

Functions

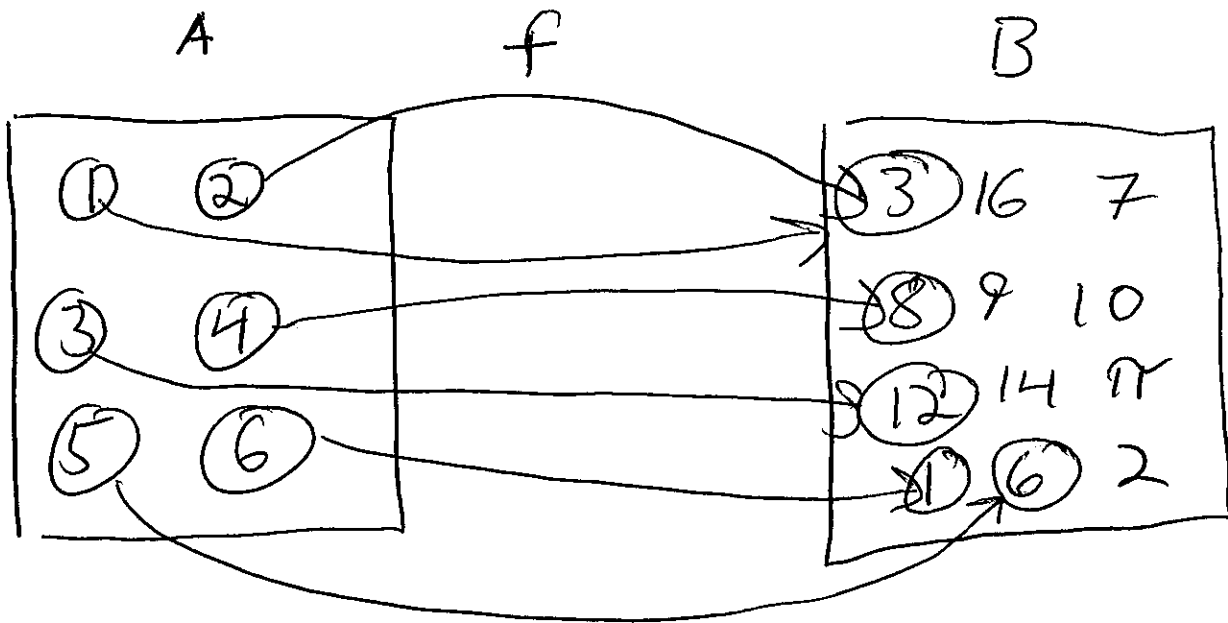
Definition

A function f from a set A to a set B is a relation that assigns each element in A to one in B with the following conditions

- ① every element in A is assigned to an element in B
- ② every element in A is assigned to ONLY ONE element in B

We call the set A the Domain of f and the assigned elements of B the Range of f .

Lets visualize this!

