

Sec 7.5 Solving equations involving radicals

To set the general idea let's start with

$2=2$ would you agree that

$2^3=2^3$ or $2^4=2^4$? Clearly yes

What about if $\sqrt{2}=\sqrt{2}$, well

Clearly $(\sqrt{2})^2=(\sqrt{2})^2=2$, What if

We say $x=2$? then is $x^2=2^2$?

Yes! They are equal so integer powers of two equal things are still equal,

What if we ask:

Solve for x :

$$\sqrt{x+1}=4$$

How could we solve this?

well if

$$\sqrt{x+1} = 4 \quad \text{then} \quad (\sqrt{x+1})^2 = 4^2$$

$$\Rightarrow x+1=16 \Rightarrow x=16-1 \Rightarrow x=15$$

How would we solve

$$\sqrt[3]{4x+1} = 2$$

$$\text{well } \sqrt[3]{4x+1} = (4x+1)^{1/3} \text{ so } \left((4x+1)^{1/3}\right)^3$$

$$= \left((4x+1)^{1/3}\right)^3 = (4x+1)^{3/3} = 4x+1$$

So if we raise both sides to the third power then we can get x out of the radical and solve

so:

$$(4x+1)^{1/3} = 2 \Rightarrow \left((4x+1)^{1/3}\right)^3 = 2^3$$

$$\Rightarrow 4x+1=8 \Rightarrow 4x=7 \Rightarrow \boxed{x=7/4}!$$

In class clicker examples

Solve:

$$1) \sqrt[3]{2x} = -2$$

$$(\sqrt[3]{2x})^3 = (-2)^3$$

$$\frac{2x}{2} = \frac{-8}{2}$$

$$x = -4$$

$$2) (2x)^{2/3} = 4$$

$$((2x)^2)^{1/3} = 4$$

$$((2x)^2)^{1/3})^3 = 4^3$$

$$(2x)^2 = 64$$

$$\sqrt{(2x)^2} = \sqrt{64}$$

$$2x = \pm 8$$

$$2x = 8$$

$$2x = -8$$

$$x = 4$$

$$x = -4$$

Story Problem!

A couple drop an egg from their hot air balloon. it hits the ground with a speed of 35 m/s ^{← meters} how high up were they?

solve

$$v = \sqrt{2gh}$$

$$g = 9.8 \text{ m/s}^2$$

$$v = \sqrt{2(9.8)h}$$

we know $v = 35$ so

$$35 = \sqrt{19.6h}$$

$$35^2 = (\sqrt{19.6h})^2$$

$$35^2 = 19.6h$$

$$\frac{35^2}{19.6} = \frac{19.6h}{19.6}$$

$$h = \frac{35^2}{19.6} \approx 62.5 \text{ m}$$

Some more types or "radical" equations

Ex

$$\sqrt{z} - 4 = 0$$

$$\sqrt{z} = 4$$

$$(\sqrt{z})^2 = 4^2$$

$$\boxed{z = 16}$$

Ex

$$\sqrt{-4y} = 4$$

$$(\sqrt{-4y})^2 = 4^2$$

$$-4y = 16$$

$$y = \frac{16}{-4}$$

$$\boxed{y = -4}$$

work $y \leq 0$

for this so that

$$-4y \geq 0$$

They solve correctly.

Ex

$$\sqrt{2t-7} = -5$$

No solution

Since

$$\sqrt{\text{anything}} \geq 0$$

What If we have 2 radicals

Ex ← They try first to
solve:

$$\sqrt{3t+1} - \sqrt{t+5} = 0$$

get a radical on each side

$$\sqrt{3t+1} = \sqrt{t+5}$$

$$(\sqrt{3t+1})^2 = (\sqrt{t+5})^2$$

$$3t+1 = t+5$$

$$3t - t = 5 - 1$$

$$2t = 4$$

$$t = \frac{4}{2} = 2$$

Ex

$$2\sqrt[3]{10-3x} = \sqrt[3]{2-x}$$

$$(2\sqrt[3]{10-3x})^3 = (\sqrt[3]{2-x})^3$$

$$\Rightarrow 2^3(10-3x) = (2-x)$$

$$\Rightarrow 8(10-3x) = (2-x)$$

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$$\Rightarrow \begin{array}{rcl} 80 - 24x & = & 2 - x \\ +x & & +1 \end{array}$$

