

Combinations of Functions

If we have more than one function we can combine them to make other functions.

Suppose we have two functions $f(x)$ $g(x)$
Then we can define:

$$① (f+g)(x) = f(x) + g(x)$$

Ex

$$f(x) = 2x + 1 \quad g(x) = x^2 + x$$

$$(f+g)(x) = f(x) + g(x) = \underbrace{2x+1}_{f(x)} + \underbrace{x^2+x}_{g(x)} = x^2 + 3x + 1$$

$$② (f-g)(x) = f(x) - g(x)$$

$$\text{EX } f(x) = 2x + 1 \quad g(x) = x^2 + x$$

$$(f-g)(x) = f(x) - g(x) = \underbrace{2x+1}_{f(x)} - \underbrace{(x^2+x)}_{g(x)} = 2x + 1 - x^2 - x \\ = -x^2 + 3x + 1$$

$$③ (fg)(x) = f(x) \cdot g(x)$$

$$\text{EX } f(x) = 2x + 1 \quad g(x) = x^2 + x$$

$$(fg)(x) = f(x) \cdot g(x) = \underbrace{(2x+1)}_{f(x)} \underbrace{(x^2+x)}_{g(x)} \overset{\text{Foil}}{=} 2x^3 + x^2 - 2x^2 - x \\ = 2x^3 - x^2 - x$$

④ $\frac{f}{g}(x) = \frac{f(x)}{g(x)} \quad g(x) \neq 0$

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Ex $f(x) = 2x + 1 \quad g(x) = x^2 + x$

$$\frac{f}{g}(x) = \frac{f(x)}{g(x)} = \frac{2x+1 \leftarrow f(x)}{x^2+x \leftarrow g(x)}$$

⑤ $f \circ g(x) = f(g(x)) \leftarrow \text{composition of functions}$

Ex

replace x with g(x)

$$f \circ g(x) = f(g(x)) = 2(g(x)) + 1$$

$$= 2(\underbrace{x^2+x}_{g(x)}) + 1 = 2x^2 + 2x + 1$$

replace x with f(x)

Ex

$$g \circ f(x) = g(f(x)) = (f(x))^2 + (f(x))$$

$$= (2x+1)^2 + (2x+1) = 4x^2 + 4x + 1 + 2x + 1$$

$$= 4x^2 + 6x + 1$$

FOIL
(2x+1)(2x+1)

you can think of composition of functions as putting one function inside another

HW Example like # 54 HW #1

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Suppose $f(x) = x^9 + 7$ $g(x) = \sqrt{x}$ Then

$$f(g(x)) = (g(x))^9 + 7 = \boxed{(\sqrt{x})^9 + 7}$$

replace every $x^{\text{in } f(x)}$ with $g(x)$

replace $g(x)$ with what it is equal to

$$g(f(x)) = \sqrt{f(x)} = \boxed{\sqrt{x^9 + 7}}$$

replace every x in $g(x)$ with $f(x)$

replace $f(x)$ with what it is equal to

HW Example like # 55

Express $y(x) = 3x^3 - 4$ as a composition of $f(u)$ $g(x)$ where $y(x) = f(g(x))$

Soln

Let

$$\boxed{g(x) = x^3 \quad f(u) = 3u - 4}$$

Check
Then

$$f(g(x)) = 3(g(x)) - 4 = 3x^3 - 4$$

replace u with $g(x)$

replace $g(x)$ with what it is = to

HW example like #56 HW #1

If $f(g(x)) = 4(x^2 + 4)^6$ and $g(x) = x^2 + 4$

find ~~$f(g(x))$~~ $f(x)$,

$$f(g(x)) = 4(\underbrace{x^2 + 4}_{g(x)})^6 = 4(g(x))^6 = f(g(x))$$

So we replaced
what was an
x with g(x)

$$\Rightarrow f(x) = 4(x)^6 = 4x^6$$

HW Example like #57

Suppose $V(r) = \frac{4}{3}\pi r^3$ $P(t) = V(3t)$ Find

$P(4)$

$$P(t) = V(3t) = \frac{4}{3}\pi (3t)^3 = \frac{4}{3}\pi 3^3 t^3$$

replace r
with 3t

$$= \frac{4}{3}\pi \cancel{27} t = 4 \cdot 9 \pi t = 36\pi t$$

$$P(t) = 36\pi t \Rightarrow P(4) = 36\pi(4)$$

$$= (36 \times 4)\pi = \boxed{144\pi}$$

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In class examples

$$\text{If } f(x) = 3x + 2 \quad g(x) = x^2 + 2x + 1$$

find

$$(f+g)(x) = \overbrace{3x+2}^{f(x)} + \overbrace{x^2+2x+1}^{g(x)}$$

$$= \boxed{x^2 + 5x + 3}$$

$$(f-g)(x) = \overbrace{3x+2}^{f(x)} - \overbrace{x^2+2x+1}^{g(x)}$$

$$= 3x + 2 - x^2 - 2x - 1$$

$$= \boxed{-x^2 + x + 1}$$

distribute
to each term

$$fg(x) = \underbrace{(3x+2)}_{f(x)} \underbrace{(x^2+2x+1)}_{g(x)}$$

$$= (3x+2)x^2 + (3x+2)(2x) + (3x+2)(1)$$

$$= 3x^3 + 2x^2 + 6x^2 + 4x + 3x + 2$$

$$= \boxed{3x^3 + 8x^2 + 7x + 2}$$

$$f \circ g(x) = f(g(x))$$

$$= 3(g(x)) + 2$$

$$= 3(x^2 + 2x + 1) + 2$$

$$= 3x^2 + 6x + 3 + 2$$

$$= \boxed{3x^2 + 6x + 5}$$

$$\frac{f}{g}(x) = \frac{\overbrace{3x+2}^{f(x)}}{\overbrace{x^2+2x+1}^{g(x)}}$$

$$g \circ f(x) = g(f(x))$$

$$= (f(x))^2 + 2(f(x)) + 1$$

$$= (3x+2)^2 + 2(3x+2) + 1$$

$$= 9x^2 + 5x + 4 + 6x + 4 + 1$$

$$= \boxed{9x^2 + 11x + 9}$$