

Practice Final

Math 1050 001 Summer 2013

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Name and Unid:

Solutions

by writing my name I swear by the honor code

Read all of the following information before starting the exam:

- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Justify your answers algebraically whenever possible to ensure full credit. Sketch all relevant graphs and explain all relevant mathematics.
- Circle or otherwise indicate your final answers.
- You MAY NOT use a calculator or notes. You may use the formulas provided.
- This test has 10 problems and one bonus problem and is worth 100 points, plus some extra credit at the end. It is your responsibility to make sure that you have all of the pages!
- Good luck!

Good Luck!

Here are some Formulas and rules for you.

Distance Formula: $d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$

Midpoint Formula: $(x_m, y_m) = (\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$

Rational Root Theorem:

If the polynomial $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$ has integer coefficients, then every rational zero of f has the form $\frac{p}{q}$ where p and q have no common factors other than 1, and

p = a factor of the constant term a_0

q = a factor of the leading coefficient a_n

Descartes rule of signs

Let $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$ be a polynomial with real coefficients and $a_0 \neq 0$ then:

1. The number of positive real zeros of f is either equal to the number of variations in sign of $f(x)$ or less than that by an even integer.
2. The number of negative real zeros of f is either equal to the number of variations in sign of $f(-x)$ or less than that by an even integer

Rules for asymptotes: Let f be the rational function

$$f(x) = \frac{N(x)}{D(x)} = \frac{f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0}{f(x) = b_n x^n + b_{n-1} x^{n-1} + \dots + a_2 x^2 + b_1 x + b_0}$$

Where $N(x)$ and $D(x)$ have no common factors Then:

1. The graph of f has vertical asymptotes at the zeros of $D(x)$ that is the x values where the bottom function is zero.
2. The graph of f has 1 or 0 horizontal asymptotes determined by comparing the degrees of $N(x)$ and $D(x)$
 - When the degree of the bottom is larger than the top the graph of f has a horizontal asymptote at $y=0$
 - When the degree of the bottom is equal to the degree of the top the graph of f has a horizontal asymptote at $y = \frac{a_n}{b_n}$ the ratio of the leading coefficients of the top to the bottom.
 - When the degree of the top is larger than the degree of the bottom there are no horizontal asymptotes.
 - If the degree of the top is one larger than the degree of the bottom there is an oblique asymptote given by the equation of the quotient after polynomial long division.

Good Luck!

Here are some formulas to help you along:

Properties of Logarithms

$$\text{Log}(AB) = \text{Log}(A) + \text{Log}(B)$$

$$\text{Log}\left(\frac{A}{B}\right) = \text{Log}(A) - \text{Log}(B)$$

$$\text{Log}(A^n) = n\text{Log}(A)$$

Compound Interest:

$$A = P_o \left(1 + \frac{r}{n}\right)^{nt}$$

Where A is the amount in the account after t years, r the interest rate as a decimal, P_o the initial investment and n the number of times per year interest is compounded.

Exponential Models

$$P = P_o e^{rt}$$

where P is the population after a time t , P_o the initial population, r the growth rate and $e = 2.718281828459045235360287471352662497757\dots$

Inverse of a 2X2 matrix

If

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Then

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Arithmetic Sequence

$$n^{\text{th}} \text{ term: } a_n = a_1 + (n-1)d$$

Partial Sum:

$$S_n = \frac{n}{2}(a_1 + a_n)$$

Geometric Sequence

$$n^{\text{th}} \text{ term } a_n = a_1 r^{n-1}$$

$$S_n = a_1 \left(\frac{1-r^n}{1-r} \right)$$

$$S_{\infty} = a_1 \left(\frac{1}{1-r} \right)$$

1. (15 points)

a. (5 pts) Find the midpoint between the points (1, 4) and (5, 4)

