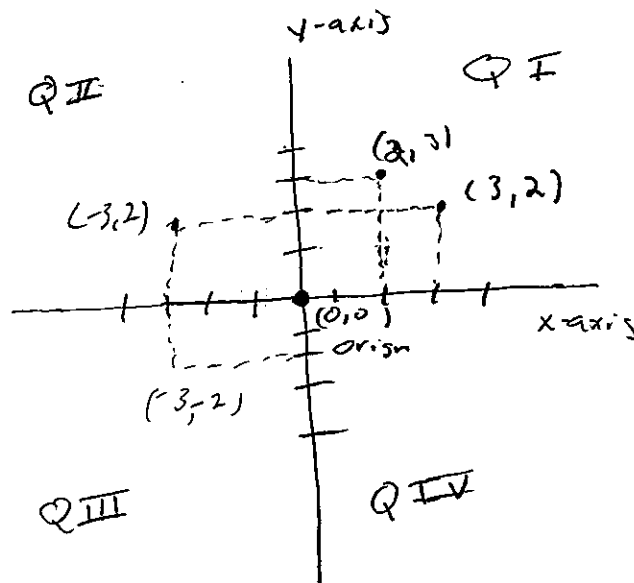


Sec 1.1

①

Sec 1.1

The cartesian plane.

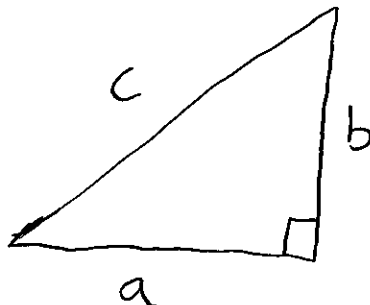


(x,y) is called an ordered pair

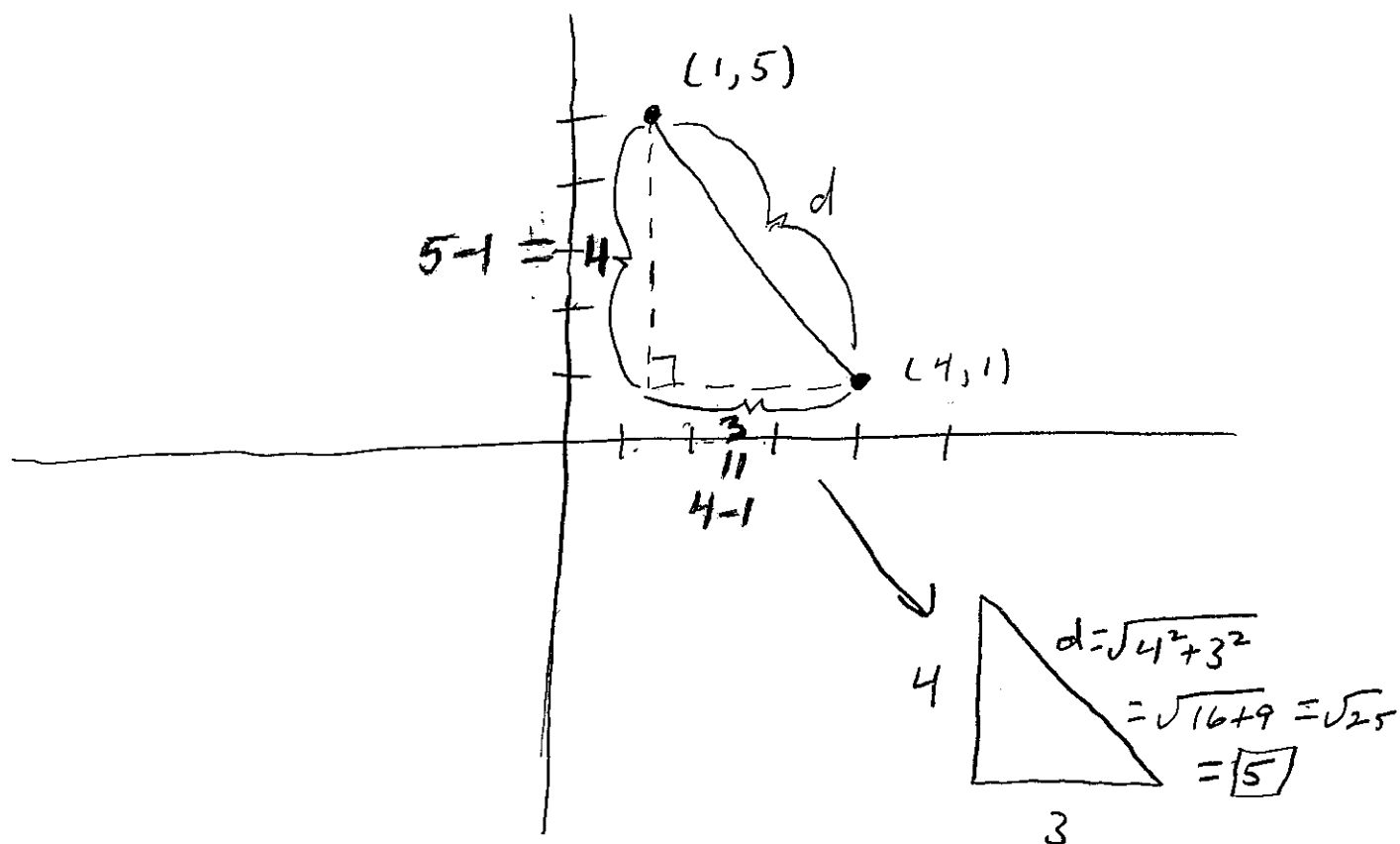
(x, y)
distance from origin from x-axis distance from origin from y-axis

