

Graphs of equations

We can think of an equation as a rule that pairs a given number with a second number to create an ordered pair which we can then plot in the (x, y) plane. Typically we take x -values apply the rule and get the corresponding y -value.

Consider the equation:

$$y = 2x + 1$$

here we choose an x value any one we want and obtain a y value The rule is take x multiply by 2 and add 1 to get y

$$y = 2x + 1 \leftarrow \text{add 1}$$

↑
multiply
by 2

So if we chose $x = 1$ then we get the corresponding y using the rule

$$y = 2(1) + 1 = 2 + 1 = 3$$

so when $x = 2$ $y = 3$ giving us the point

$$(2, 3)$$

↑ ↑
 x y

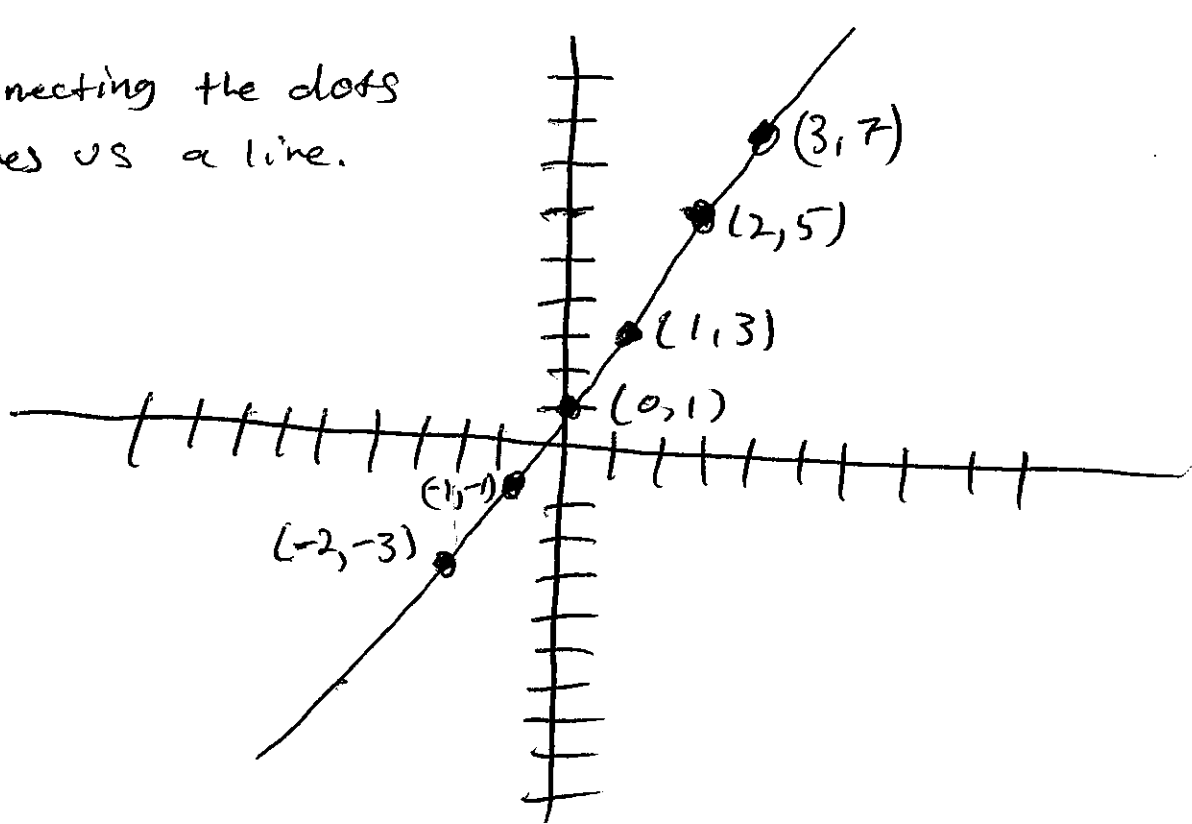
we call this point a solution point.

we can plot the solution points to make a graph. Let's plot $y = 2x + 1$ we will choose some points for x and find corresponding y -values.

x	$y = 2x + 1$	(x, y)
-2	$y = 2(-2) + 1 = -4 + 1 = -3$	$(-2, -3)$
-1	$y = 2(-1) + 1 = -2 + 1 = -1$	$(-1, -1)$
0	$y = 2(0) + 1 = 0 + 1 = 1$	$(0, 1)$
1	$y = 2(1) + 1 = 2 + 1 = 3$	$(1, 3)$
2	$y = 2(2) + 1 = 4 + 1 = 5$	$(2, 5)$
3	$y = 2(3) + 1 = 6 + 1 = 7$	$(3, 7)$

~~Plot~~ Plotting the points

connecting the dots gives us a line.