$x_1 \ y_1 \ x_2 \ y_2$

$$(x_m, y_m) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$= \left(\frac{1+5}{2}, \frac{4+4}{2} \right) = \left(\frac{6}{2}, \frac{8}{2} \right) = \boxed{(3, 4)}$$

b. (10 pts) Find the point on the y-axis that is equidistant between the points (1, 4) and (5, 4)

A point on the y axis is like (0, b)

$$d_1 = \sqrt{(4-b)^2 + (1-0)^2}$$

$$= \sqrt{(4-b)^2 + 1}$$

$$d_2 = \sqrt{(4-b)^2 + (5-0)^2}$$

$$= \sqrt{(4-b)^2 + 25}$$

$$d_1 = d_2 \Rightarrow \sqrt{(4-b)^2 + 1} = \sqrt{(4-b)^2 + 25}$$

$$\Rightarrow (4-b)^2 + 1 = (4-b)^2 + 25$$

$$\Rightarrow 16 - 8b + \cancel{b^2} + 1 = \cancel{16} + 8b + \cancel{b^2} + 25$$

$$\Rightarrow -8b + 1 = 8b + 25$$

$$\Rightarrow -24 = 16b$$

$$\Rightarrow \frac{-24}{16} = b$$

$$\Rightarrow \boxed{b = -\frac{3}{2}}$$

$$\boxed{(0, -3/2)}$$

2. (15 points)

a. (5 pts) Find the slope of the line going through the points (-1, 2) and (4, 5)

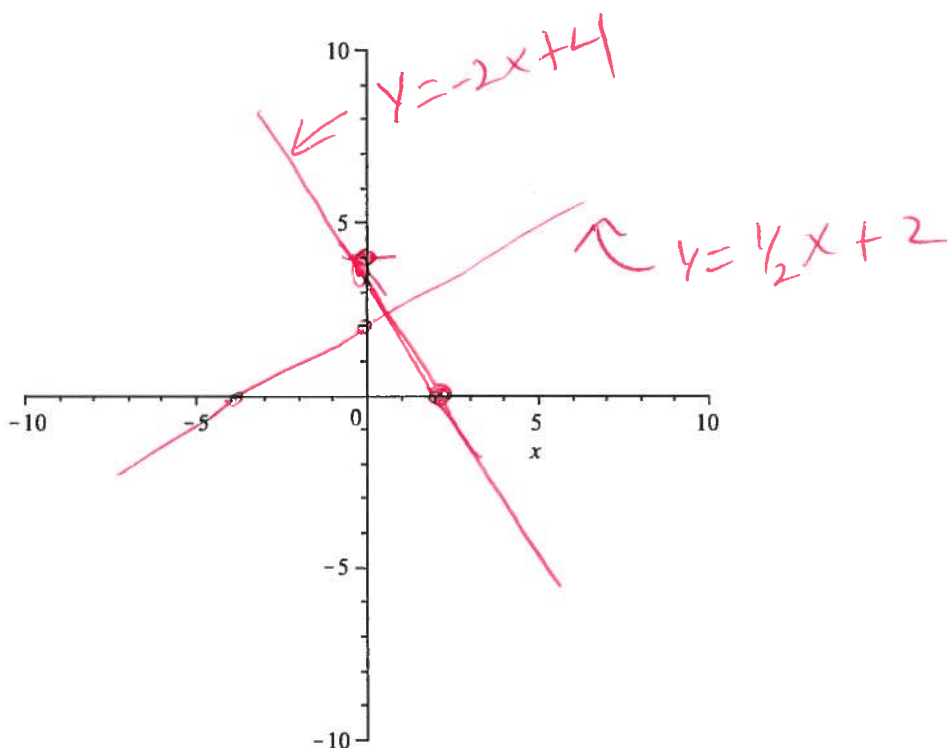
$x_1 \ y_1 \ x_2 \ y_2$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 2}{4 - (-1)} = \boxed{\frac{3}{5}}$$

b. (10 pts) Find the equation of the line perpendicular to the line $y = -2x + 4$ going through $(0, 2)$ Sketch both lines below:

$$m_{\perp} = \frac{-1}{m_{\text{given}}} = \frac{-1}{-2} = \frac{1}{2} \quad (0, 2) \text{ is } +b \text{ in } t$$

$$y = mx + b \Rightarrow y = \frac{1}{2}x + 2$$



3. (15 points)