The Pythagorean theorem and the distance between two points

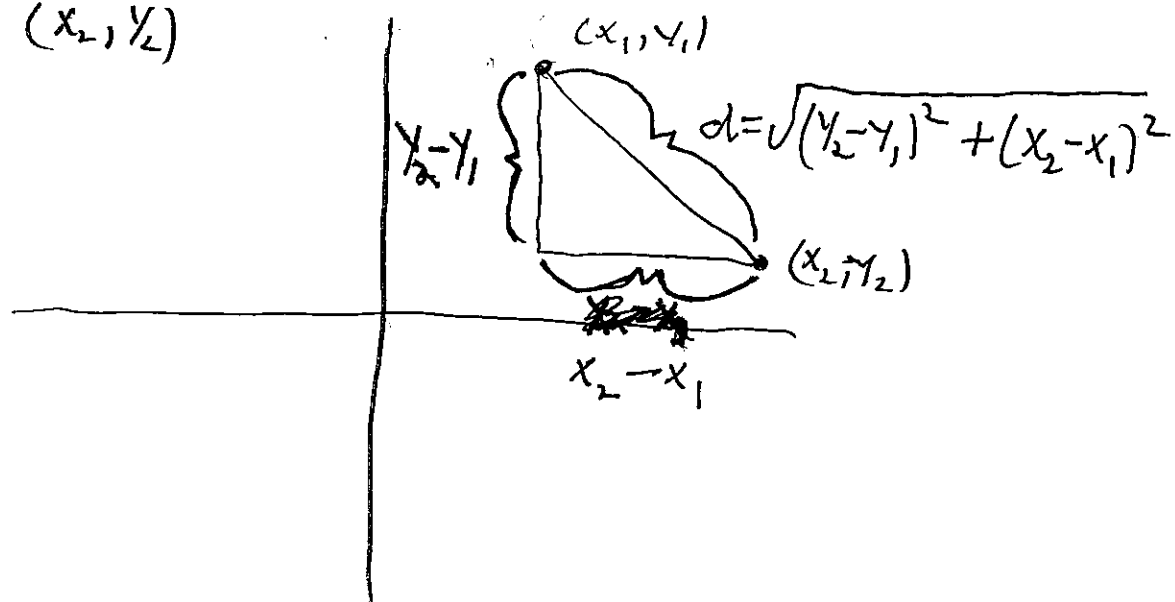
We know that for a right triangle $a^2 + b^2 = c^2$



How could we use this to find the distance between two points?



Note that the side length 4 was $5 - 1$ the difference in the y values of the points and 3 was $4 - 1$ the difference in the x values, suppose we have two points (x_1, y_1) (x_2, y_2)