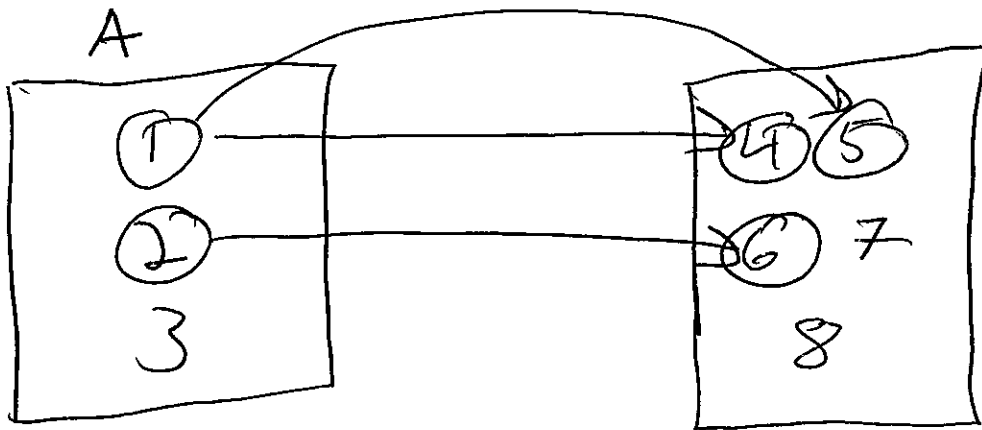


Here we have matched 1 to 3, 2 to 3, 3 to 12
4 to 8, 5 to 6 and 6 to 1

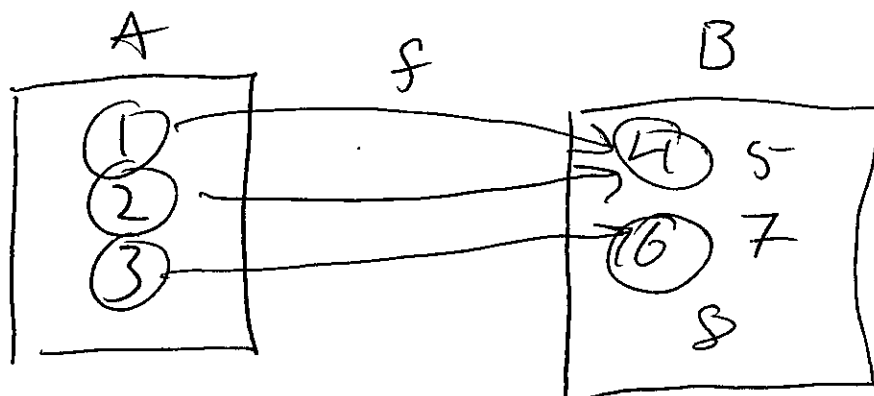
We would call the numbers in A 1, 2, 3, 4, 5, 6
The Domain while the circled (matched)
numbers in B would be the range the
numbers not circled in B are NOT
in the range. we also note that
Each element in A is matched to
only one element in B .

Example of a non function

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Here we have two problems first 3 in the set A has not been assigned a value further the number 1 in A has been assigned 2-values 1 to 4 and 1 to 5. In this case we do not have a function. Note though we can assign two values in A to the same in B and be OK.



∈ This is a function.

A function as a rule

consider the equation $y = x^2 + 1$

Here we can think of the Domain Set A as all the real numbers, using the rule that $y = x^2 + 1$ we can assign real numbers x to other real numbers y

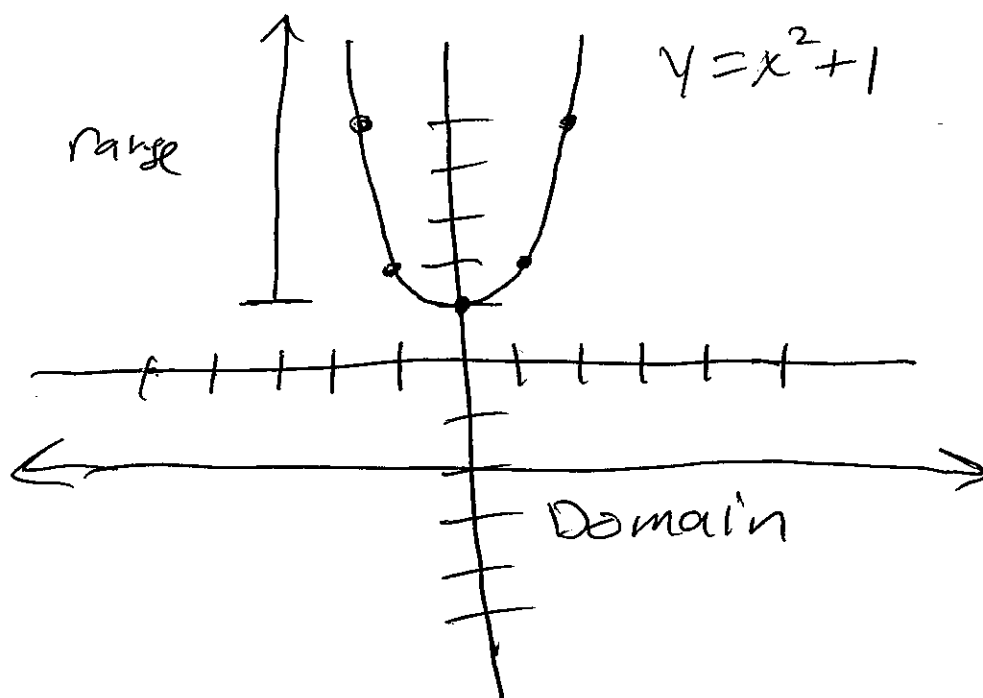
The resulting set of real numbers y will be the range, we have done this when drawing graphs.

