^4 - 4x^3 = 0$$

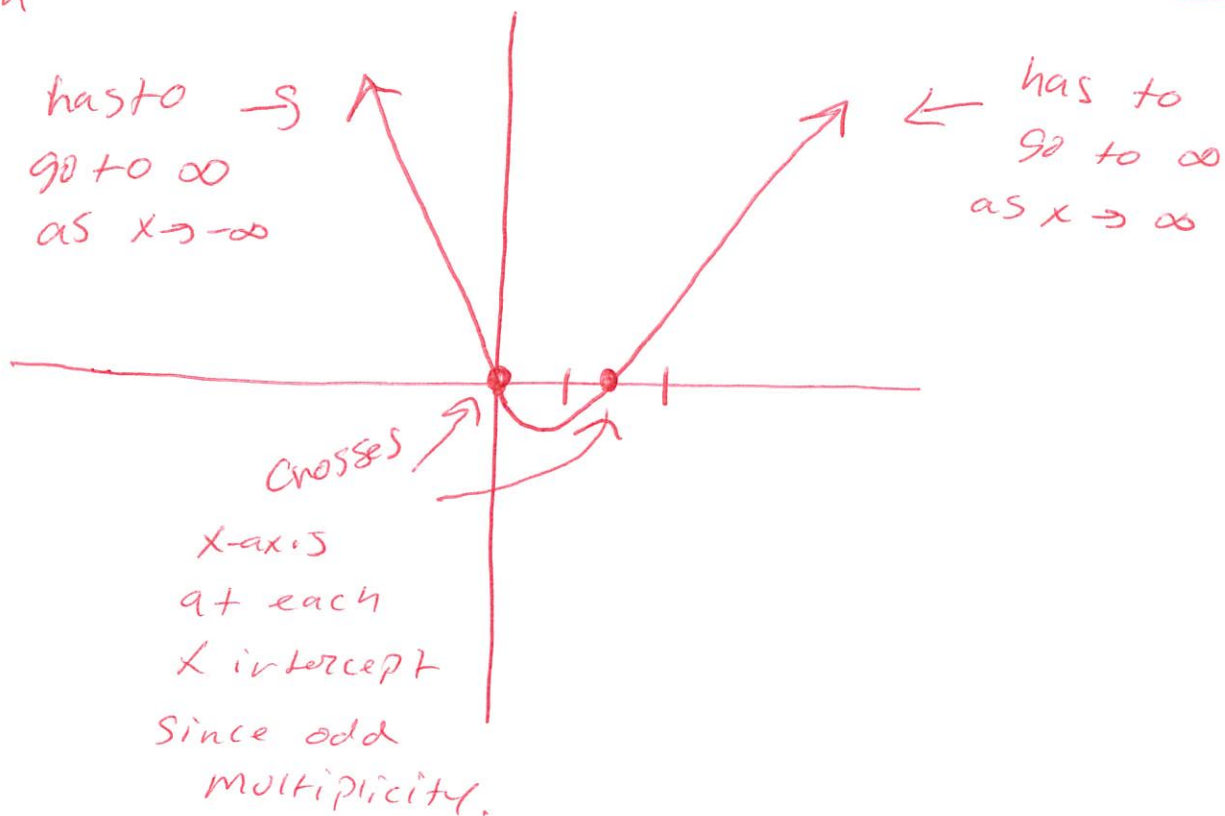
$$x^3(3x - 4) = 0$$

$$\Rightarrow x^3 = 0 \longrightarrow x^3 = (x - 0)^3 \quad \leftarrow \begin{array}{l} \text{odd multiplicity} \\ \text{crosses the } x\text{-axis} \end{array}$$

$$(3x - 4)^1 = 0 \longrightarrow (3x - 4)^1 \quad \leftarrow \begin{array}{l} \text{odd multiplicity} \\ \text{crosses the } x\text{-axis} \end{array}$$

$\Rightarrow x = 0$
 $x = \frac{4}{3}$ are x-intercepts.

Sketch

EX

Sketch $f(x) = (x-2)^2 (x+3)^3 (4x+2)^4$

① Leading coefficient test.

This time we first need to find the leading coefficient

$$\begin{array}{ccc} (x-2)^2 & (x+3)^3 & (4x+2)^4 \\ \downarrow & \downarrow & \downarrow \\ x^2 & x^3 & 4x^4 \end{array}$$

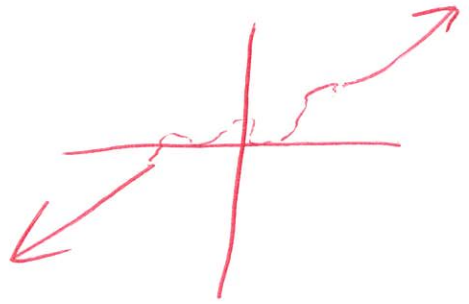
$$\text{so } x^2 x^3 4^4 x^4 = 256 x^9$$

So the leading coefficient is 25670 and the degree is 9 odd so

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$$f(x) \rightarrow \infty \text{ as } x \rightarrow \infty$$

$$f(x) \rightarrow -\infty \text{ as } x \rightarrow -\infty$$



② Find the zeros.

This one is factored for us!

$(x-2)$ ②	$(x+3)$ ③	$(4x+2)$ ④
$\Downarrow$	$\Downarrow$	$\Downarrow$
$x-2=0$	$x+3=0$	$4x+2=0$
$\Downarrow$	$\Downarrow$	$\Downarrow$
$x=2$	$x=-3$	$x=-\frac{1}{2}$
multiplicity 2 even touches	multiplicity 3 odd crosses	multiplicity 4 even touches.

Now we put it all together.