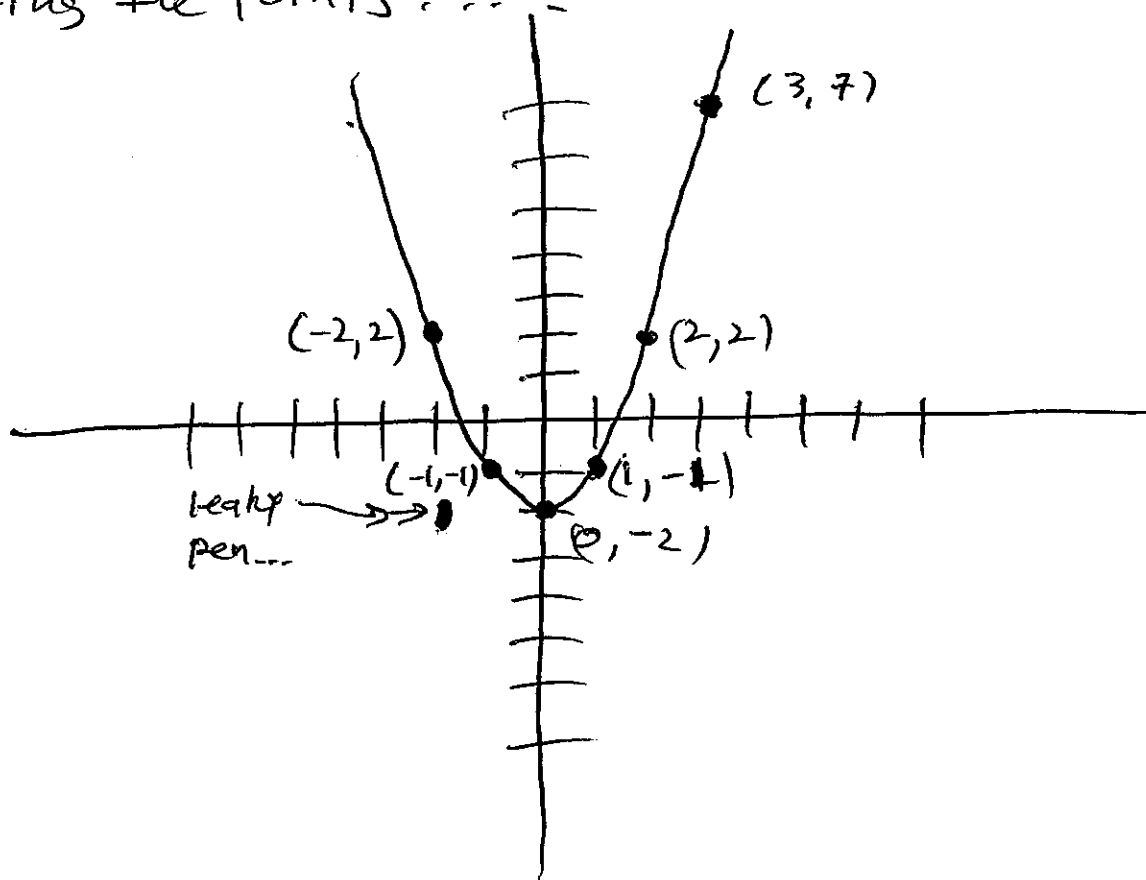
lets sketch

④

$$y = x^2 - 2$$

x	$y = x^2 - 2$	(x, y)
-2	$y = (-2)^2 - 2 = 4 - 2 = 2$	$(-2, 2)$
-1	$y = (-1)^2 - 2 = 1 - 2 = -1$	$(-1, -1)$
0	$y = 0^2 - 2 = 0 - 2 = -2$	$(0, -2)$
1	$y = 1^2 - 2 = 1 - 2 = -1$	$(1, -1)$
2	$y = 2^2 - 2 = 4 - 2 = 2$	$(2, 2)$
3	$y = 3^2 - 2 = 9 - 2 = 7$	$(3, 7)$

Plotting the points . . .



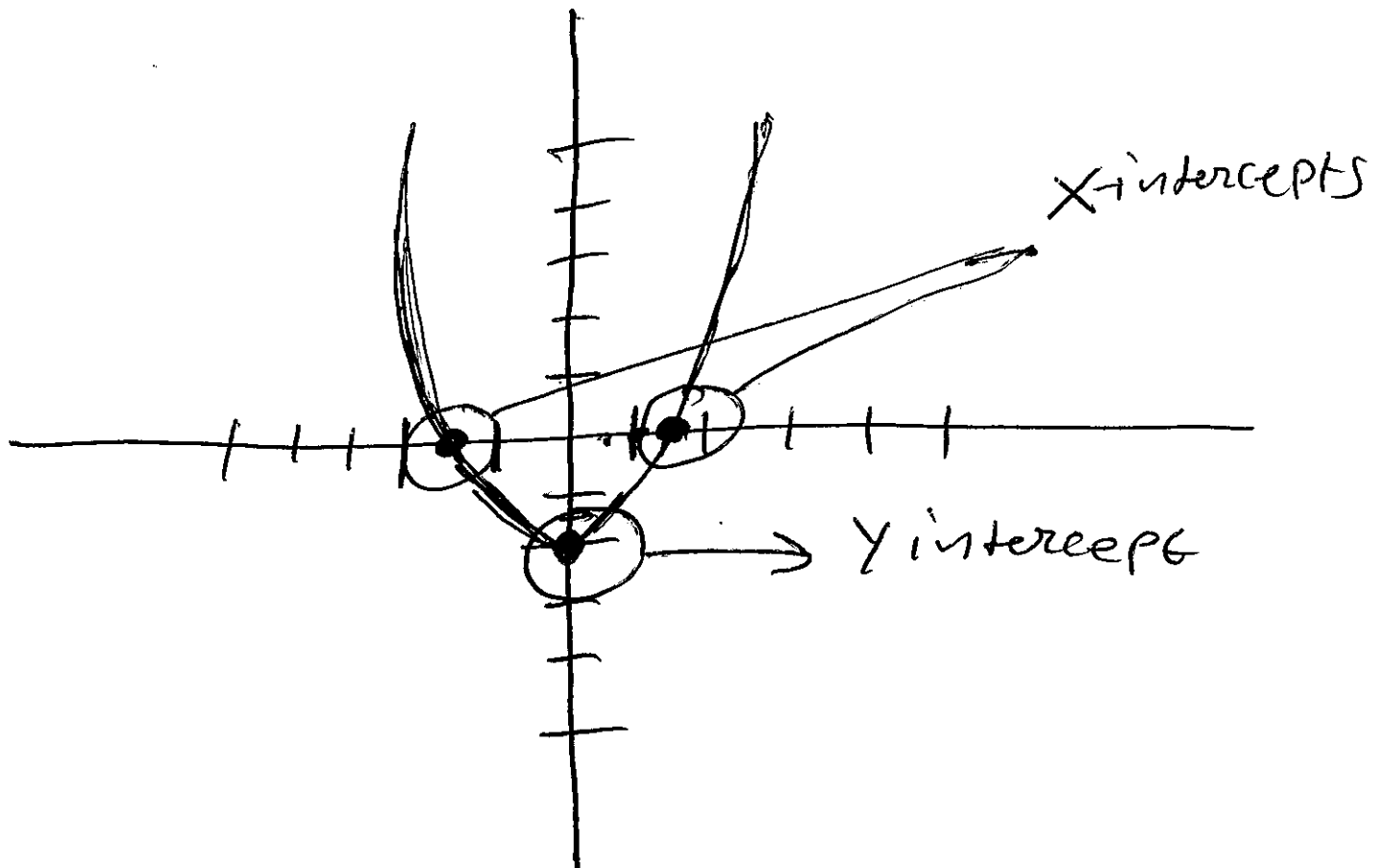
Important points on a graph.