$$\begin{array}{rcl} 80 - 23x & = & 2 \\ -80 & & -80 \end{array}$$

$$-23x = -78$$

$$23x = 78 \quad \leftarrow \text{multiply both sides by } -1$$

$$\boxed{x = \frac{78}{23}}$$

Ex

$$\sqrt{3x+7} = x+3$$

Here we will square both sides and get a quadratic to solve!

$$(\sqrt{3x+7})^2 = (x+3)^2$$

$$\begin{array}{rcl} 3x+7 & = & x^2+6x+9 \\ -7 & & -7 \end{array}$$

$$\begin{array}{rcl} 3x & = & x^2+6x+2 \\ -3x & & -3x \end{array}$$

$$0 = x^2 + 3x + 2$$

$$x^2 + 3x + 2 = 0 \quad \text{factor}$$

$$(x+1)(x+2) = 0$$

$$\boxed{x = -1 \quad x = -2}$$

Ex

$$\sqrt{z+2} = 1 + \sqrt{z}$$

$$(\sqrt{z+2})^2 = (1 + \sqrt{z})^2$$

$$\begin{array}{ccccccc} z+2 & = & z & + & 2\sqrt{z} & + & 1 \\ -z & -1 & -z & & -1 & & \end{array}$$

← From here get radical term alone on one side

$$1 = 2\sqrt{z}$$

← square both sides again

$$1^2 = (2\sqrt{z})^2$$

$$1 = 4z$$

$$\boxed{z = \frac{1}{4}}$$

Ex

$$\sqrt[3]{x+4} = \sqrt{6-x}$$

Here we have different roots so let's convert to rational powers

$$(x+4)^{1/3} = (6-x)^{1/2}$$

First I'll raise both sides to the third power

$$\left((x+4)^{1/3}\right)^3 = \left((6-x)^{1/2}\right)^3$$

$$(x+4) = (6-x)^{3/2}$$

Now both to the second power

$$(x+4)^2 = \left((6-x)^{3/2}\right)^2$$

$$(x+4)^2 = (6-x)^{3/2 \cdot 2}$$

$$(x+4)^2 = (6-x)^3 \quad \Leftarrow \text{multiply out}$$

$$x^2 + 8x + 16 = x^3 - 6x^2 - 108x + 216 \quad \Leftarrow \text{get everything on one side}$$

$$x^3 - 7x^2 - 116x + 200 = 0$$

Ex

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$$\frac{4}{\sqrt{x}} = 3\sqrt{x} - 4$$

Times both sides by $\sqrt{x}$

$$\sqrt{x} \frac{4}{\sqrt{x}} = (3\sqrt{x} - 4)\sqrt{x}$$

$$4 = 3\sqrt{x}\sqrt{x} - 4\sqrt{x}$$

$$4 = 3x - 4\sqrt{x}$$

← Get radical alone

$$4 - 3x = -4\sqrt{x}$$

← Square both sides

$$(4 - 3x)^2 = (-4\sqrt{x})^2$$

$$\begin{array}{r} 9x^2 - 24x + 16 = 16x \\ -16x \quad -16x \end{array}$$

← Set equal to zero

$$9x^2 - 40x + 16 = 0$$

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Imaginary #'s

In the past we have said $\sqrt{-1}$ does not exist. That it is against the rules of math to take the $\sqrt{}$ of a negative number. Lets change the rules.

