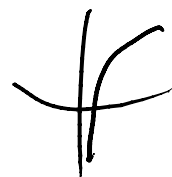


5_12_14

Limits and Continuity

Find the following Limits:

$$\lim_{x \rightarrow \infty} \frac{1}{x^n} = 0$$



$$\lim_{x \rightarrow \infty} \frac{3x^2 + 2x}{x^2 + 4} = \frac{\infty}{\infty}$$

$$\lim_{x \rightarrow 0} (x^{\frac{1}{2}} \ln(x))$$

$$\begin{aligned} & \lim_{x \rightarrow \infty} \frac{\frac{1}{x^2} (3x^2 + 2x)}{\frac{1}{x^2} (x^2 + 4)} \\ &= \lim_{x \rightarrow \infty} \frac{(3 + \frac{2}{x})}{(1 + \frac{4}{x^2})} = \frac{3}{1} = 3 \end{aligned}$$

$$\lim_{x \rightarrow 0} \frac{\ln(x)}{\frac{1}{x}} = \frac{\infty}{\infty}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim_{x \rightarrow 0} -\frac{x}{1} = 0$$

$$= \lim_{x \rightarrow 0} -\frac{x}{1} = 0$$

$$\lim_{x \rightarrow -3} \frac{x+3}{x^2+5x+6}$$

$$\lim_{x \rightarrow 0} x^+ = L = 1$$

$$\lim_{x \rightarrow -3} \frac{(x+3)}{(x+3)(x+2)}$$

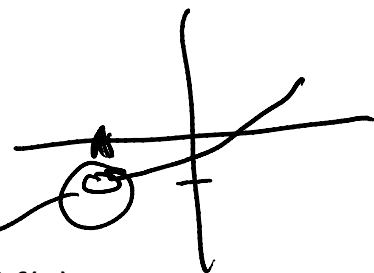
$$= \lim_{x \rightarrow -3} \frac{1}{x+2} = -\frac{1}{1}$$

$$\begin{aligned} \ln(L) &= \lim_{x \rightarrow 0} \ln(x^+) \\ &= \lim_{x \rightarrow 0} x \ln(x) \end{aligned}$$

$$\begin{aligned} \ln(L) &= 0 \\ e^0 &= 1 \\ L &= e^0 = 1 \end{aligned}$$

Continuity:

$$f(x) = \frac{x+3}{(x+3)(x+2)} = -1$$



A Function $f(x)$ is continuous at $x=a$ if the limit L of f at a exists and $f(a)=L$

Which of the following functions are continuous at the specified value?

$$f(x) = \begin{cases} x^2 & x < 1 \\ \sqrt{x} & x \geq 1 \end{cases} \quad x=1$$

$$f(x) = \begin{cases} 2x+1 & x < 2 \\ -x & x \geq 2 \end{cases} \quad x=2$$

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} \sqrt{x} = 1$$

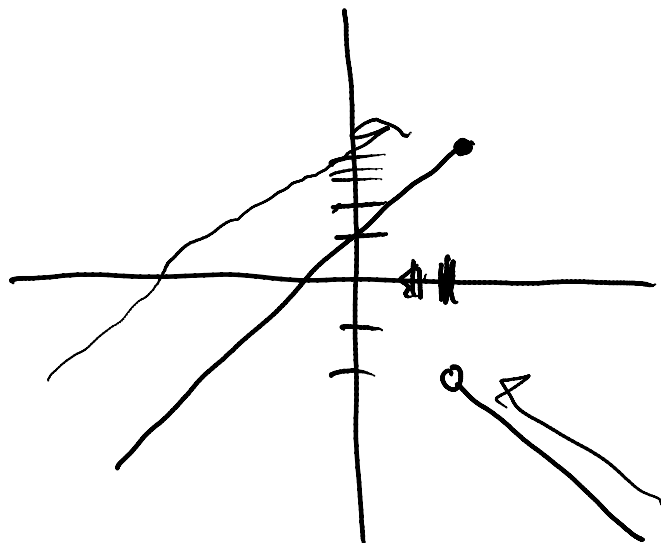
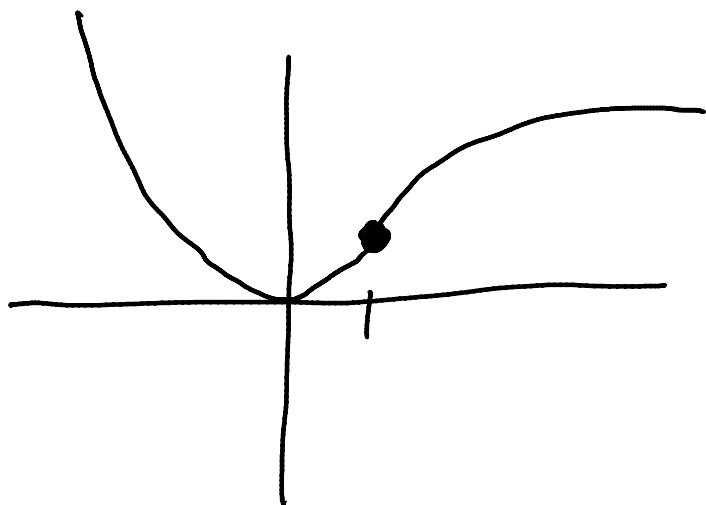
$$\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} -x = -2$$

