

So $x=+2$ and $y=-3$

(127)

So our solution is $(2, -3)$

Question 3

Method 2

Elimination ~~Sec 4.1~~ Sec 4.1

This method Goes back to the real number

Property: If $a=b$ then $a+c=b+c$

consider the system of equations

$$\begin{cases} 3x + 2y = 4 \\ 5x - 2y = 8 \end{cases}$$

Look at

$$3x + 2y = 4$$

certainly we could add 8 to both sides
and the equation will still be true

$$3x + 2y + 8 = 4 + 8$$

but from the second equation

$8 = 5x - 2y$ so let's replace
8 on the left hand side with $5x - 2y$

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$$3x + 2y + 8 = 4 + 8$$

||
 $5x - 2y$

so

$$3x + 2y + (5x - 2y) = 4 + 8$$

combine like terms

$$3x + 5x + 2y - 2y = 12$$

$8x = 12$ woah! we got rid of the y's and can solve

$$\Rightarrow x = \frac{12}{8} = \frac{3}{2} \quad \text{for } x!$$

so $x = \frac{3}{2}$ plugging that in for y we get

$$3\left(\frac{3}{2}\right) + 2y = 4$$

$$\frac{9}{2} + 2y = 4$$

$$9 + 2y = 4$$

$$2y = -1/2$$

$$y = -1/4$$

Lets look at a bit differently

$$\begin{cases} 3x + 2y = 4 \\ 5x - 2y = 8 \end{cases}$$

Lets just add the equations together

$$\begin{array}{r} 3x + 2y = 4 \\ + 5x - 2y = +8 \\ \hline 8x + 0y = 12 \end{array}$$

$$8x = 12$$

$$x = 12/8 = 3/2$$

so we were able to get rid of the y's which made life easy.

The point is is that we can add two equations in a system of equations to get a new equation which is easier to solve. It does not change the problem because we are only employing the property that if $a=b$ then $a+c=b+c$.

When we have to multiply before adding,

Ex

$$\begin{cases} 4x - 5y = 13 \\ 3x - y = 7 \end{cases}$$

Here if we JUST add our equations we get

$$\begin{array}{r} 4x - 5y = 13 \\ + 3x - y = 7 \\ \hline \end{array}$$

$$7x - 6y = 20 \quad \leftarrow \text{Nothing went away} \quad \therefore$$

Lets look carefully at

$3x - y = 7$ If I multiply both sides by
 -5 the equation is still true...

$$-5(3x - y) = -5(7)$$

$$-15x + 5y = -35$$

Now look at

$$\begin{cases} 4x - 5y = 13 \\ -15x + 5y = -35 \end{cases} \quad \leftarrow \text{Still same as } 3x - y = 7 \text{ since we multiplied both sides by } -5$$

Now lets add them

$$\begin{array}{r} 4x - 5y = 13 \\ + -15x + 5y = -35 \\ \hline -11x + 0y = -22 \end{array}$$

$$-11x = -22$$

Bam! take that y terms

$$\Rightarrow x = \frac{-22}{-11} = 2$$

$$\text{so } x = 2$$

Then we can plug x into either of the original equations to solve for y lets do both.

$$4(2) - 5y = 13$$

$$8 - 5y = 13$$

$$-5y = 13 - 8 = 5$$

$$-5y = 5$$

$$\boxed{y = -1}$$

$$-3(2) - y = 7$$

$$\text{or } 6 - y = 7$$

$$-y = 7 - 6 = 1$$

$$-y = 1$$

$$\boxed{y = -1}$$

Sweet!

So with elimination we aim to multiply one of the given equations by a constant such that when we add it to the other we remove one of the variable terms,

RQ Question 4 in class.

Lets do examples so you can get a feel for it.

Ex 1

$$\begin{cases} 4x + 3y = 8 \\ x - 2y = 13 \end{cases}$$

Notice If I multiply the second equation by -4 and add it to the first the x terms will go away

$$-4(x - 2y) = -4(13)$$

$$-4x + 8y = -52$$

$$+4x + 3y = 8$$

$$0x + 11y = -44$$

$$\Rightarrow 11y = -44$$

$$\Rightarrow \boxed{y = \frac{-44}{11} = -4}$$

Then we can solve for x using either original equation

$$\text{I'll choose } x - 2y = 13 \quad \text{if } y = -4$$

$$x - 2(4) = 13$$

$$x + 8 = 13$$

$$x = 13 - 8 = 5$$

$$x = 5$$

$$\text{So } x = 5 \quad y = -4 \Rightarrow (x, y) = (5, -4)$$

EX 2

$$\begin{cases} 3x + 9y = 8 \\ 2x + 6y = 7 \end{cases}$$

This time I should multiply both equations as none of the coefficients are multiples of each other,