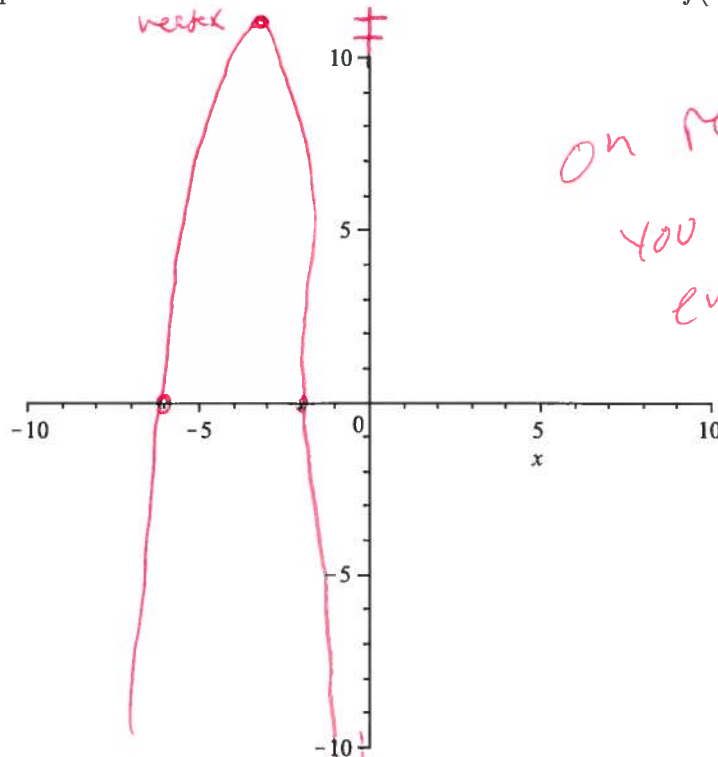
a. (5 pts) For the parent function $f(x) = |x|$ find the equation that results from a reflection over the x-axis a shift to the right 3 and down 4

reflect $-f(x) = -|x|$

shift right 3 $-f(x-3) = -|x-3|$

down 4 $-f(x-3) - 4 = \boxed{-|x-3| - 4}$

b. (10 pts) Sketch the graph of $f(x) = -3(x+4)^2 + 12$. Include the x and y intercepts as points on your graph. Label the vertex. What is the maximum value of $f(x)$?



On real final
you will have
enough points
on the axis...

x, y int

x int set $y=0$

$$0 = -3(x+4)^2 + 12$$

$$-12 = -3(x+4)^2$$

$$\frac{-12}{-3} = (x+4)^2$$

$$4 = (x+4)^2$$

$$\pm 2 = x+4$$

$$x = -4 \pm 2$$

$$x = -4 + 2 = -2$$

$$x = -4 - 2 = -6$$

y int $\neq -36$

Set $x=0$

$$y = -3(0+4)^2 + 12$$

$$= -3(16) + 12$$

$$= -48 + 12$$

$$= -36$$

vertex

$$a(x-h) + k$$

$$-3(x+4) + 12$$

$$-3(x - (-4)) + 12$$

$$h = -4 \quad k = 12$$

$$(4, 12)$$

$$\boxed{\max = k = 12}$$

4. (15 points)

a. (5 pts) Write the equation of a circle centered at $(-2, 3)$ with radius 4.