Ex

③

Find the distance between $(-4, 2)$ and $(3, 6)$

Soln

The distance formula is

$$d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

$$(-4, 2) \quad (3, 6)$$

↓ ↓

$$(x_1, y_1) \quad (x_2, y_2)$$

or

$$(3, 6) \quad (-4, 2)$$

↓ ↓

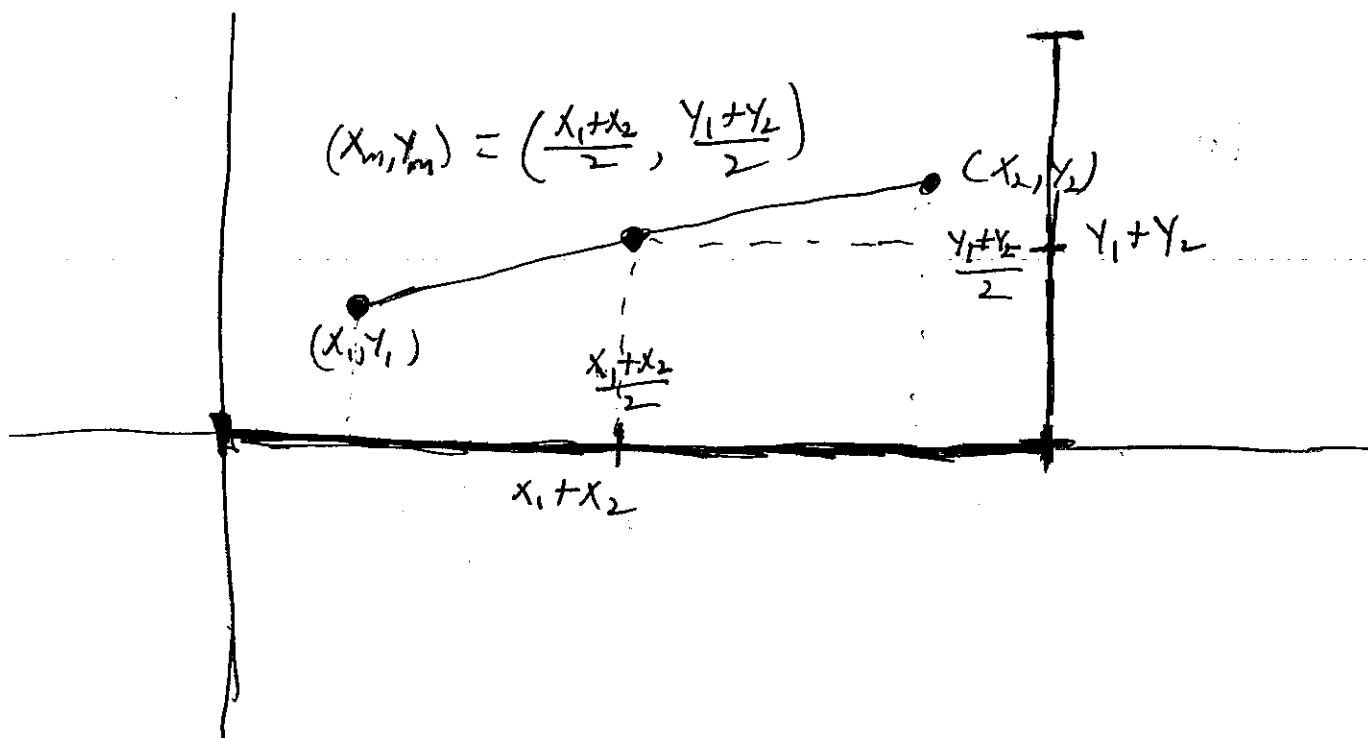
$$(x_1, y_1) \quad (x_2, y_2)$$

Then

$$\begin{aligned} d &= \sqrt{(6-2)^2 + (3-(-4))^2} \\ &= \sqrt{4^2 + 7^2} \\ &= \sqrt{16 + 49} \\ &= \sqrt{65} \end{aligned}$$

$$\begin{aligned} d &= \sqrt{(2-6)^2 + (-4-3)^2} \\ &= \sqrt{(-4)^2 + (-7)^2} \\ &= \sqrt{16 + 49} \\ &= \sqrt{65} \end{aligned}$$

Mid Point formula



EX Find the mid point between $(-4, 2)$ and $(3, 6)$

(4)

$$\begin{array}{cc} (-4, 2) & (3, 6) \\ \downarrow \downarrow & \downarrow \downarrow \\ (x_1, y_1) & (x_2, y_2) \end{array}$$

$$\text{or} \quad \begin{array}{cc} (3, 6) & (-4, 2) \\ \downarrow & \downarrow \\ (x_1, y_1) & (x_2, y_2) \end{array}$$