we define

$$\sqrt{-1} = i \quad \text{we just created}$$

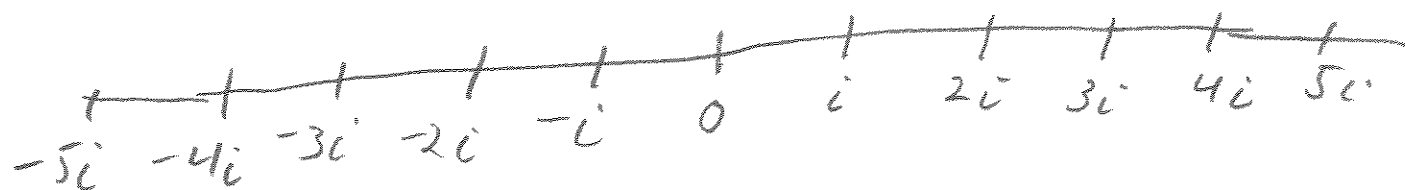
a new number! Mathemagic!

In doing so we created a whole new number system called the imaginary numbers.

In the imaginary number system i is line 1 just like $1+1=2$ $i+i=2i$

In fact there is a whole new number line

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Yay! we have a whole new system to do math on!

lets find

$$\sqrt{-4}, \sqrt{-36}, \sqrt{-16}$$

$$\sqrt{-4} = \sqrt{4}\sqrt{-1} = \pm 2i$$

$$\sqrt{-36} = \sqrt{36}\sqrt{-1} = \pm 6i$$

$$\sqrt{-16} = \sqrt{16}\sqrt{-1} = \pm 4i$$

In fact if c is a real #

$$\sqrt{-c} = \sqrt{c}\sqrt{-1} = \pm \sqrt{c}i$$

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EXAMPLES

Ex 1 $\sqrt{-9} = \sqrt{9(-1)} = \sqrt{9}\sqrt{-1} = \pm 3i$ or $3i$ principle root

Ex 2

Write $\sqrt{-7}$ in i form

$$\sqrt{-7} = \sqrt{7(-1)} = \pm \sqrt{7}i \quad \sqrt{7}i \text{ principle root}$$

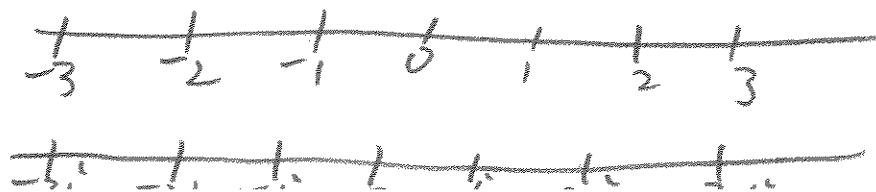
Ex 3

$$\sqrt{\frac{-20}{49}} = \sqrt{\frac{20}{49}(-1)} = \sqrt{\frac{20}{49}}\sqrt{-1} = \pm \frac{2\sqrt{5}}{7}i$$