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} x^2 = 1$$

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} 2x+1 = 5$$

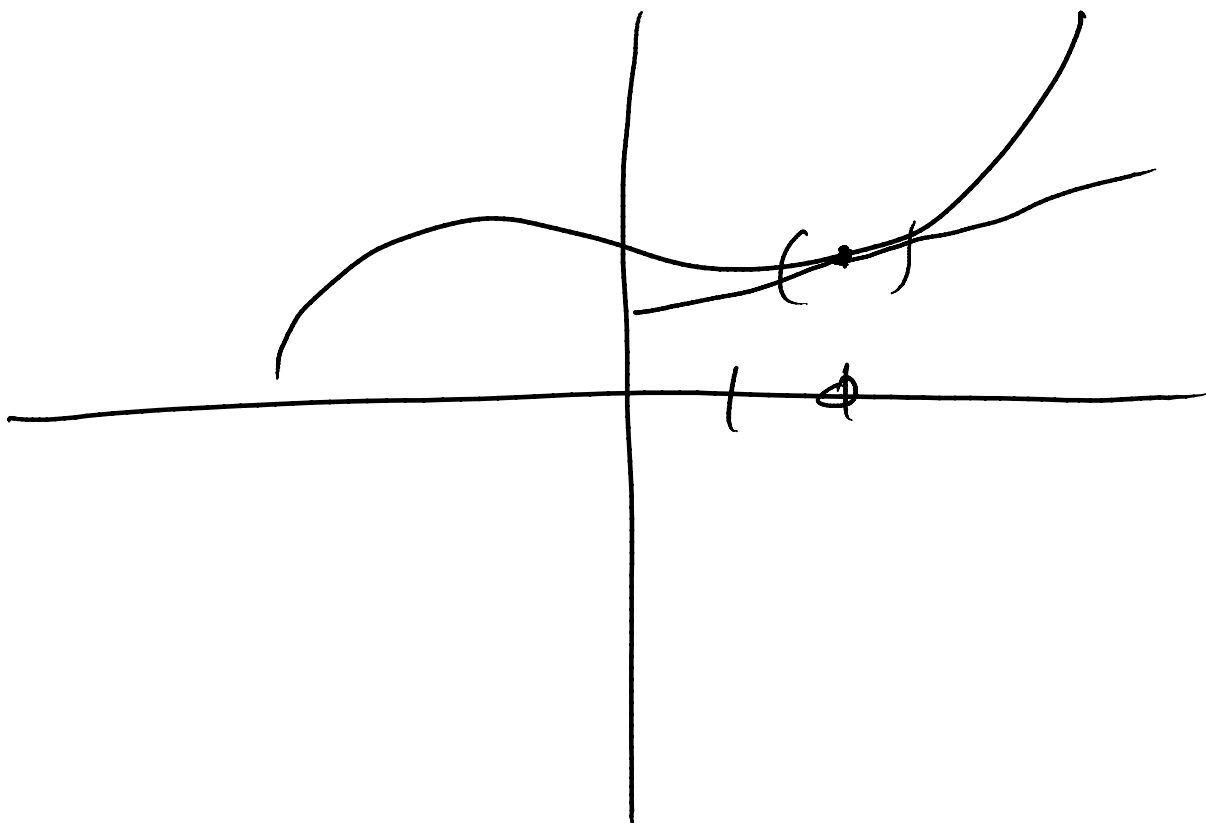
$$f(1) = \sqrt{1} = 1 \quad \checkmark$$

f is cont at 1



Derivatives!