std form circ $(x-h)^2 + (y-k)^2 = r^2$

(h, k) : center

r : radius

$$(x - (-2))^2 + (y - 3)^2 = 4^2$$

$$(x + 2)^2 + (y - 3)^2 = 4$$

b. (10 pts) Write the quadratic equation $2x^2 + 4x - 8$ in standard form

complete square

$$2x^2 + 4x - 8$$

$$2(x^2 + 2x - 4) \rightarrow \frac{2}{2} = 1 \quad 1^2 = 1$$

$$2(x^2 + 2x + 1 - 1 - 4)$$

$$2((x+1)^2 - 5)$$

$$2((x+1)^2 - 5)$$

$$\boxed{2(x+1)^2 - 10}$$

5. (20 points) For the polynomial $f(x) = 2x^3 + 4x^2 - 2x - 4$
 a. (5 pts) List all of the possible rational zeros of $f(x)$

$$2x^3 + 4x^2 - 2x - 4$$

here we can factor out a 2

$$2(\overset{\downarrow \text{q's}}{x^3} + 2x^2 - x - \overset{\downarrow \text{p's}}{2})$$

apply rational root Thm.

P: 2: 2, 1 $\rightarrow \pm \frac{2}{1} \pm \frac{1}{1}$
 Q: 1: 1
 $\boxed{\pm 2, \pm 1}$

- b. (5 pts) How many possible numbers of positive roots are there?

Descartes rule of signs.

$$2x^3 + 4x^2 - 2x - 4$$

1 sign change

1 ~~possible~~ positive root

- c. (5 pts) How many possible numbers of negative roots are there?

Check $f(-x)$

$$2(-x)^3 + 4(-x)^2 - 2(-x) - 4$$

$$\underbrace{-2x^3 + 4x^2} + \underbrace{+2x - 4}$$

2 sign changes

2 or $2-2=0$ possible #'s
of negative roots

- d. (5 pts) Find the zeros of $f(x)$ and write it as a product of its linear factors.

Try roots from part a)

$$f(1) = 2(1)^3 + 4(1)^2 - 2(1) - 4 = 2 + 4 - 2 - 4 = 0$$

1 is a root ~~$(x-1)$~~ a linear factor

$$f(-1) = 2(-1)^3 + 4(-1)^2 - 2(-1) - 4$$

$$= -2 + 4 + 2 - 4 = 0$$

-1 a root $(x - (-1)) = (x+1)$ a factor

$$f(-2) = 2(-2)^3 + 4(-2)^2 - 2(-2) - 4$$

$$= -16 + 16 + 4 - 4 = 0 \quad -2 \text{ a root}$$

$(x - (-2)) = (x+2)$ a factor

$$f(x) = 2(x+1)(x-1)(x+2)$$

6. (15 points) For the function $f(x) = \frac{2x^2-8}{x^2-9}$

a. (5 pts) Find all of the vertical and horizontal asymptotes if there are any.

$$VA: x^2 - 9 = 0 \Rightarrow x^2 = 9 \Rightarrow \boxed{x = \pm 3}$$

HA: degree of top = degree of Bottom

HA is ratio of leading coeff's

$$HA: y = \frac{2}{1} = 2$$

b. (5 pts) Find the x and y intercepts if there are any.

