

## Sec 6.5

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Dividing Polynomials and synthetic division.

We have been dealing with rational expressions like

$$\frac{x^2 - 2x - 3}{(x - 3)} \leftarrow \text{division}$$

In this case we have that:

$$= \frac{(x^2 - 2x - 3)}{(x - 3)} = \frac{\cancel{(x - 3)}(x + 1)}{\cancel{(x - 3)}} = x + 1 \quad x \neq 3$$

but what if what is on top does not factor easily, or does not have any factors of the bottom in its factored form.

In these cases we will use polynomial long division.

Reminder of long division.

Ex

$$\begin{array}{r} 235 \\ 28 \overline{) 6584} \\ \underline{56} \phantom{00} \\ 98 \phantom{00} \\ \underline{84} \phantom{00} \\ 144 \phantom{00} \\ \underline{140} \phantom{00} \\ 4 \end{array}$$

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$$\Rightarrow 6584 \div 28 = 235 + \frac{4}{28} = 235 + \frac{1}{7}$$

We take much the same approach with poly nomials  
where powers of  $x$  are like the place values or  
10's 100's 1000's for numbers.

Ex

$$x-1 \overline{) 2x^2 + 4x + 8}$$

Here we look at the highest  
powers of  $x$  in both the  
dividend and divisor

$$\text{dividend: } 2x^2$$

$$\text{divisor: } x$$

Now think what do I multiply  
 $x$  by to get  $2x^2$ ?

$$(2x)x = 2x^2 \text{ so we take}$$

$2x$  and put it above

$4x$  in the dividend since  
those are the same power  
of  $x$ ,

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$$\begin{array}{r}
 \phantom{x-1} \overline{2x} \\
 x-1 \overline{) 2x^2 + 4x + 8} \\
 \underline{-(2x^2 - 2x)} \quad \leftarrow \text{note my} \\
 \phantom{0} + 6x + 8 \quad \text{signs}
 \end{array}$$

Then take  $2x(x-1) = 2x^2 - 2x$   
 and subtract it from  
 the dividend, Remember  
 to put ( ) around it  
 You are subtracting the  
 whole thing  
 $-(2x^2 - 2x) = -2x^2 + 2x$

So now we have:

$$\begin{array}{r}
 \phantom{x-1} \overline{2x} \\
 x-1 \overline{) 2x^2 + 4x + 8} \\
 \underline{-(2x^2 - 2x)} \\
 \phantom{0} + 6x + 8
 \end{array}$$

Now we look at the  
 highest power of the  
 divisor:  $x$  and of  
 what is left after subtracting  
 $6x$  and think what  
 do I multiply  $x$  by  
 to get  $6x$

$6(x) = 6x$  So we put  
 a 6 above the constant  
 term in the dividend,  
 and subtract

$$6(x-1) = 6x - 6$$

So we have

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$$\begin{array}{r} \phantom{X-1} \overline{2x+6} \\ X-1 \overline{) 2x^2+4x+8} \\ \underline{-(2x^2-2x)} \phantom{+8} \\ 6x+8 \\ \underline{-(6x-6)} \\ 0x+14 \end{array}$$

Now here we see that we can't multiply  $x$  by anything to get 14 so we are done and 14 is our remainder.

So we can now say

$$\frac{2x^2+4x+8}{x-1} = \overset{\text{Quotient:}}{\underset{\downarrow}{2x+6}} + \frac{14}{x-1}$$

← remainder  
← divisor

We can check our work by multiplying

$$\begin{aligned} (x-1) \left( \frac{2x+6}{x-1} + \frac{14}{x-1} \right) &= (x-1)(2x+6) + \frac{\cancel{x-1}(14)}{\cancel{x-1}} \\ &= x^2 - 2x + 6x - 6 + 14 = x^2 + 4x + 8 \quad \checkmark \end{aligned}$$

Example

$$\text{Divide } \underline{-2x+3} \overline{) 10x^4 - 13x^3 + 8x - 7x^2 + 4}$$

Looking at  $-2x$  I can multiply by  $-5x^3$  to get  $10x^4$

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So  $-5(x^3)(-2x+3) = +10x^4 - 15x^3$  Now subtract it

$$\begin{array}{r}
 -5x^3 + x^2 + 2x - 1 \\
 -2x+3 \overline{) 10x^4 - 13x^3 - 7x^2 + 8x + 4} \\
 \underline{-10x^4 + 15x^3} \phantom{- 7x^2 + 8x + 4} \\
 2x^3 - 7x^2 + 8x + 4 \\
 \underline{-2x^3 + 3x^2} \phantom{+ 8x + 4} \\
 -4x^2 + 8x + 4 \\
 \underline{+4x^2 - 6x} \phantom{+ 4} \\
 2x + 4 \\
 \underline{-2x + 3} \\
 7
 \end{array}$$