(12)

The x -intercept is the point where a graph crosses the x -axis.

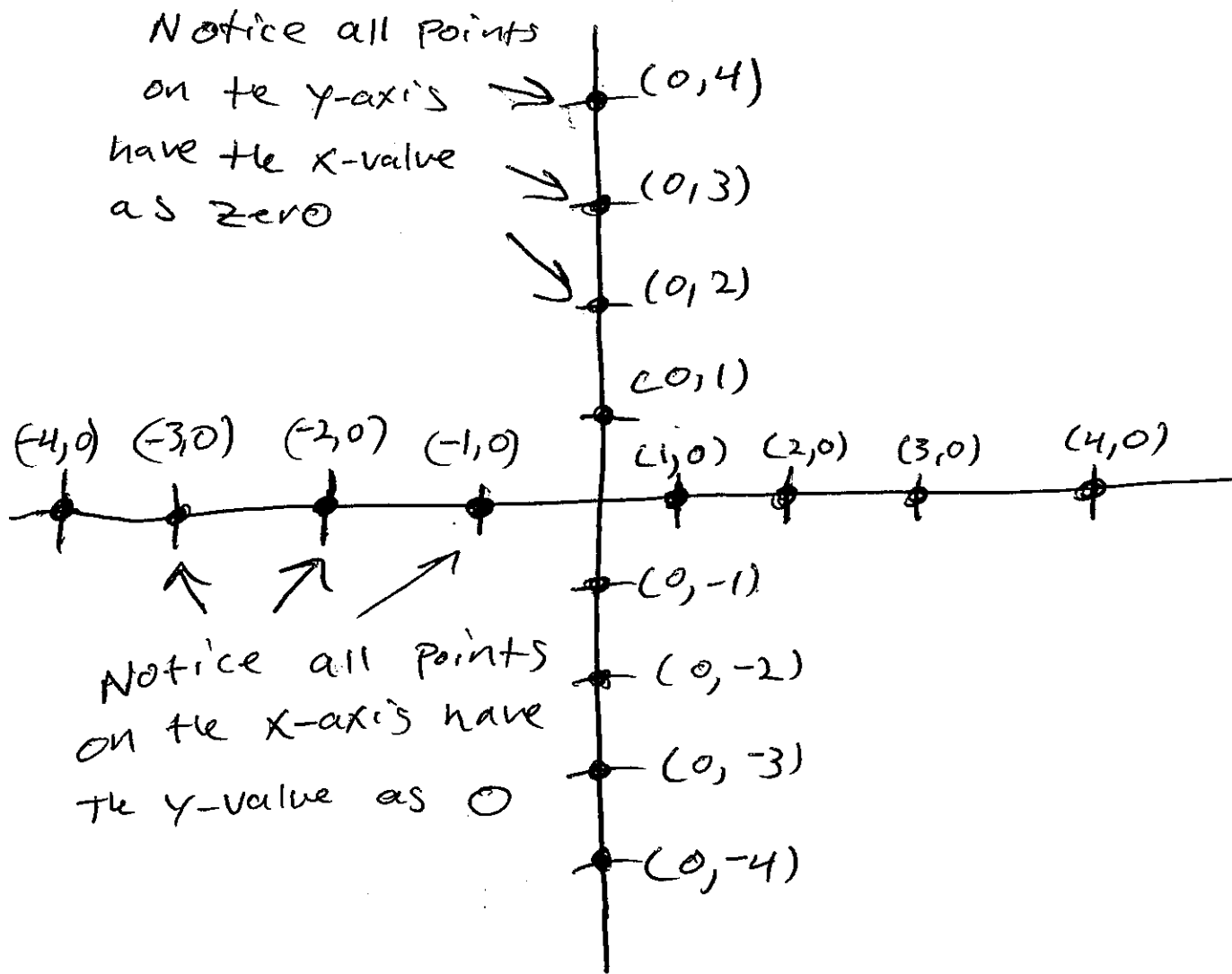
The y -intercept is the point where a graph crosses the y -axis.

Consider $y = x^2 - 2$ again



How can we find the actual values for the x and y intercepts other than looking at the graph.

Let's examine the points that line on the x and y axes



So if a graph crosses the x -axis at that point its y -value will be zero and if it crosses the y -axis it must have its x -value as zero.

To find the y-intercept Set $x=0$ and calculate y . (14)

Ex

$y = x^2 - 2$ crosses the y axis when $x = 0$

so $y = 0^2 - 2 = -2$ so the y intercept is the point $(0, -2)$

~~$y = x^2 - 2$ crosses the x~~

To find the x-intercept set $y=0$ and solve for x .