I know $3 \cdot 2 = 6$ so $3 \cdot 2x = 6x$
and $2x \cdot 3 = 6x$

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So I will do

$$2(3x + 9y) = (8) 2$$

$$+ -3(2x + 6y) = (7) 3$$

$$\Rightarrow 6x + 18y = 16$$

$$-6x - 18y = 21$$

$$\hline 0x + 0y = -5$$

$\Rightarrow 0 = -5$ $\Leftarrow$ woops this is an untrue
statement This means
there is no solution!

NO SOLUTION

Ex 3

$$\begin{cases} -2x + 6y = 3 \\ 4x - 12y = -6 \end{cases}$$

I notice $2(-2x) = -4x$
so that would cancel
with the $4x$ in eq 2
so I'll multiply eq 1 by
2 and add to eq 2,

$$2(-2x + 6y) = -2.3$$

$$+4x - 12y = -6$$

$$-4x + 12y = 6$$

$$+4x - 12y = -6$$

$$0x + 0y = 0$$

$\Rightarrow 0=0$ $\Leftarrow$ this is a true statement

but we have no x's or y's this means the equations are the same equation and

so we have infinitely many solutions.

Notice

$$-2(-2x + 6y) = (2)(-2)$$

$$= 4x - 12y = -6 \quad \text{which is eq 2}$$

so eq₁ was a multiple of eq₂

this also means we have infinitely many solutions.

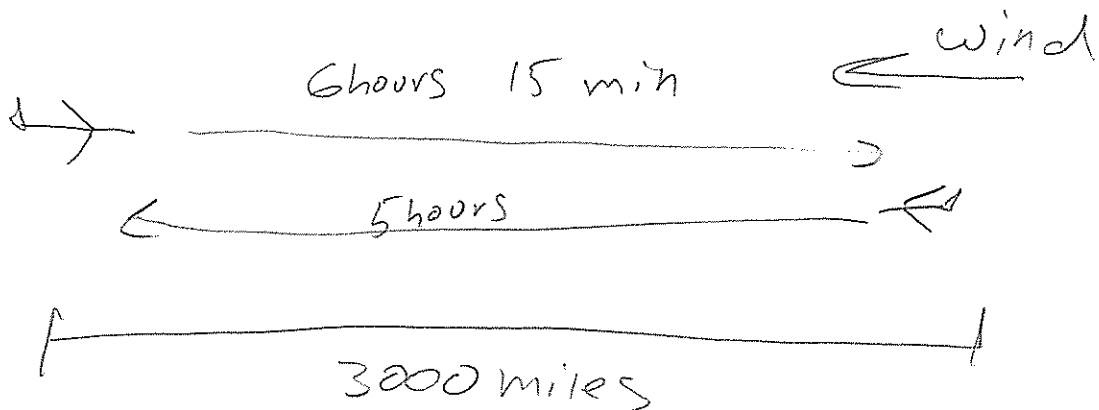
Linear equations in 2 variables Story/ Problem examples.

Air Speed

An Airplane flying into a head wind travels 3000 miles in 6 hours and 15 minutes. On the return flight the same distance is traveled in 5 hours. Find the speed of the ~~wind~~ plane in still air and the speed of the wind assuming they both remain constant for the trips.

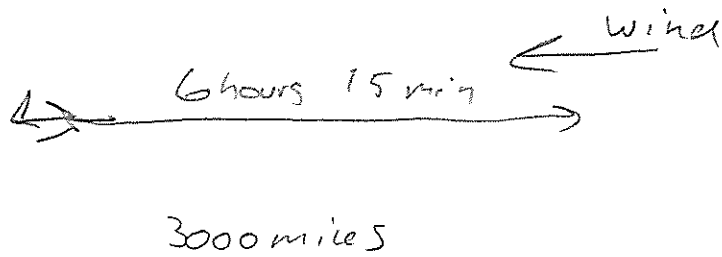
Soln

Picture



We will need 2 equations!