$\frac{2\sqrt{5}}{7}$ principle

Complex #'s

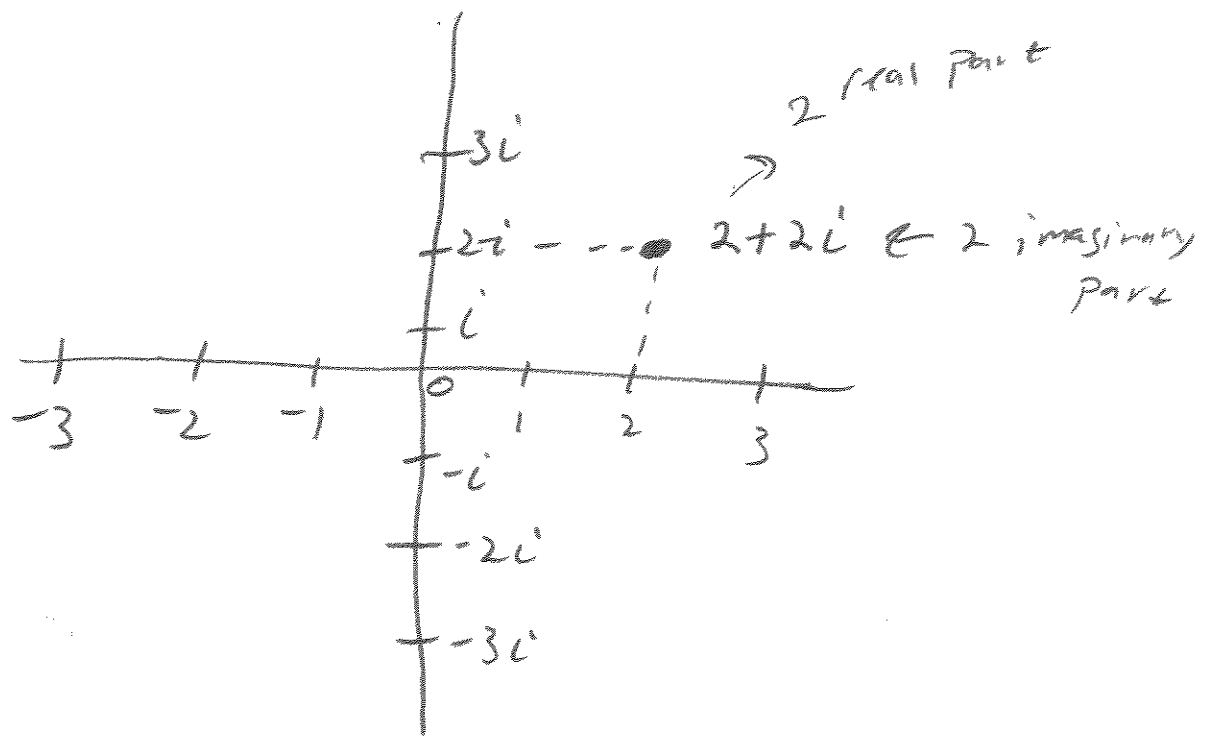
Lets look at the real line and imaginary lines



Notice they share 0 in common

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Let's create a new number system by
Joining the two lines together at 0



Complex #'s are #'s of the form

$$\begin{array}{cc} a + bi \\ \uparrow \quad \uparrow \\ \text{Real part} \quad \text{Imaginary part} \end{array}$$

They live in the complex plane shown
above but we don't need to worry
too much about that.

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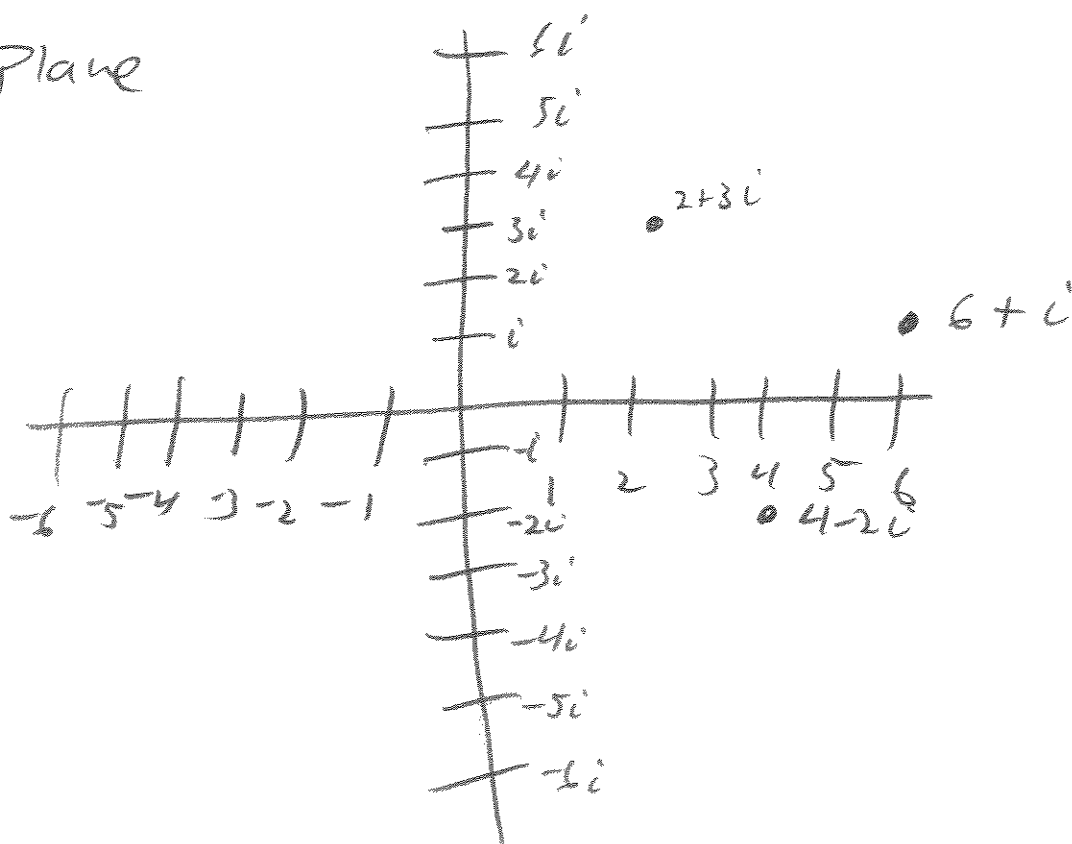
lets learn how to add and multiply complex
#s