Ex

$y = x^2 - 2$ crosses the x-axis when $y = 0$

so:

$$0 = x^2 - 2$$

+2 +2

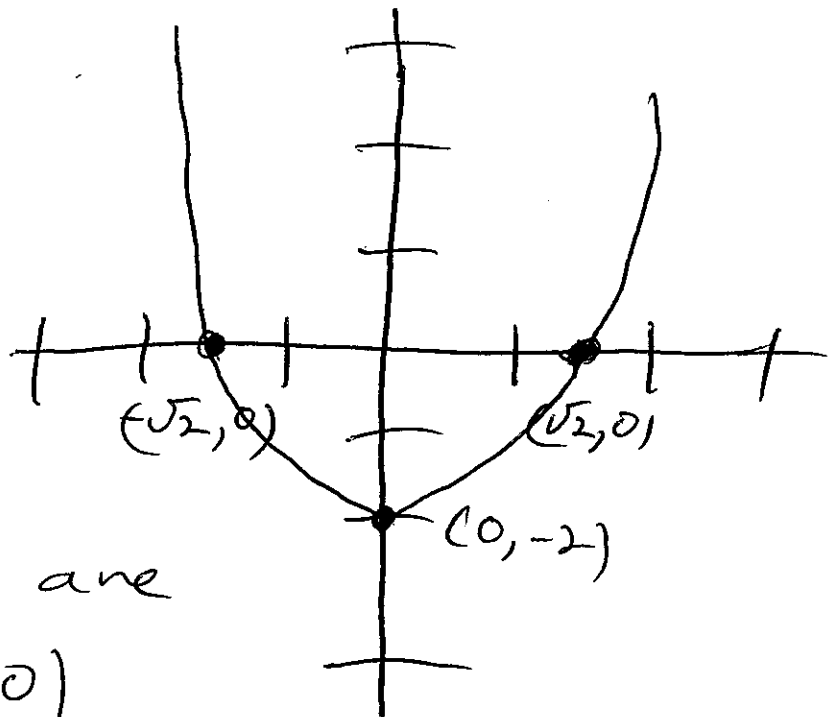
$$\sqrt{2} = \sqrt{x^2}$$

$$\pm\sqrt{2} = x$$

so the x intercepts are

$(-\sqrt{2}, 0)$ and $(\sqrt{2}, 0)$

$$\sqrt{2} \approx 1.4$$



Ex

(15)

Find the x and y intercepts for $y = x^3 - 4x$

To find the y intercept set $x = 0$

$y = 0 - 4(0) = 0$ so the y intercept is $(0, 0)$ Notice that since the x value is also 0 this is an x intercept as well.