7  
↑  
remainder.

$$-x^2(-2x) = 2x^3$$

So take

$$\begin{aligned}
 -x^2(-2x+3) \\
 = 2x^3 - 3x^2
 \end{aligned}$$

$$\text{Then } 2x(-2x) = -4x^2$$

So take

$$\begin{aligned}
 2x(-2x+3) \\
 = -4x^2 + 6x
 \end{aligned}$$

$$\text{Then } (-1)(-2x) = 2x$$

So

$$\begin{aligned}
 (-1)(-2x+3) \\
 = 2x - 3
 \end{aligned}$$

So we have

$$\frac{10x^4 - 13x^3 - 7x^2 + 8x + 4}{-2x + 3} =$$

$$\boxed{-5x^3 - x^2 + 2x - 1 + \frac{7}{-2x+3}}$$

$$\underline{x^2 + 2x - 3} \overline{) x^4 + 6x^3 + 6x^2 - 10x - 3}$$

What if powers of  $x$  are missing

like  $x-1 \overline{) x^3 - 2}$

rewrite with all powers

$$\underline{x-1} \overline{) x^3 + 0x^2 + 0x - 2}$$

Then divide.

Using division to factor.

Factor completely

$$x^3 - 7x + 6$$

Let's see what happens when we plug in  $x=1$

$$1^3 - 7(1) + 6 = -6 + 6 = 0 \quad \text{Thus } x-1 \text{ is a}$$

factor of  $x^3 - 7x + 6$  so  $\underline{x-1} \overline{) x^3 - 7x + 6}$

will divide evenly.

lets div/ up

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$$\begin{array}{r} x^2 + x - 6 \\ x-1 \overline{) x^3 + 0x^2 - 7x + 6} \\ \underline{-x^3 + x^2} \phantom{+ 6} \\ x^2 - 7x + 6 \\ \underline{-x^2 + x} \phantom{+ 6} \\ -6x + 6 \\ \underline{6x - 6} \\ 0 \end{array}$$

So

$$x^3 - 7x + 6 = (x-1)(x^2 + x - 6)$$

and

$$x^2 + x - 6 = (x+3)(x-2)$$

So

$$x^3 - 7x + 6 = (x-1)(x+3)(x-2)$$

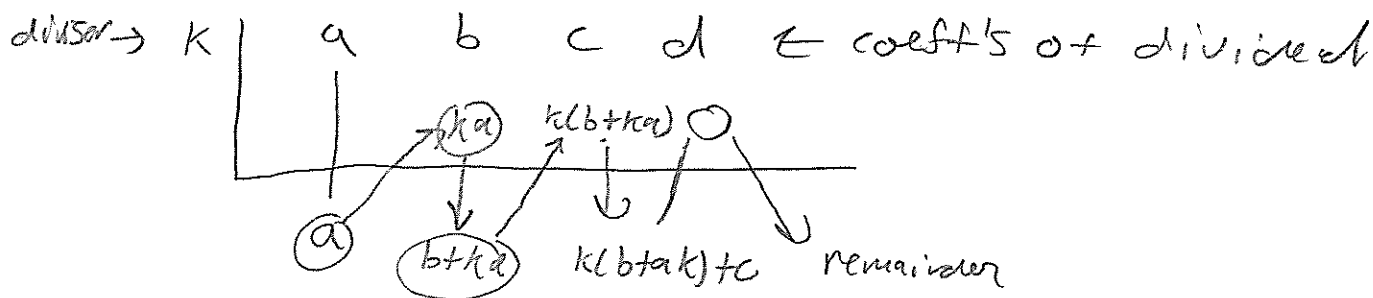
So if we know a factor we can divide  
by it reduce the degree to something  
we can factor. you will do this alot  
in 1050.

## Synthetic division

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This only works when you are dividing by something like  $x-k$

To divide something like  $ax^3+bx^2+cx+d$  by  $x-k$ :



Going up! multiply by  $k$

Going down add terms.

Ex

divide  $x^4 - 10x^2 - 2x + 3$  by  $x+3$

Note no  $x^3$  term so we rewrite as

$$x^4 + 0x^3 - 10x^2 - 2x + 3 \quad \text{also}$$

We need  $x-k$  but we have  $x+3$  so