x	$y = x^2 + 1$	(x, y)
-2	$y = (-2)^2 + 1 = 5$	$(-2, 5)$
-1	$y = (-1)^2 + 1 = 2$	$(-1, 2)$
0	$y = 0^2 + 1 = 1$	$(0, 1)$
1	$y = 1^2 + 1 = 2$	$(1, 2)$
2	$y = 2^2 + 1 = 5$	$(2, 5)$

So we assign -2 to 5, -1 to 2
0 to 1, 1 to 2, 2 to 5

we can plot this to better understand the function,

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we note that the Domain is all real numbers but that the range is only ~~$y \geq 1$~~ $y \geq 1$.
we will revisit the idea of Domain and range again.

The Difference between a graph and a function

we have drawn quite a few graphs using rules like $y = 2x + 1$ $y = x^2$

$x = y^2$ $x^2 + y^2 = 1$ etc but not all represent functions.

To understand why we need to define a few things.

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Def

We say that y is a function of x if each x value in the domain is paired to only one y value in the range.

We call x the independent variable and y the dependent variable.

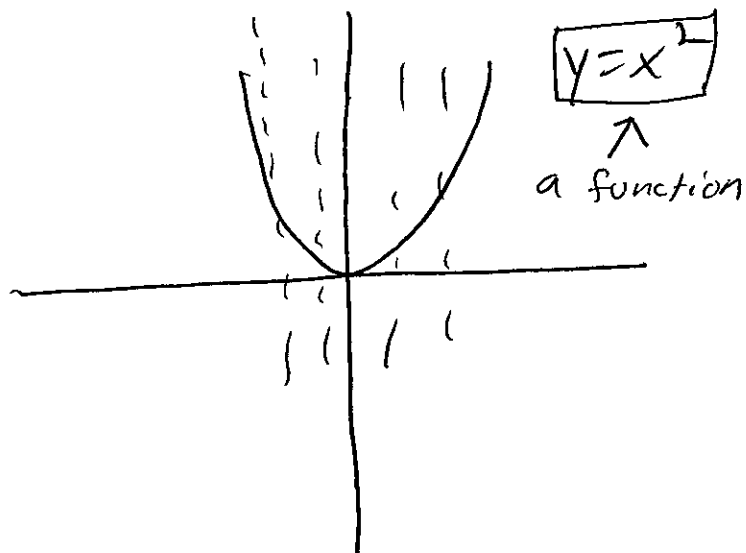
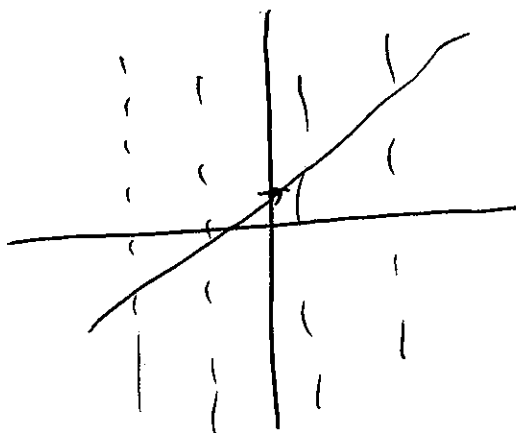
This leads us to the vertical line test
VLT: we can say y is a function of x if there are no vertical lines that cross the graph more than once.

This might make sense since if a vertical line did cross a graph twice one x would be paired with two y 's

This can be easily seen graphically

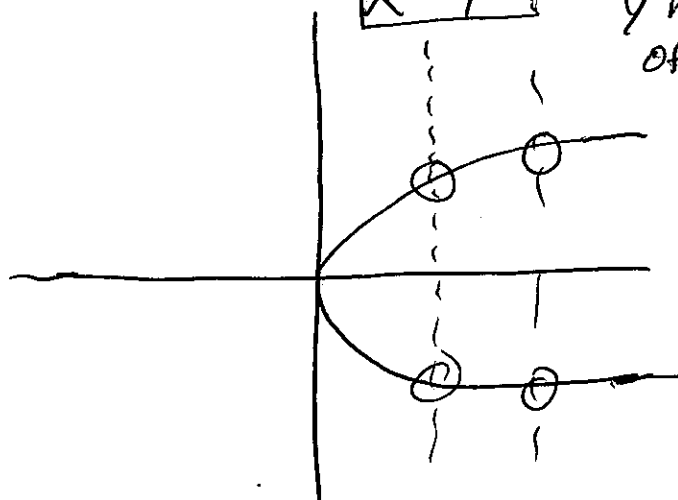
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$y = 2x + 1$ is a function