To find the x intercepts set $y = 0$

$$0 = x^3 - 4x$$

$$0 = x(x^2 - 4)$$

To solve we set each part of the product equal to zero

$$0 = x$$

$$x = 0 \Rightarrow (0, 0)$$

is an x intercept

$$0 = x^2 - 4$$

$$+4 \quad +4$$

$$4 = x^2$$

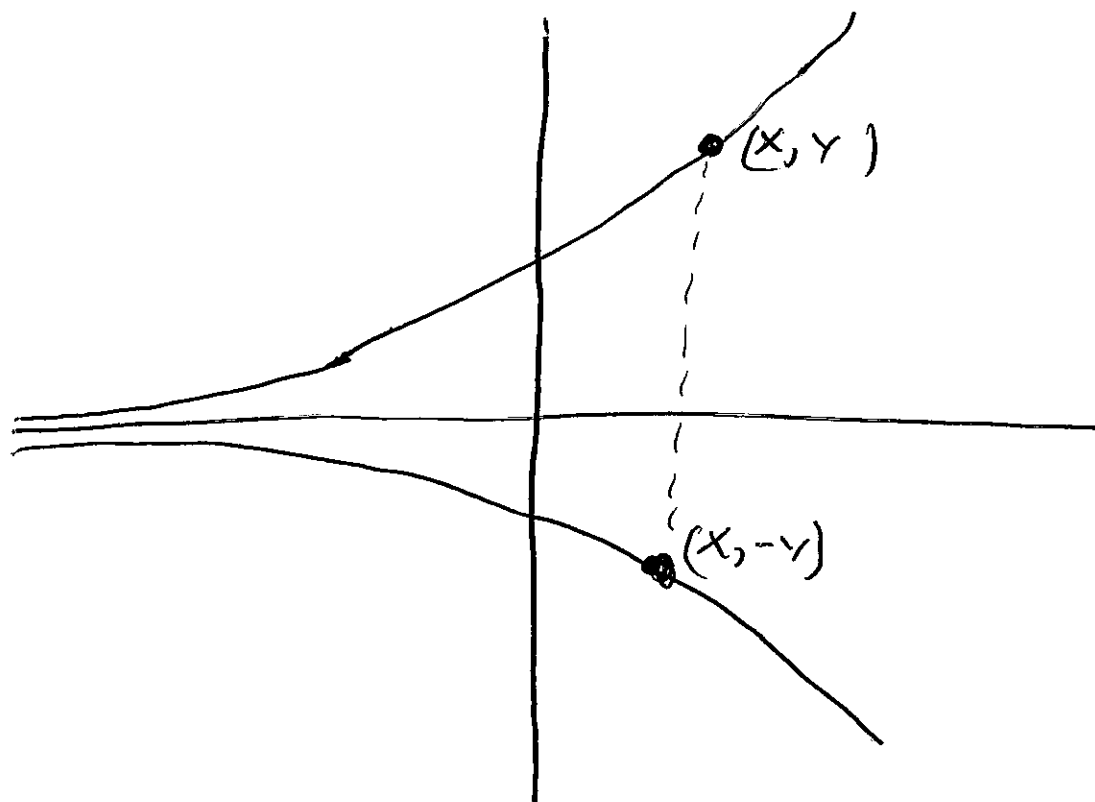
$$\pm\sqrt{4} = \sqrt{x^2}$$

$$\pm 2 = x$$

so $(-2, 0)$ and $(2, 0)$ are x intercepts

Symmetry

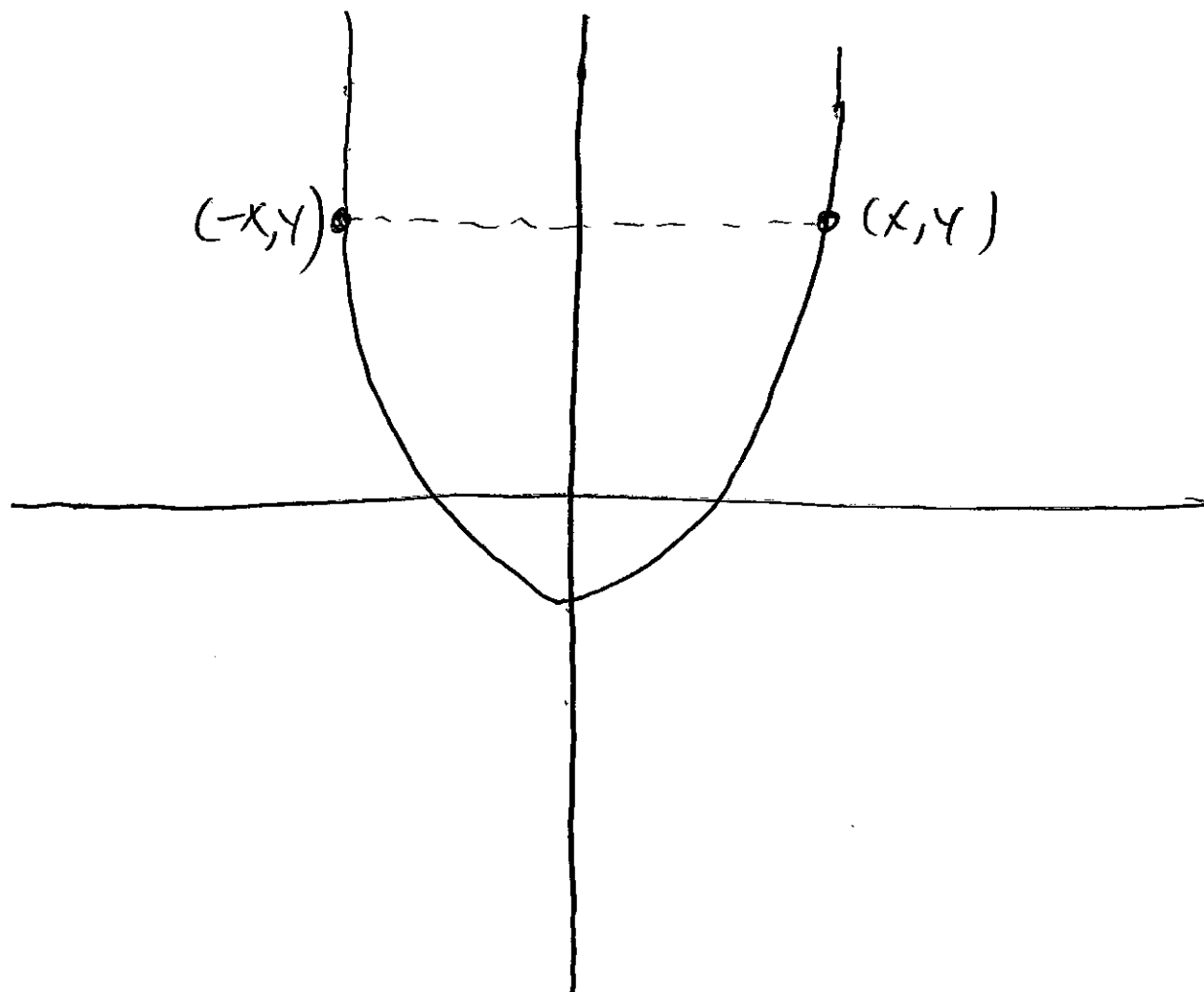
X-axis Symmetry: If we fold the $x-y$ plane over the x -axis the part of the graph above the x -axis coincides with the portion below the x -axis.



Another way to say this is that whenever the point (x, y) is on the graph so is the point $(x, -y)$.

y-axis symmetry

When the xy plane is folded over the y -axis the part of the graph to the right of the y -axis coincides with the part on the left

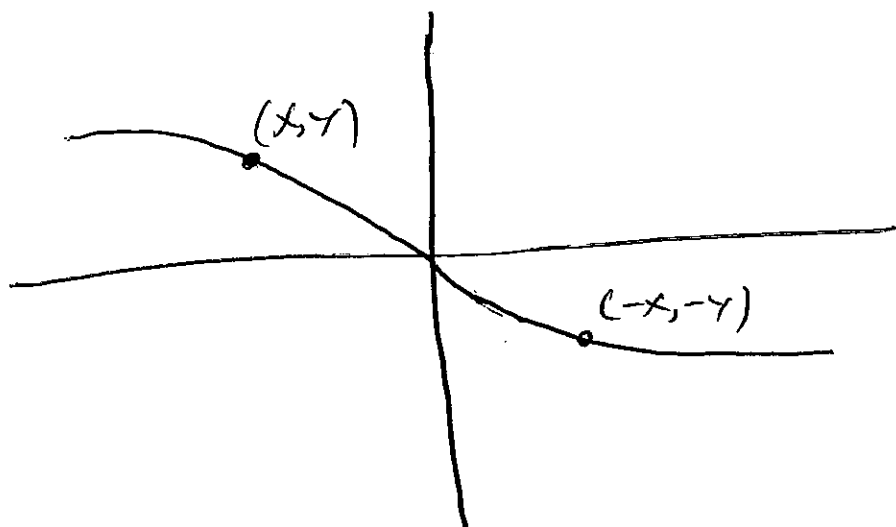
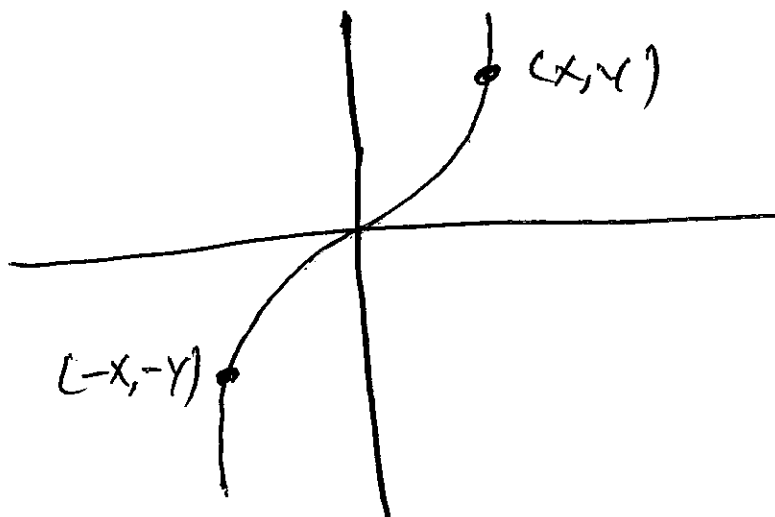