Ex

$(2+3i) + (4-2i)$ $\Leftarrow$ we combine like terms
real and imaginary

$$(2+4) + (3-2)i = 6 + i$$

In the Plane



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clicker Example

Ex $(-10+2i) + (4-7i) = ?$

$$= (-10+4) + (2i-7i) = -6 - 5i$$

Ex

$$(-1+i) - 2 - \sqrt{2} = ?$$

$$-1+i - \sqrt{2} - \sqrt{2}i$$

$$= (-1-\sqrt{2}) + (i-\sqrt{2}i)$$

$$= -(1+\sqrt{2}) + (1-\sqrt{2})i$$

Ex

Try with dickens

$$6 - (3-4i) + 2i = ?$$

$$(6-3) + (-4i+2i)$$

$$= \boxed{3 - 2i}$$

Ex multiplying complex #'s

$$(9-2i)\sqrt{-4} = ?$$

$$(9-2i)2i = 9(2i) - 2i(2i) = 18i - 4(i)^2$$

but $i^2 = -1$ so

$$= 18i - 4(-1) = 18i + 4 = \boxed{4 + 18i}$$

Ex

$$(3+5i)(3+5i) \leftarrow \text{distribute}$$

$$(3+5i)(3) + (3+5i)5i$$

$$= 9 + 15i + 15i + 25i^2$$

$$= 9 + 15i + 15i + 25(-1) \quad \text{since } i^2 = -1$$

$$= \cancel{9} \cancel{+ 15i} \quad 9 - 25 + 15i + 15i$$

$$= \boxed{-16 + 30i}$$

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Ex

calculate $(3+5i)(3-5i)$ $\leftarrow$ have conjugates

$$= 3^2 + 15i - 15i - 5^2(i^2)$$

$$= 9 + 25 = \textcircled{34} \leftarrow \text{real!}$$

$3+5i$, $3-5i$ are complex conjugates

when multiplying complex conjugates
you always get a real number.

dividing complex #'s

Ex

$$\frac{4i}{5-3i}$$

Trick is to multiply
by the complex conjugate
of the denominator on the
top and bottom,

$$\frac{4i(5+3i)}{(5-3i)(5+3i)} = \frac{20i-12}{25+9} = \frac{20i-12}{34} \quad (25)$$

$$= \frac{-12}{34} + \frac{20}{34}i \leftarrow \text{standard form}$$

Ex

$$\frac{5}{3+i} + \frac{1}{3-i}$$

we need a common denominator

$(3+i)(3-i)$ sounds good

$$(3+i)(3-i) = 3^2 - 3i + 3i - i^2$$

$$= 9 - (-1) = 9 + 1 = 10$$

$$\frac{5(3-i)}{(3+i)(3-i)} + \frac{(3+i)}{(3+i)(3-i)} = \frac{15-5i+3+i}{(3+i)(3-i)}$$

$$= \frac{18-4i}{10} = \boxed{\frac{18}{10} - \frac{4}{10}i}$$