x int

$$(x^2-9) \cdot 0 = \frac{2x^2-8}{(x^2-9)} \cdot (x^2-9)$$

$$\begin{array}{cc} 0 = 2x^2 - 8 \\ +8 \quad \quad +8 \end{array}$$

$$8 = 2x^2$$

$$4 = x^2 \rightarrow (2, 0)$$

$$\boxed{\pm 2 = x} \rightarrow (-2, 0)$$

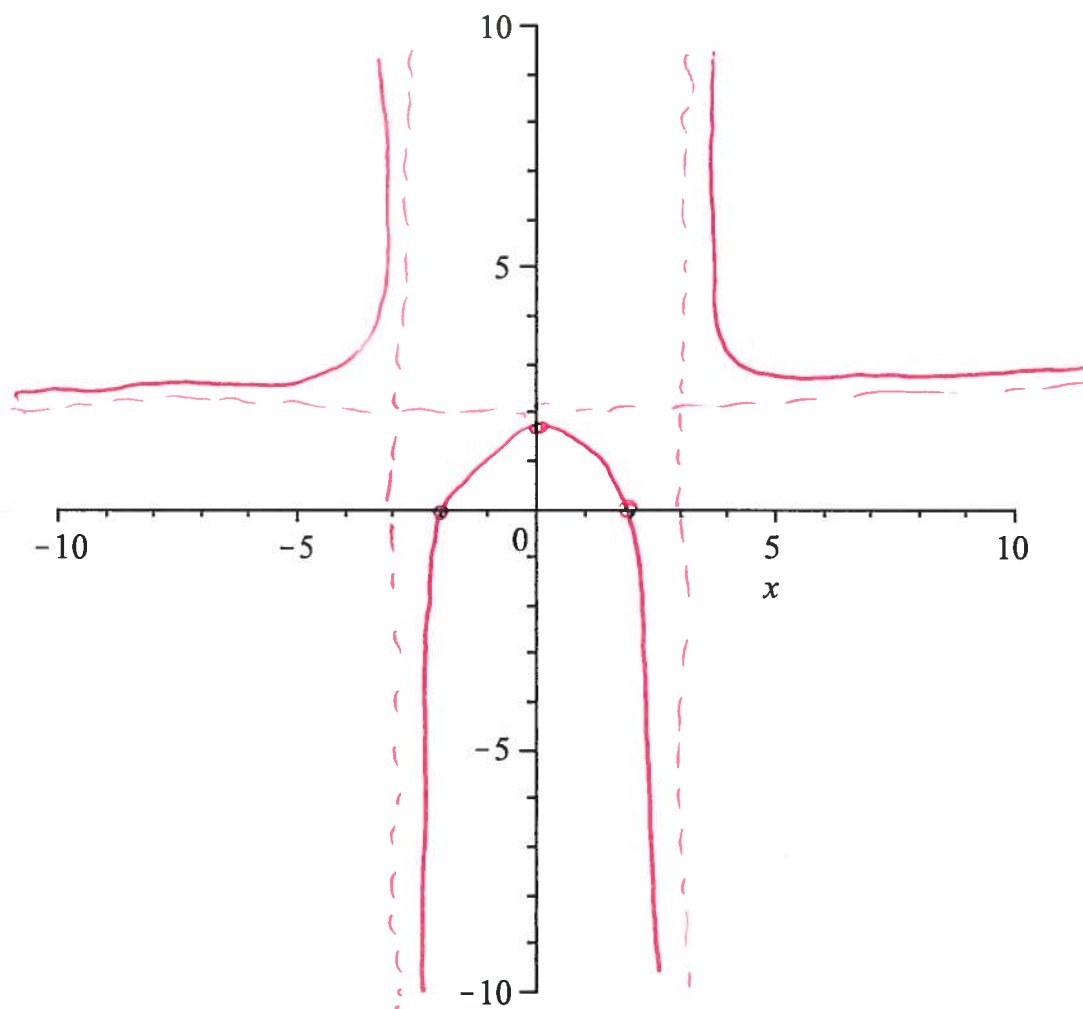
y int

$$f(0) = \frac{2(0)^2 - 8}{0^2 - 9}$$

$$= \frac{-8}{9}$$

$$= \boxed{\frac{8}{9}} \rightarrow (0, \frac{8}{9})$$

c. (5 pts) Sketch the graph of $f(x)$. The space below the axes can be used to calculate any additional solution points or find out if the function crosses any horizontal asymptotes etc... to help you sketch the graph.



Does it cross the H.A.:

$$(x^2 - 9)2 = \frac{2x^2 - 8}{x^2 - 9} \quad \text{cancel } (x^2 - 9)$$

$$2x^2 - 18 = 2x^2 - 8$$

$$-18 = -8$$

never true does not cross.