Another way to say this is that if the point (x, y) is on the graph so is the point $(-x, y)$

Origin Symmetry

(18)

When the plane is folded over the x -axis and then the y -axis the part of the graph in one of the quadrants coincides with the part in the other.



Another way to say this is that if (x, y) is on the graph so is the point $(-x, -y)$

Hw Example like #7 hw #1

(19)

For the point $(-2, 2)$ Find the points that exhibit x-axis, y-axis, and origin symmetry

Soln

For x-axis symmetry we can simply change the sign of the y value for $(-2, 2)$

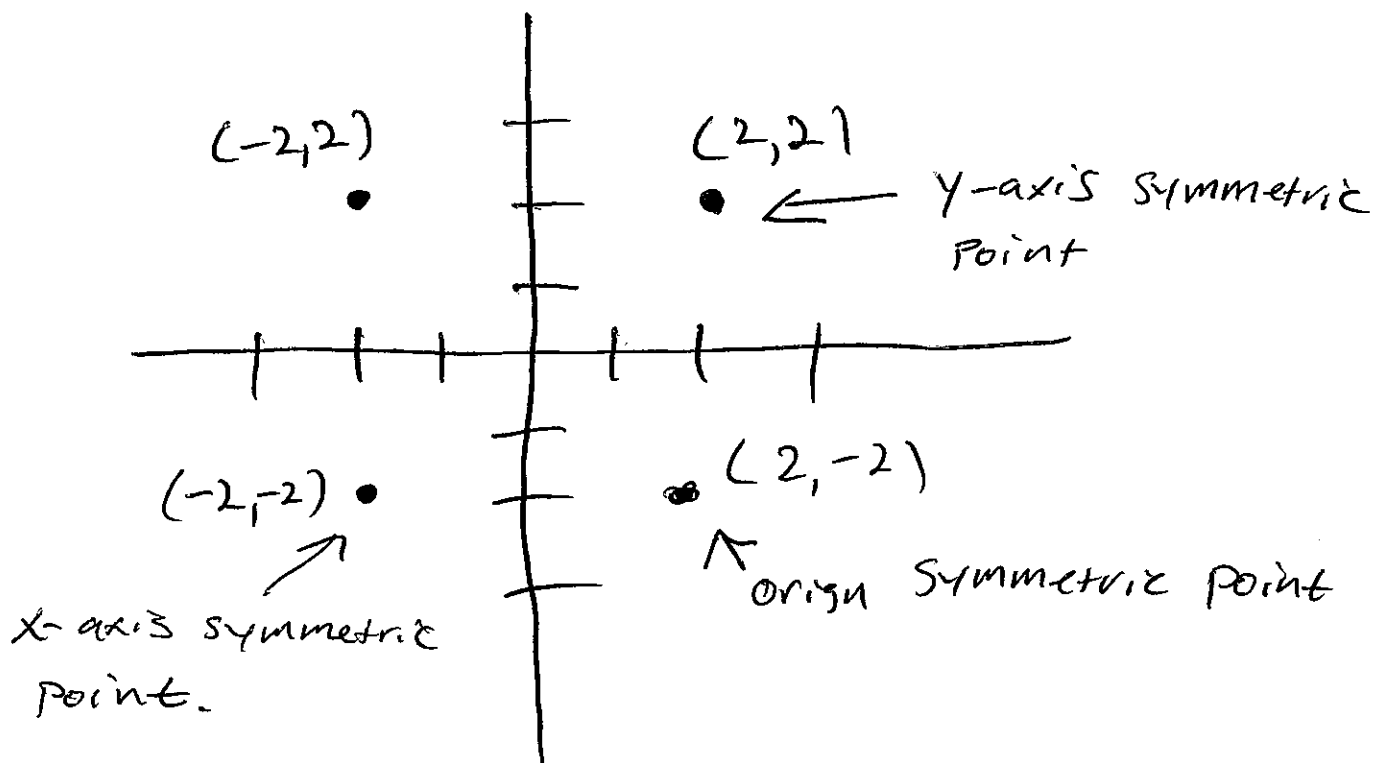
we get the point $(-2, -2)$

For y-axis symmetry we can simply change the sign of the x value so for $(-2, 2)$

we get the point ~~$(-2, 2)$~~ $(2, 2)$

For the origin symmetry we change the sign on both the (x, y) values so

for $(-2, 2)$ we get $(2, -2)$



Algebraic tests for symmetry.