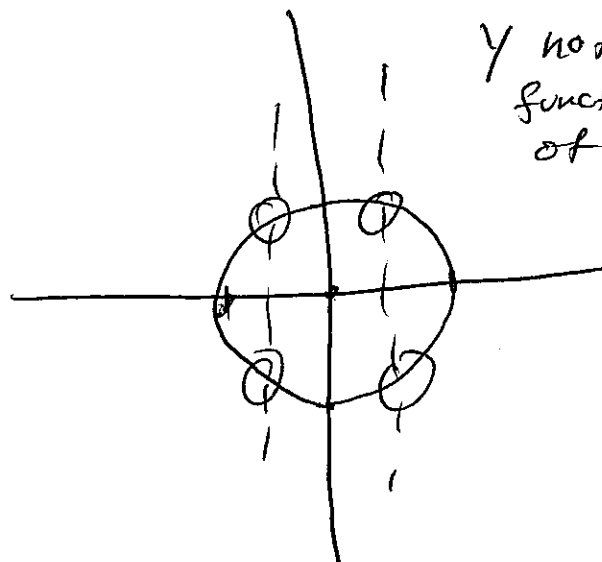
$x = y^2$

crosses more
than once
y not a function
of x

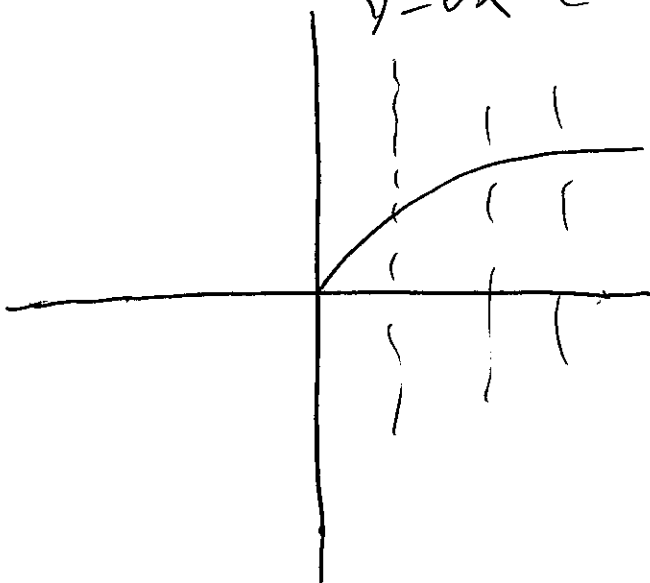


$x^2 + y^2 = 1$

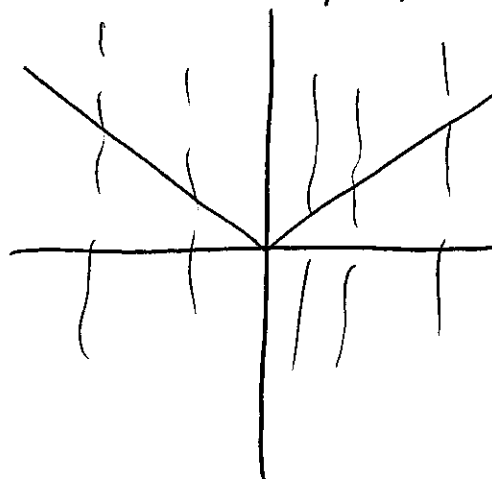
y not a
function
of x



$y = \sqrt{x}$ is a function



$y = |x|$ is a function



function notation $f(x)$ $g(x)$ $g(t)$ $h(z)$ (49)

Ex

$$f(x) = x^2 + 1$$

We can replace y with $f(x)$ $f(x)$ means what follows is a function of x we can use other variables as well for example

Function name $\rightarrow f(t) = t^2 + 1$ where f is a function of t
Independent variable $\rightarrow t$
for assigning values

$$h(z) = 2z + 6 \quad h \text{ is a function of } z$$

Using this notation makes it clear which is the dependent and which is the Independent variable