A Derivative gives the rate of change of a function



Rules for Derivatives

Constant Multiple Rule

$$\frac{d}{dx}(c f(x)) = c \frac{d}{dx} f(x)$$

$$f(x) = x^2$$

$$f'(x) = 2x$$

$$\frac{d}{dx} 4x^2 = \frac{d}{dx} 4 f(x)$$

Power Rule

$$\frac{d}{dx} x^n = n x^{n-1}$$

Ex $\frac{d}{dx} x^5 = 5x^4$ $\frac{d}{dx} x^{-2} = -2x^{-3}$ $\frac{d}{dx} x^1 = 1x^0 = 1$

Sum Rule

$$\frac{d}{dx} (\underline{f(x)} + \underline{g(x)}) = \underline{\frac{d}{dx} f(x)} + \underline{\frac{d}{dx} g(x)}$$

Ex

$$\frac{d}{dx} (\underline{x^2} + \underline{x^4} - \underline{3x^2} + \underline{x^{-1/2}}) = 2x + 4x^3 - 6x - \frac{1}{2}x^{-3/2}$$

Product Rule

$$\frac{d}{dx} (\underline{f(x)} \underline{g(x)}) = \underline{f(x) \frac{d}{dx} g(x)} + \underline{g(x) \frac{d}{dx} f(x)}$$

Ex $\frac{d}{dx} (\underline{x^2} \underline{\cos(x)}) = \underline{x^2 \frac{d}{dx} \cos(x)} + \underline{\cos(x) \frac{d}{dx} x^2}$

$$= 2x \cos(x) + (-\sin(x)) x^2$$

Quotient Rule

$$\frac{d}{dx} \left(\frac{f(x)}{g(x)} \right) = \frac{g(x) \frac{d}{dx} f(x) - f(x) \frac{d}{dx} g(x)}{(g(x))^2} \quad \text{when } g(x) \neq 0$$

Ex

$$\frac{d}{dx} \left(\frac{x^3}{\sin(x)} \right) = \frac{3x^2(\sin(x)) - (\cos(x))x^3}{\sin^2(x)}$$

$$(x^2)' = 2x \cdot \frac{d}{dx} x$$

Chain Rule

$$f'(g(x)) \cdot g'(x)$$

$$\frac{d}{dx} (f(g(x))) = \frac{d}{dx} (f(g(x))) \cdot \frac{d}{dx} g(x)$$

$$f(x) = x^2$$

$$g(x) = \cos(2x+1)$$

Ex

$$\frac{d}{dx} (\cos^2(2x+1)) = \frac{d}{dx} ((\cos(2x+1))^2)$$

$\cos(x) = f(x)$
 $\nearrow g(x) = 2x+1$

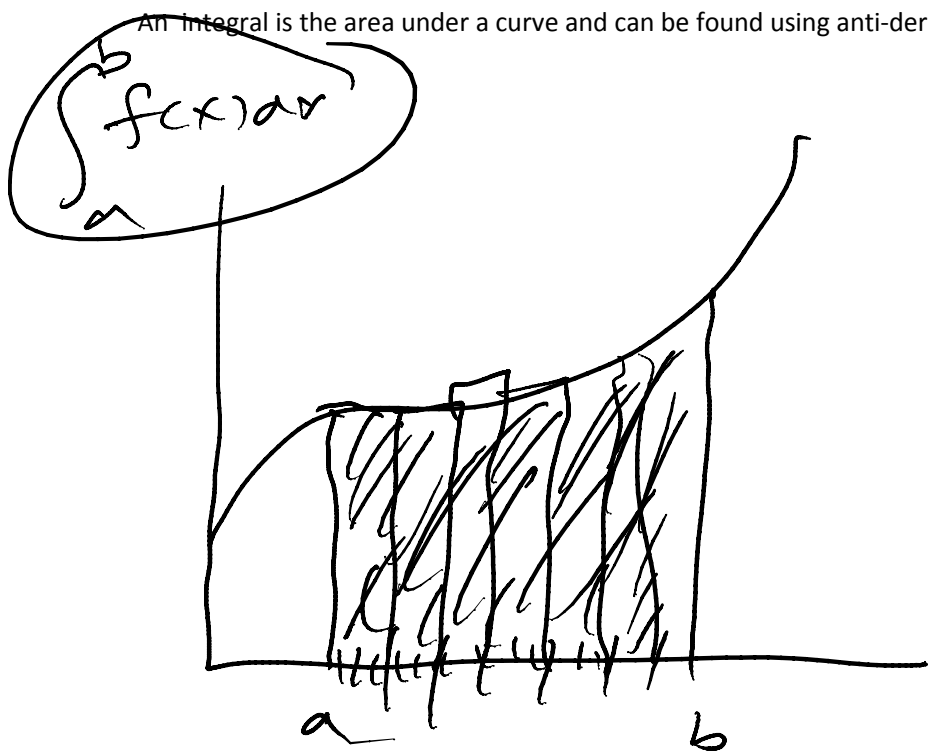
$$= 2(\cos(2x+1)) \cdot \frac{d}{dx} \cos(2x+1)$$