Then

$$(x_m, y_m) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$= \left(\frac{-4 + 3}{2}, \frac{2 + 6}{2} \right)$$

$$= \left(-\frac{1}{2}, \frac{8}{2} \right)$$

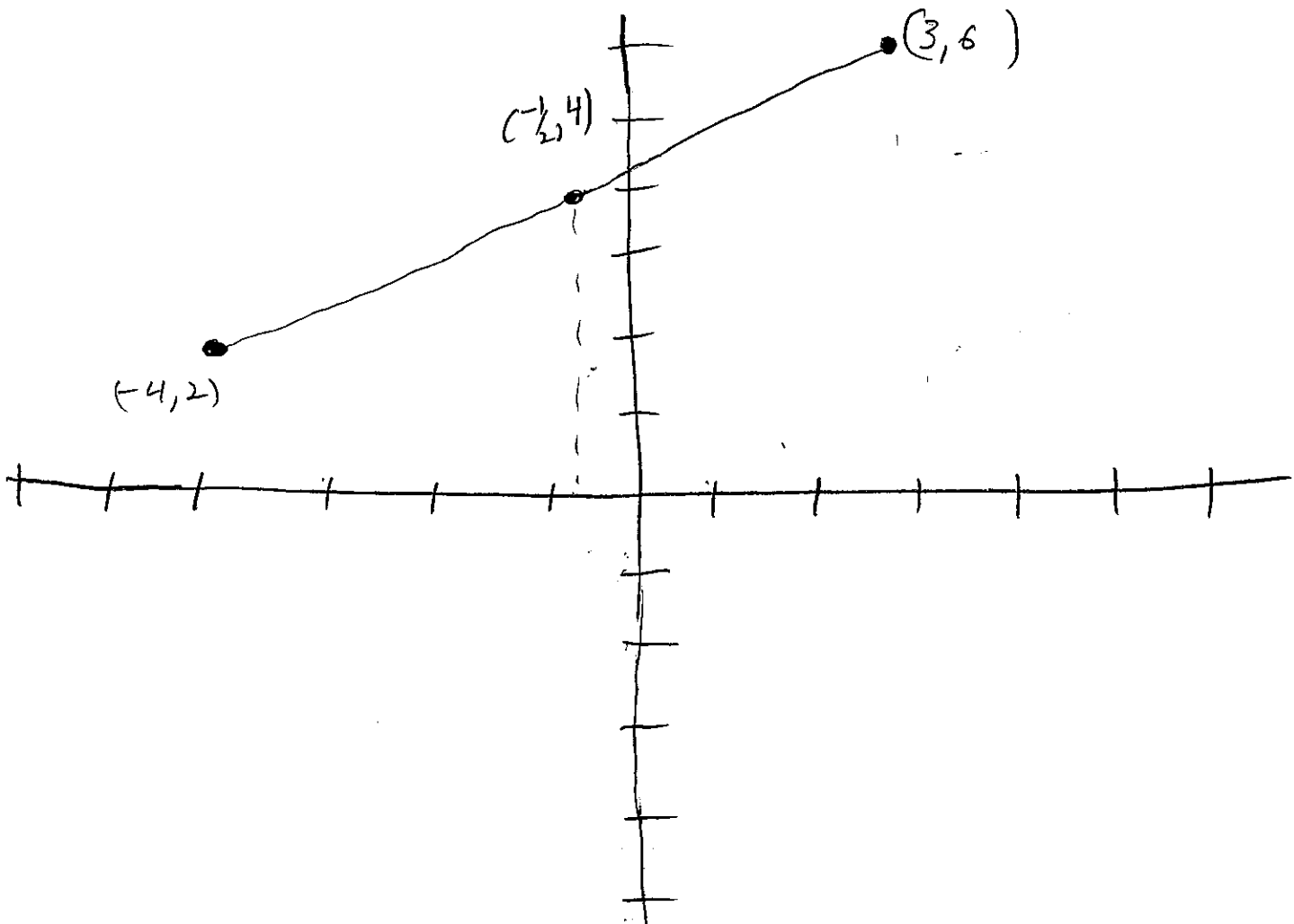
$$= \left(-\frac{1}{2}, 4 \right)$$

$$(x_m, y_m) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$= \left(\frac{3 - 4}{2}, \frac{6 + 2}{2} \right)$$

$$= \left(-\frac{1}{2}, \frac{8}{2} \right)$$

$$= \left(-\frac{1}{2}, 4 \right)$$



(5)