Evaluating functions

Let $f(x) = x^2 + 1$ find $f(2)$, $f(-3)$, $f(x+1)$

$$f(2) = 2^2 + 1 = 5 \leftarrow \text{replace } x \text{ with } 2$$

$$f(-3) = (-3)^2 + 1 = 10 \quad \text{replace } x \text{ with } -3$$

$$f(x+1) = (x+1)^2 + 1 \leftarrow \text{replace } x \text{ with } x+1$$

Piecewise Functions

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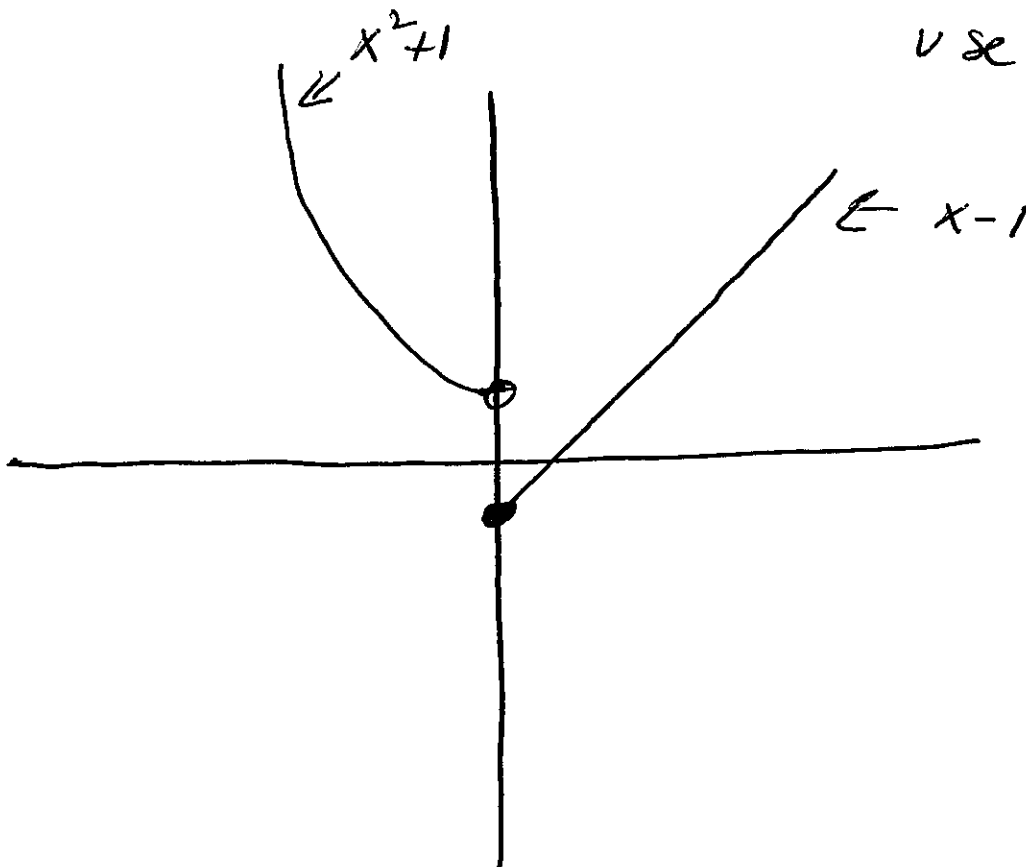
Sometimes we use two or more functions to define a single function. These are called piecewise functions since we use different functions for different parts of the domain.

Ex

when $x < 0$ we
use $x^2 + 1$

$$f(x) = \begin{cases} x^2 + 1 & x < 0 \\ x - 1 & x \geq 0 \end{cases}$$

↑ when $x \geq 0$ we
use $x - 1$



Given the function like #36 HW #/ (50)

$$f(x) = \begin{cases} x^2 - 2 & x < 1 \\ 2x^2 - 2 & x \geq 1 \end{cases}$$

Find

$f(-2)$: $-2 < 1$ so we use $x^2 - 2$

$$f(-2) = (-2)^2 - 2 = 4 - 2 = 2$$

$f(-1)$: $-1 < 1$ so we use $x^2 - 2$

$$f(-1) = (-1)^2 - 2 = 1 - 2 = -1$$

$f(2)$: $2 > 1$ so we use $2x^2 - 2$

$$f(2) = 2(2)^2 - 2 = 2(4) - 2 = 8 - 2 = 6$$

HW Example #37 HW 1

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$$f(x) = 8x - 5 \quad \text{find } f(a) \quad f(a+h)$$

$$\frac{f(a+h) - f(a)}{h}$$

Soln

$$f(a) = 8a - 5$$

$$f(a+h) = 8(a+h) - 5$$

$$\frac{f(a+h) - f(a)}{h} = \frac{8(a+h) - 5 - (8a - 5)}{h}$$

$$= \frac{\cancel{8a} + 8h - \cancel{5} - \cancel{8a} + \cancel{5}}{h} = \frac{8h}{h} = 8$$

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Finding Domain and range

For a given function there are 3 things to consider when finding the Domain

① Is the domain specified?