7. (15 points) Solve the following system of equations any way you like:

$$x + y + z = 2$$

$$3x + 2y + z = 4$$

$$2x + 4y - 2z = 8$$

Augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 3 & 2 & 1 & 4 \\ 2 & 4 & -2 & 8 \end{array} \right] \xrightarrow{-3R_1 + R_2} \begin{array}{ccc|c} -3 & -3 & -3 & -6 \\ 3 & 2 & 1 & 4 \\ \hline 0 & -1 & -2 & -2 \end{array}$$

$$\downarrow$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -1 & -2 & -2 \\ 2 & 4 & -2 & 8 \end{array} \right] \xrightarrow{-2R_1 + R_3} \begin{array}{ccc|c} -2 & -2 & -2 & -4 \\ + 2 & 4 & -2 & 8 \\ \hline 0 & 2 & -4 & 4 \end{array}$$

$$\downarrow$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -1 & -2 & -2 \\ 0 & 2 & -4 & 4 \end{array} \right] \xrightarrow{2R_2 + R_3} \begin{array}{ccc|c} 0 & -2 & -4 & -4 \\ + 0 & 2 & -4 & 4 \\ \hline 0 & 0 & -8 & 0 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -1 & -2 & -2 \\ 0 & 0 & -8 & 0 \end{array} \right] \text{ Now back subs.}$$

$$\begin{aligned} \Rightarrow x + y + z &= 2 \\ -y - 2z &= -2 \\ -8z &= 0 \end{aligned}$$

$$\Rightarrow -8z = 0 \Rightarrow \boxed{z = 0}$$

$$\Rightarrow -y - 2(0) = -2 \Rightarrow -y = -2 \Rightarrow \boxed{y = 2}$$

$$\Rightarrow x + 2 + 0 = 2 \Rightarrow x + 2 = 2 \Rightarrow \boxed{x = 0}$$

8. (15 points)

a. (5 pts)

Does the following system have a solution? Determine the answer by taking the determinant of the coefficient matrix.

$$2x - 3y = 7$$

$$4x - 12y = 14$$

$$\begin{vmatrix} 2 & -3 \\ 4 & -12 \end{vmatrix} = 2(-12) - 4(-3) = -24 + 12 \\ = -12 \neq 0 \quad \text{unique solution}$$

b. (10 pts) Solve the matrix equation by using the inverse of the coefficient matrix.

$$\begin{bmatrix} 2 & 3 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 3 \\ -4 & 1 \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{2(1) - (-4)(3)} \begin{bmatrix} 1 & -3 \\ 4 & 2 \end{bmatrix}$$

$$\Rightarrow = \frac{1}{14} \begin{bmatrix} 1 & -3 \\ 4 & 2 \end{bmatrix}$$

$$\Rightarrow \frac{1}{14} \begin{bmatrix} 1 & -3 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{14} \begin{bmatrix} 1 & -3 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} 5 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} 10 \\ 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{14} \begin{bmatrix} 14 \\ 14 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \begin{array}{l} x = 1 \\ y = 1 \end{array}$$

9. (15 points) Let $A = \begin{bmatrix} 2 & 4 & -2 \\ 3 & -1 & 3 \\ 0 & 1 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 4 & -1 \\ -3 & 2 & 10 \\ 5 & 1 & 5 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 2 & 5 \end{bmatrix}$

a. (5 pts) Calculate $2A-B$

$$2 \begin{bmatrix} 2 & 4 & -2 \\ 3 & -1 & 3 \\ 0 & 1 & -3 \end{bmatrix} - \begin{bmatrix} 2 & 4 & -1 \\ -3 & 2 & 10 \\ 5 & 1 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 8 & -4 \\ 6 & -2 & 6 \\ 0 & 2 & -6 \end{bmatrix} - \begin{bmatrix} 2 & 4 & -1 \\ -3 & 2 & 10 \\ 5 & 1 & 5 \end{bmatrix} = \begin{bmatrix} 4-2 & 8-4 & -4-(-1) \\ 6-(-3) & -2-2 & 6-10 \\ 0-5 & 2-1 & -6-5 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 & -3 \\ 9 & -4 & -4 \\ -5 & 1 & -11 \end{bmatrix}$$

b. (5 pts) Which is calculable CB or BC what is the size of the resulting matrix for the one that can be done?