HW Example like #3 HW 1

Find the point $(0, b)$ which is equidistant from the points $(5, 5)$ $(6, -5)$

Soln

We can find the distance between $(0, b)$ and our two given points $(5, 5)$ $(6, -5)$ by using the distance formula we then set them equal to each other and solve for b .

$$(0, b) \quad (5, 5)$$

$$(x_1, y_1) \quad (x_2, y_2)$$

$$(0, b) \quad (6, -5)$$

$$(x_1, y_1) \quad (x_2, y_2)$$

$$d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

$$= \sqrt{(5 - b)^2 + (5 - 0)^2}$$

$$= \sqrt{(5 - b)^2 + 25}$$

$$d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

$$= \sqrt{(-5 - b)^2 + (6 - 0)^2}$$

$$= \sqrt{(-5 - b)^2 + 36}$$

For the distances to be equal we require

$$\sqrt{(5 - b)^2 + 25} = \sqrt{(-5 - b)^2 + 36}$$

$$\Rightarrow \left(\sqrt{(5 - b)^2 + 25} \right)^2 = \left(\sqrt{(-5 - b)^2 + 36} \right)^2$$

$$\Rightarrow (5 - b)^2 + 25 = (-5 - b)^2 + 36$$

$$\begin{array}{ccc} \downarrow & & \downarrow \\ 25 - 10b + b^2 + 25 & = & 25 + 10b + b^2 + 36 \end{array}$$

For to find

$$(5 - b)^2 = 25 - 10b + b^2$$

$$(-5 - b)^2 = 25 + 10b + b^2$$

Now we can solve our equation for the value of b which satisfies the requirement that the two distances be equal,

$$\begin{array}{r} 25 - 10b + b^2 + 25 = 25 + 10b + b^2 + 36 \\ -b^2 \qquad \qquad \qquad -b^2 \end{array}$$

$$\begin{array}{r} 25 - 10b + 25 = 25 + 10b + 36 \\ -25 \quad -25 \end{array}$$

$$\begin{array}{r} 25 - 10b = 10b + 36 \\ -25 \qquad \qquad \qquad -25 \end{array}$$

$$\begin{array}{r} -10b = 10b + 11 \\ -10b \quad -10b \end{array}$$

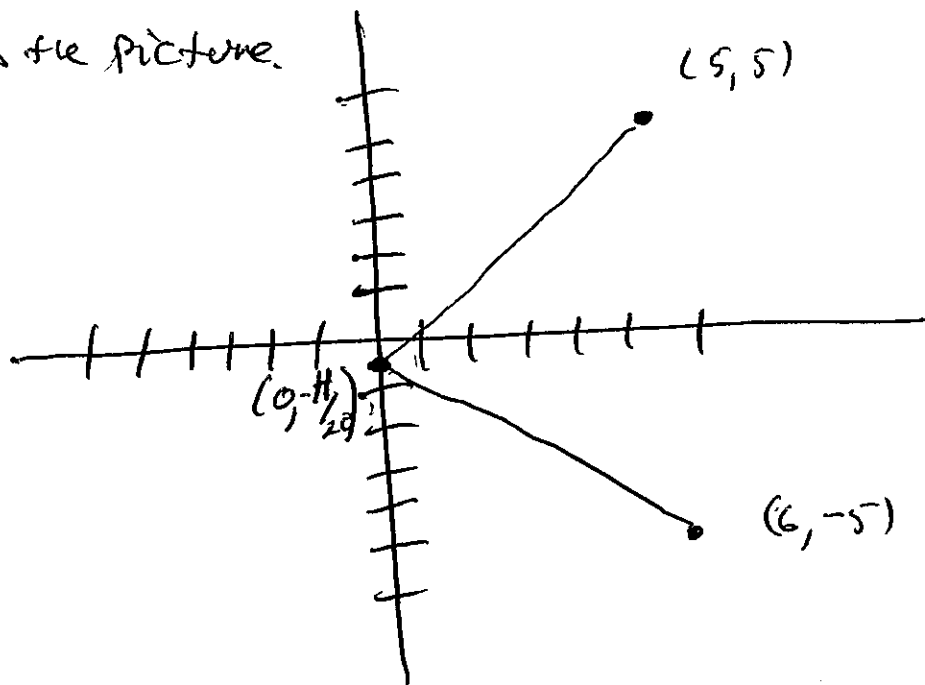
$$\begin{array}{r} -20b = 11 \\ \hline -20 \quad -20 \end{array}$$

$$b = -11/20$$

close to

$$-11/20 \approx -\frac{1}{2}$$

Here is the picture.