Ex $f(x) = x^2 + 1 \quad x \geq 0$

↑
here we are told
the domain is $x \geq 0$

② Are there any $\sqrt{\quad}$'s in the function
If so make sure what is underneath
is positive

Ex $f(x) = \sqrt{2x+1}$ to avoid taking
the square root of a negative number
we require $2x+1$ to be non-negative

so $2x+1 \geq 0$ solving for x tells
 $-1 \quad -1 \quad$ us ...

$$\boxed{x \geq -\frac{1}{2}} \leftarrow \text{our domain}$$

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③ Are there any variables in the denominator? If so find where it is zero and exclude the values from the domain

Ex | $f(x) = \frac{1}{x^2 - 1}$

we can't divide by zero so we check where $x^2 - 1 = 0$

$$x^2 - 1 = 0$$

$$x^2 = 1$$

$$\sqrt{x^2} = \sqrt{1}$$

$$x = \pm 1$$

} so when $x = 1$ or -1 we get zero on the bottom

So our domain is

all reals except $x = 1$ $x = -1$

In interval notation

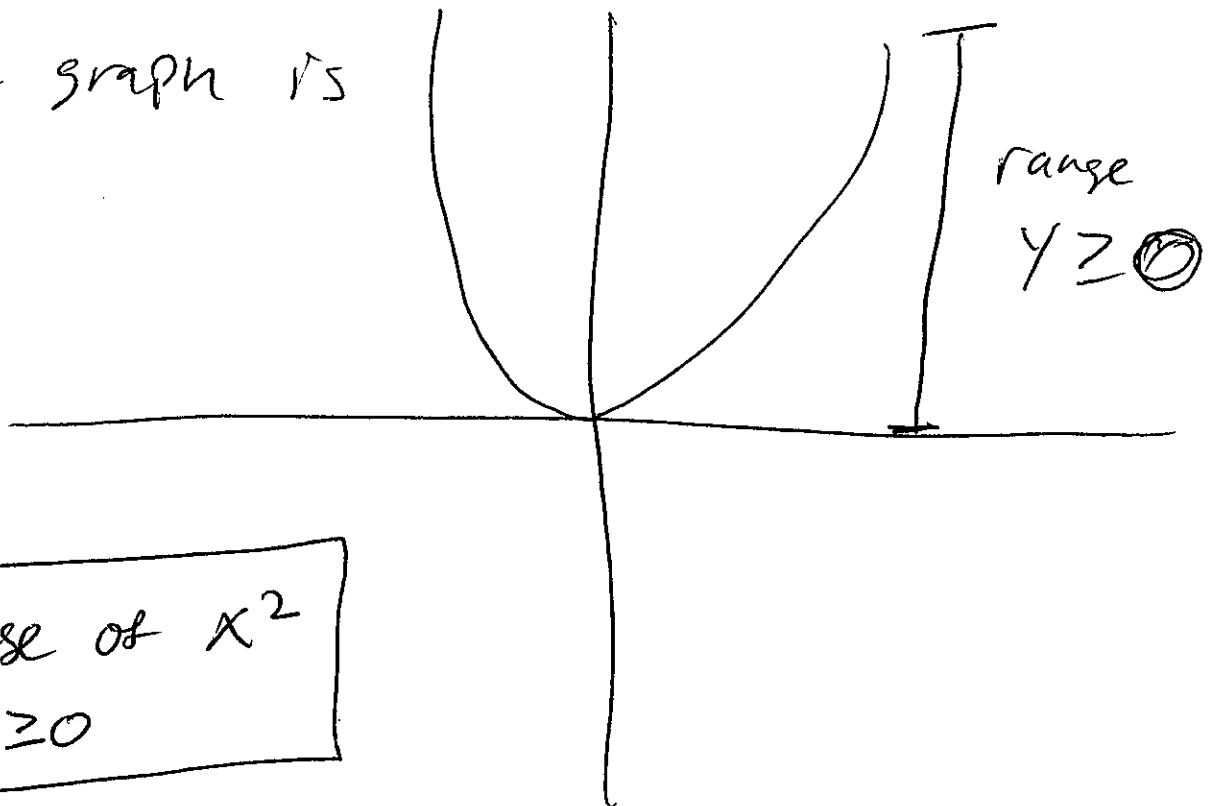
$$(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$$