C B
 2×3 3×3
 same
 new matrix size

B C
 3×3 2×3
 not same
 no GO

CB will be a 2×3

c. (5 pts) Calculate CA

$$CA = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 2 & 5 \end{bmatrix} \begin{bmatrix} 2 & 4 & -2 \\ 3 & -1 & 3 \\ 0 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 2+0+0 & 4+0+2 & -2+0-6 \\ 4+6+0 & 8-2+5 & -4+6-15 \end{bmatrix}$$
$$= \begin{bmatrix} 2 & 6 & -8 \\ 10 & 11 & -13 \end{bmatrix}$$

10. (15 points)

a. (10 pts) Take the determinant of $A = \begin{bmatrix} 0 & 1 & -1 \\ -4 & 2 & 1 \\ 0 & 3 & -3 \end{bmatrix}$

Expand across
row or column with
most zeros

$$0 \begin{vmatrix} 2 & 1 \\ 3 & -3 \end{vmatrix} - 4 \begin{vmatrix} 1 & -1 \\ 3 & -3 \end{vmatrix} + 0 \begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix}$$

$$= 0 - 4(-3 - (-3)) + 0$$

$$= -4(0) = \boxed{0}$$

b. (5 pts) If the determinant of a matrix is zero what does that say about the inverse of that matrix?

The inverse won't exist

11. (15 points)

a. (5 pts) If $f(x) = \sqrt{x-3} + 4$ find $f^{-1}(x)$

$$y = \sqrt{x-3} + 4$$

$$x = \sqrt{y-3} + 4 \quad \text{swap } x, y$$

Solve for y

$$x-4 = \sqrt{y-3}$$

$$(x-4)^2 = y-3$$

$$\rightarrow (x-4)^2 + 3 = y$$

$$\boxed{f^{-1}(x) = (x-4)^2 + 3}$$

b. (5 pts) Given $f(x) = (x+2)^2 + 1$ verify that $f^{-1} = \sqrt{x-1} - 2$ by calculating $f(f^{-1})$ and $f^{-1}(f)$

$$f(f^{-1}(x)) = (f^{-1}(x) + 2)^2 + 1 = (\sqrt{x-1} - 2 + 2)^2 + 1$$

$$= (\sqrt{x-1})^2 + 1 = x - 1 + 1 = x \quad \checkmark$$

$$f^{-1}(f(x)) = \sqrt{f(x) - 1} - 2 = \sqrt{(x+2)^2 + 1 - 1} - 2$$

$$= \sqrt{(x+2)^2} - 2 = x + 2 - 2 = x \quad \checkmark$$

c. (5 pts) Simplify using the inverse property: $4^{\log_4(x^2+3x+2)} + \log_3(3^{-x^2-3x+5})$

$$\underbrace{4^{\log_4(x^2+3x+2)}}_{x^2+3x+2} + \underbrace{\log_3(3^{-x^2-3x+5})}_{-x^2-3x+5}$$

$$= 2 + 5 = \boxed{7}$$

12. (15 points)

a. (5 pts) Calculate $\log_2\left(\frac{1}{32}\right)$

$$\log_2\left(\frac{1}{32}\right) = \log_2\left(\frac{1}{2^5}\right) = \log_2(2^{-5}) = \boxed{-5}$$