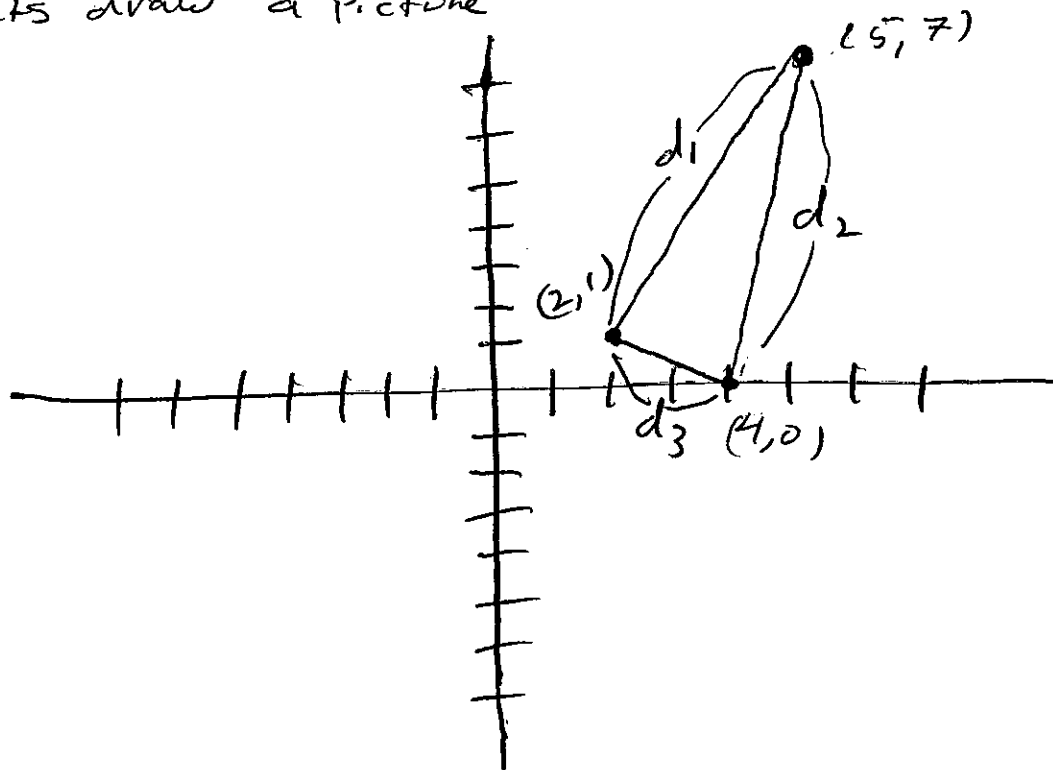
HW Example like #11 HW #1

⑦

Find the perimeter of the triangle with the vertices $(2,1)$ $(4,0)$ and $(5,7)$, is it a right triangle?

Soln

Let's draw a picture



We can use the distance formula to find d_1, d_2, d_3 then sum them to find the perimeter.

$$d_1 = \sqrt{(7-1)^2 + (5-2)^2} = \sqrt{6^2 + 3^2} = \sqrt{36+9} = \sqrt{45}$$

$$d_2 = \sqrt{(7-0)^2 + (5-4)^2} = \sqrt{7^2 + 1^2} = \sqrt{49+1} = \sqrt{50}$$

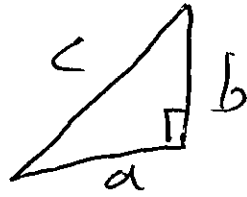
$$d_3 = \sqrt{(1-0)^2 + (2-4)^2} = \sqrt{1^2 + (-2)^2} = \sqrt{1+4} = \sqrt{5}$$

So the perimeter is $d_1 + d_2 + d_3 = \sqrt{45} + \sqrt{50} + \sqrt{5}$

To see if this is a right triangle we can apply the Pythagorean theorem.

Recall for right triangles

⑧



$$a^2 + b^2 = c^2$$

* a, b are the shorter sides and c the long side so we need to check

$$\begin{array}{ccc} (\sqrt{45})^2 & + & (\sqrt{5})^2 = (\sqrt{50})^2 \Rightarrow 45 + 5 = 50 \checkmark \\ \downarrow & & \downarrow \quad \downarrow \\ a & & b \quad c \end{array}$$

Since the sides of our triangle satisfy the pythagorean theorem we know we have a right triangle.