Finding the range

The range of a function is the set of y -values it takes on.

For example $f(x) = x^2$ only takes positive numbers for x values since we are squaring everything so even though we can plug in negative numbers like $x = -4 \Rightarrow f(-4) = (-4)^2 = 16$ we only get positive numbers out

The graph is



Range of x^2
 $y \geq 0$

Hw example Like # 33 Hw # 1 (55)

Find the Domain and range for

$$f(x) = 3x^2 - 18$$

Domain

There is no specified domain $\sqrt{\quad}$'s or variables in the denominator so the domain is all real numbers ($\mathbb{R}$)

In interval notation $(-\infty, \infty)$

Range

Let's think about the smallest and largest possible y-values we can get out of $3x^2 - 18$

If we plug in a negative number say -2

we get $3(-2)^2 - 18 = 3(4) - 18 = 12 - 18$, we

get a positive value from x^2 ~~so~~ If we

plug in a positive number say 2

we get $3(2)^2 - 18 = 3(4) - 18 = 12 - 18$ we

get the same as when $x = -2$

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So for any positive or negative number we plug in we are adding that onto -18 . Thus the smallest y value we can get is -18 when $x=0$

$$3(0)^2 - 18 = -18$$

Choosing anything else gives us something larger

$$3(-2)^2 - 18 = 12 - 18 = -6$$

$$3(2)^2 - 18 = 12 - 18 = -6$$

~~$$3(-3)^2 - 18 = 27 - 18 = 9$$~~

$$3(-3)^2 - 18 = 27 - 18 = 9$$

etc. we notice we can get as large a y -value as we want as well. so

here the range is $-18 \leq y < \infty$

In interval notation

$$[-18, \infty)$$

HW example like #34 HW1

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Find the domain and range for

$$f(t) = \sqrt{t-31}$$

Domain

Here we have a $\sqrt{\quad}$ so we make sure what is underneath is non-negative.

$$t-31 \geq 0$$

$$+31 \quad +31$$

$$t \geq 31 \leftarrow \text{Domain}$$

In interval notation $[31, \infty)$

Range

Here we notice we cannot get negative numbers from $\sqrt{t-31}$ but at $t=31$

we get $\sqrt{31-31} = \sqrt{0} = 0$ and if we take

large t -values say ~~31~~ $\sqrt{131-31} = \sqrt{100} = 10$

we can get large y -values so here

our range is $y \geq 0$ or $[0, \infty)$

HW example

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like # 35 HW # 1

Find the Domain for $f(x) = \frac{x+13}{x^2-169}$

Soln

Here we have variables on the bottom of a fraction so we need to make sure that the bottom is never zero to find out we set the denominator equal to zero and exclude the values where it is

$$x^2 - 169 = 0 \Rightarrow x^2 = 169 \Rightarrow x = \pm \sqrt{169}$$

$\Rightarrow x = \pm 13$ so for our domain

we require ~~$x = 13$~~ $x \neq 13$ $x \neq -13$

In interval notation

$$(-\infty, -13) \cup (-13, 13) \cup (13, \infty)$$

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one last domain example

~~Step~~ Find the domain of

$$f(x) = \frac{1}{\sqrt{x+1}}$$

Here we have both a $\sqrt{\quad}$ and variables on the bottom. First make sure what is under the $\sqrt{\quad}$ is not negative

$$x+1 \geq 0$$

$$-1 \quad -1$$

$$x \geq -1$$

Find out where the bottom is zero and throw it out

$$x+1 = 0$$

$$-1 \quad -1$$

$$x = -1$$

$$\text{so } x \neq -1$$

combining both gives $x \neq -1$ $x \geq -1$

$$\Rightarrow x > -1$$

$\uparrow$ strictly greater since $x \neq -1$