b. (5 pts) Solve $2\log_3(x+2) - \log_3(x+2) + \log_3(x+1) = 0$

$$= \log_3((x+2)^2) - \log_3(x+2) + \log_3(x+1) = 0$$

$$\Rightarrow \log_3\left(\frac{(x+2)^2}{x+2}\right) + \log_3(x+1) = 0$$

$$\Rightarrow \log_3\left(\frac{(x+2)^2}{(x+2)}(x+1)\right) = 0 \Rightarrow \log_3((x+2)(x+1)) = 0$$

$$\Rightarrow 3^{\log_3((x+2)(x+1))} = 3^0$$

c. (5 pts) Solve $e^{2x} + 3e^x + 1 = -1$

$$e^{2x} + 3e^x + 1 = -1$$

$$y = e^x$$

$$y^2 + 3y + 1 = -1$$

$$y^2 + 3y = -2$$

$$y(y+3) = -2$$

$$y = 0 \quad y = -3$$

$$\rightarrow e^x = 0$$

$$e^x = -3$$

$$\ln(e^x) = \ln(0)$$

$$x = \ln(0)$$

$$\ln(e^x) = \ln(-3)$$

$$x = \ln(-3)$$

but $\ln(0)$, $\ln(-3)$

are undefined

NO SOLUTION

$$\Rightarrow (x+2)(x+1) = 1$$

$$\Rightarrow x^2 + 3x + 1 = 1$$

$$\Rightarrow x^2 + 3x = 0$$

$$\Rightarrow x(x+3) = 0$$

$$\Rightarrow x = 0 \quad x = -3$$

↓
In domain

↓
not in domain

13. (15 points)

a. (5 pts) Find an expression for the n^{th} term of the sequence 1, -3, 9, -27, 81, ...

$$\begin{array}{ccccccc} n=1 & n=2 & n=3 & n=4 & n=5 \\ 1 & -3 & 9 & -27 & 81 \end{array}$$

$$a_n = (-1)^{n+1} (3)^{n-1}$$

b. (5 pts) For the arithmetic sequence with $a_1 = 0$ and $d = -3$ find S_{210}

$$\text{Find } a_n \quad a_n = a_1 + (n-1)d = 0 + (n-1)(-3)$$

$$S_n = \frac{n}{2} (a_1 + a_n)$$

$$\text{So } S_{210} = \frac{210}{2} (0 + (210-1)(-3)) = 105 (209)(-3)$$

Good enough for answer.

c. (5 pts) For the Geometric sequence with $a_1 = 2$ and $r = \frac{1}{2}$ find

$$\sum_{i=1}^{\infty} a_1 r^{i-1}$$

$$\sum_{i=1}^{\infty} a_1 r^{i-1} = a_1 \left(\frac{1}{1-r} \right) \text{ when } |r| < 1$$

$$|1/2| < 1 \quad \text{so} \quad \sum_{i=1}^{\infty} 2 \left(\frac{1}{2} \right)^{i-1} = 2 \left(\frac{1}{1-1/2} \right)$$

$$= 2 \left(\frac{1}{1/2} \right) = 2(2) = \boxed{4}$$

14. (5 points) (Bonus all or nothing) Using properties of logarithms and what you know about arithmetic series find:

$$\sum_{i=0}^{100} \ln(2^i)$$

Since $\ln(A^n) = n \ln A$

$$\ln(2^i) = i \ln(2)$$

So

$$\sum_{i=0}^{100} \ln(2^i) = \sum_{i=0}^{100} i \ln(2)$$

$$= \cancel{0 \ln(2)} + 1 \ln(2) + 2 \ln(2) + 3 \ln(2) + \dots + 100 \ln(2)$$

like an Arithmetic sum with

$$a_1 = 0 \quad d = \ln(2)$$

$$S_{100} = \frac{100}{2} (0 + 100 \ln(2))$$

$$= 50(100 \ln(2))$$

$$= \boxed{5000 \ln(2)}$$