For trip 1 the speed of the plane is the still air speed ~~plus~~ minus the wind speed

Trip 1

First $d = vt$ and $v = v_{\text{plane}} - v_{\text{wind}}$

and $t = 6 \text{ hours and } 15 \text{ min} = 6.25 \text{ hours}$

$$= 6 + .25 = 6 + \frac{1}{4} = \frac{24}{4} + \frac{1}{4} = \frac{25}{4} \text{ hours}$$

so and $d = 3000 \text{ miles}$

$$3000 = (v_{\text{plane}} - v_{\text{wind}}) \frac{25}{4}$$

so $3000 = \frac{25}{4}(v_{\text{plane}} - v_{\text{wind}})$

Trip 2

Now the wind is at the back of the plane so $d = vt$ $v = v_{\text{plane}} + v_{\text{wind}}$

and $t = 5 \text{ hours}$ and $d = 3000 \text{ miles}$

so $d = vt = (v_{\text{plane}} + v_{\text{wind}})t$

$$\Rightarrow 3000 = (v_{\text{plane}} + v_{\text{wind}})5$$

so we have 2 equations and 2 unknowns

$$(1) \begin{cases} 3000 = \frac{25}{4}(v_{\text{plane}} - v_{\text{wind}}) \\ 3000 = 5(v_{\text{plane}} + v_{\text{wind}}) \end{cases}$$

So lets solve the system:

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$$(1) \quad 3000 = \frac{25}{4} (V_{\text{plane}} - V_{\text{wind}})$$

$$(2) \quad 3000 = 5 (V_{\text{plane}} + V_{\text{wind}})$$

For eq₁, I will first multiply both sides by

$$\frac{4}{25}$$

$$\frac{4}{25}(3000) = \frac{\cancel{4}}{25}(\frac{25}{\cancel{4}})(V_{\text{plane}} - V_{\text{wind}})$$

$$\Rightarrow \frac{12,000}{25} = V_{\text{plane}} - V_{\text{wind}}$$

$$\Rightarrow 480 = V_{\text{plane}} - V_{\text{wind}}$$

Then for eq₂ I will divide everything by 5

$$\frac{3000}{5} = \frac{\cancel{5}(V_{\text{plane}} + V_{\text{wind}})}{\cancel{5}}$$

$$600 = V_{\text{plane}} + V_{\text{wind}}$$

Now lets look at our system

$$\begin{cases} 480 = V_{\text{plane}} - V_{\text{wind}} \\ 600 = V_{\text{plane}} + V_{\text{wind}} \end{cases}$$

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Lets solve using elimination I see a $-V_{wind}$
 and $+V_{wind}$ in eq 1 and 2 respectively
 so I will add the two equations and
 then I can find V_{plane}

$$\begin{array}{rcl} 1180 & = & V_{plane} - V_{wind} \\ + 600 & = & + V_{plane} + V_{wind} \\ \hline 1080 & = & 2V_{plane} \end{array}$$

$$\Rightarrow \frac{1080}{2} = \frac{2V_{plane}}{2}$$

$$\Rightarrow 540 = V_{plane}$$

Now we take V_{plane} and plug it back into
 either of the top two equations to get
 V_{wind} . I choose equation (2)

$$\begin{array}{rcl} 600 & = & 540 + V_{wind} \\ -540 & - & -540 \end{array}$$

$$\boxed{60 = V_{wind}}$$

and

$$\boxed{540 = V_{plane}}$$

EX 2
The admission fee at an amusement park is \$1.50 for children and \$4 for adults, on a certain day 261 people entered the park and the admission fees collected totalled \$654. How many children and adults were admitted.

Soln (Setup)

Let C be the # of children and A be the number of Adults. Then

The park makes off the children is

$$1.5C = 1.5 \text{ dollars} \times \# \text{ children who go,}$$

for the Adults we get

$$4(A) = 4 \text{ dollars} \times \# \text{ of Adults}$$

The total amount of fees then is

$$1.5C + 4A = 654$$

The total # of people who attend the park was 261 which has to equal the # of adults + children

so

$$C + A = 261$$

Thos we have the system of equations:

$$\begin{cases} 1.5C + 4A = 654 \\ C + A = 261 \end{cases}$$

or

$$\begin{cases} \frac{3}{2}C + 4A = 654 \\ C + A = 261 \end{cases}$$

← just converting
1.5 to $\frac{3}{2}$

kicking it up a notch.

Linear systems in 3 variables!

Lets consider the system of 3 equations and 3 unknowns

$$\begin{cases} 4x + y - 3z = 11 \\ 2x - 3y + 2z = 9 \\ x + y + z = -3 \end{cases}$$

Pretty Scary!