1. The graph of an equation is symmetric with respect to the x -axis when replacing y with $-y$ yields an equivalent equation.
2. The graph of an equation is symmetric to the ~~ax~~ with respect to the y -axis when replacing x with $-x$ yields an equivalent equation.
3. The graph of an equation is symmetric with respect to the origin when replacing x with $-x$ and y with $-y$ yields an equivalent equation.

HW example like #8 HW #1

(2)

Determine whether the following functions have no symmetry (N) y-axis symmetry (y) x-axis symmetry (x) or origin symmetry (o)

$$y = x + 1$$

Testing for y axis symmetry replace x with $-x$

$$y = -x + 1$$

This is not equivalent to $y = x + 1$ so no y-axis symmetry

Testing for x-axis symmetry replace y with $-y$

$$-y = x + 1 \Rightarrow y = -x - 1 \text{ not equivalent}$$

to $y = x + 1$ No x-axis symmetry

Testing for origin symmetry replace y with

$-y$ and x with $-x$

$$-y = -x + 1 \Rightarrow y = x - 1 \text{ not equivalent}$$

to $y = x + 1$ No origin symmetry

N

$$y = x^3$$

y-axis

$$y = (-x)^3 = -x^3 \quad y = -x^3 \text{ not equivalent to } y = x^3$$

x-axis

$$-y = x^3 \Rightarrow y = -x^3 \text{ not equivalent to } y = x^3$$

$$y = x^3$$

origin

$$-y = (-x)^3 \Rightarrow -y = -x^3 \Rightarrow (-1)(-y) = (-1)(-x^3)$$

$$\Rightarrow y = x^3 \text{ equivalent to } y = x^3 \text{ so we}$$

have origin symmetry.



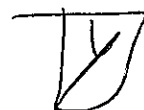
$$y = x^2$$

y-axis

$$y = (-x)^2 = (-1)^2(x^2) = x^2 \Rightarrow y = x^2$$

$$\text{equivalent to } y = x^2$$

y-axis symmetry



$$x = y^2$$

Y-axis $-x = y^2 \Rightarrow x = -y^2$ not equivalent
~~to~~ to $x = y^2$

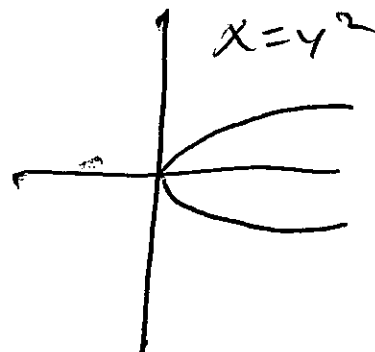
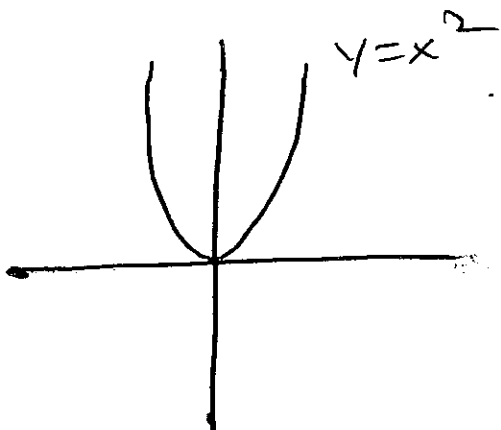
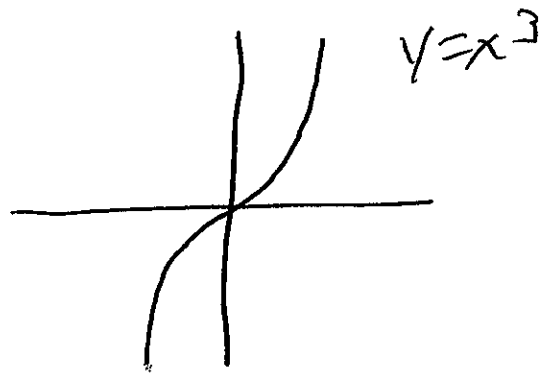
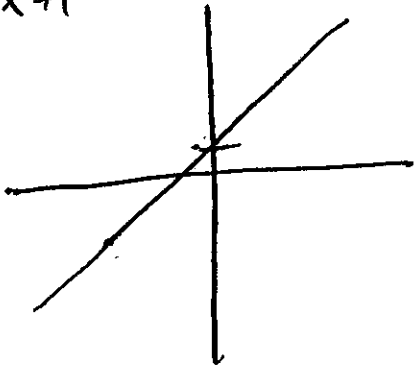
X-axis

$$x = (-y)^2 \Rightarrow x = (-1)^2 y^2 \Rightarrow x = y^2$$

equivalent to $x = y^2$ so we have x-axis
 symmetry. x

Here are the Graphs :

$$y = x + 1$$

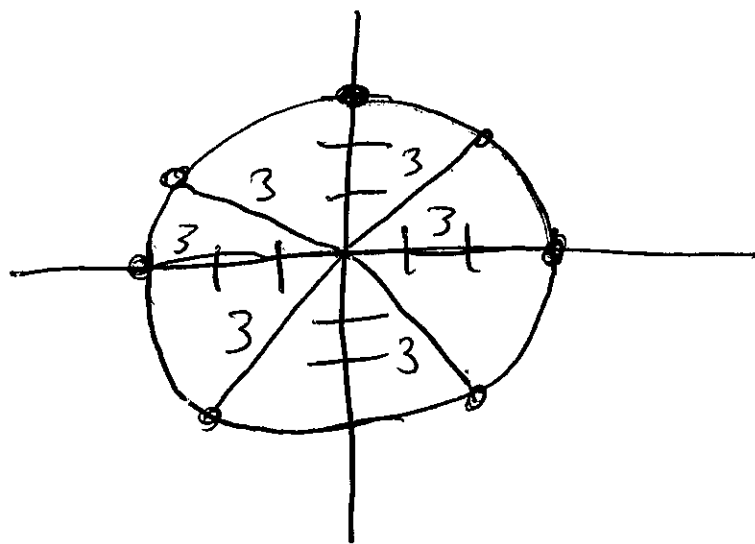


Circles

(24)

A circle can be thought of as the set of all points which are the same distance away from its center.

For example the circle of radius 3 centered at the point $(0,0)$ looks like



Every point is a distance 3 away from $(0,0)$. Let's use this idea to find the general equation of a circle.

Suppose we want to center a circle at the point (h,k) and want it to have a radius r .

(25)

Then the points on the circle will be all (x, y) such that the distance between (h, k) and (x, y) is equal to r . From the distance formula we can write the distance between (h, k) and (x, y)

$$\begin{matrix} (h, k) & (x, y) \\ x_1, y_1 & x_2, y_2 \end{matrix} \Rightarrow d = \sqrt{(y-k)^2 + (x-h)^2}$$

We want the distance to be r so we set $d = r$

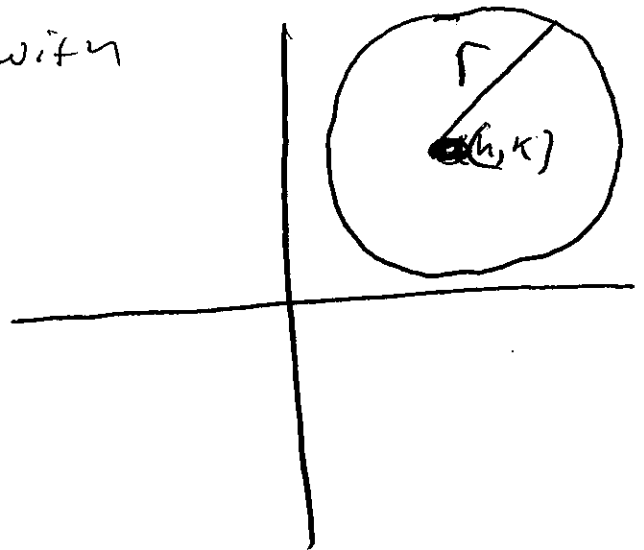
$$r = \sqrt{(y-k)^2 + (x-h)^2}$$

If we square both sides we get

$$r^2 = (y-k)^2 + (x-h)^2$$

Re-arranging gives us the general form of a circle centered at (h, k) with radius r .

$$(x-h)^2 + (y-k)^2 = r^2$$

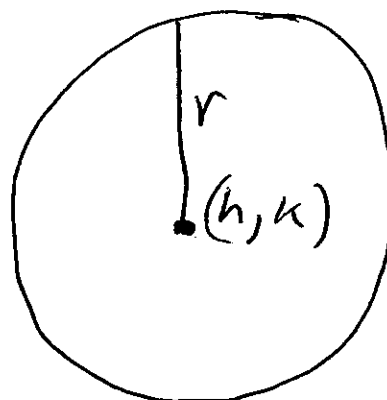


X coordinate of center

radius squared

$$(X-h)^2 + (Y-k)^2 = r^2$$

Y-coordinate of center.



Ex

What is the center of the circle

$$(x-2)^2 + (y+4)^2 = 16$$

What is the radius.

Match it

$$(x-h)^2 + (y-k)^2 = r^2$$

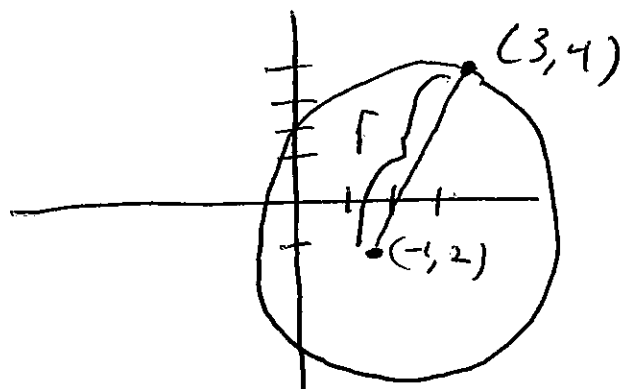
$$(x-2)^2 + (y-(-4))^2 = 4^2$$

$$h=2 \quad k=-4 \quad r=4$$

HW example like #13 HW #1

(27)

The point $(3,4)$ is on the circle centered at $(-1,2)$ find the equation of the circle.



The distance between $(-1,2)$ and $(3,4)$ is the radius of the circle.

$$r = \sqrt{(4-2)^2 + (3-(-1))^2} = \sqrt{2^2 + 4^2} = \sqrt{4+16} = \sqrt{20}$$

Then the General form of a circle is

$$(x-h)^2 + (y-k)^2 = r^2$$

$$h = -1 \quad k = 2 \quad \cdot \quad r = \sqrt{20} \Rightarrow r^2 = 20$$

So our circle is Given by

$$(x-(-1))^2 + (y-2)^2 = 20$$

$$\Rightarrow \boxed{(x+1)^2 + (y-2)^2 = 20}$$

Hw example like #10 Hw 1

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Find the (x, y) intercepts of a circle with radius 6 centered at $(0, 0)$

Soln

The equation of the circle is

$$(x-0)^2 + (y-0)^2 = 36 \Rightarrow x^2 + y^2 = 36$$

X-int Set $y=0$

$$\Rightarrow x^2 + 0^2 = 36 \Rightarrow x^2 = 36 \Rightarrow x = \pm\sqrt{36}$$

$\Rightarrow x = \pm 6$ so $(6, 0)$ and $(-6, 0)$ are the x intercepts.

Y int Set $x=0$

$$0^2 + y^2 = 36 \Rightarrow y^2 = 36 \Rightarrow y = \pm\sqrt{36} \Rightarrow y = \pm 6$$

so $(0, 6)$, $(0, -6)$ are the y intercepts.