$$= 2\cos(2x+1) \cdot (-\sin(2x+1)) \cdot \frac{d}{dx} (2x+1)$$

$$= 2 \cos(2x+1), (-\sin(2x+1))(2)$$

Integrals

An integral is the area under a curve and can be found using anti-derivatives.



Methods of Integration

Fundamental Theorem of Calculus

$$\int_a^b f(x) dx = F(b) - F(a)$$

F is the ant. derivative of f

Anti-Derivative Rule

$$\int x^n = \frac{x^{n+1}}{n+1} + C$$

Ex

$$\int x^3 dx = \frac{x^4}{4} + C \quad \int \frac{1}{\sqrt{x}} dx = \int \frac{1}{x^{1/2}} = \int x^{-1/2} = \frac{x^{1/2}}{1/2} + C$$

Sum Rule

$$\int f(x) + g(x) dx = \int f(x) dx + \int g(x) dx$$

Ex

$$\begin{aligned} \int x^2 + 2x + x^{-1} dx &= \int x^2 + \int 2x + \int x^{-1} = \int \frac{1}{x} = \\ &= \frac{x^3}{3} + \frac{2x^2}{2} + \ln(x) \end{aligned}$$

Integration by parts

$$\int u dv = u \cdot v - \int v du$$

Ex

$$\int x \sin(x) dx =$$

- 1 Inverse
 - 2 Log
 - 3 Algebraic (x^3)
 - 4 Trig
 - 5 Exp

$$\int u \cdot v - \int v du$$

$$u = x$$

$$dv = \sin(x)$$

$$du = dx$$

$$v = -\cos(x)$$

$$x \cos(x) - \int -\cos(x) dx$$

$$-x \cos(x) + \int \cos(x) dx$$

$$= -x \cos(x) + \sin(x) + C$$

u-substitution

$$\frac{d}{dx} F(g(x)) = f(g(x)) g'(x)$$

$$\int f(g(x)) g'(x) dx = \int f(u) du \quad \text{when } u = g(x)$$

Ex

$$\int x e^{x^2} dx =$$

$u = x^2$

$$\frac{du}{dx} = 2x \quad dx$$

$$du = 2x dx$$

$$\int x e^u \frac{du}{2x} = \frac{1}{2} \int e^u du$$

$\frac{du}{2x} = dx$

$$= \frac{1}{2} e^u + C \rightarrow \frac{1}{2} e^{x^2} + C$$

Implicit Differentiation and Relating Rates:

Find $\frac{dy}{dx}$ if $y + x^2 = xy + y^2$ $y(x)$

$$\frac{d}{dx}(y + x^2) = \frac{d}{dx}(xy + y^2)$$

$$\frac{d}{dx} y + \frac{d}{dx} x^2 = \frac{d}{dx} xy + \frac{d}{dx} y^2$$

$$\frac{dy}{dx} + 2x = y + x \frac{dy}{dx} + 2y \frac{dy}{dx}$$

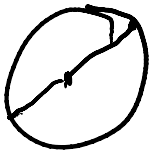
$$\begin{array}{ccccccc} \frac{dy}{dx} & +2x & = & y & +x & \frac{dy}{dx} & +2y & \frac{dy}{dx} \\ -\frac{dy}{dx} & -y & & y & & & -\frac{dy}{dx} & \\ \hline & & & & & & \frac{dy}{dx} & \end{array}$$

$$2x - y = -\frac{dy}{dx} + x\frac{dy}{dx} + 2y\frac{dy}{dx}$$

$$2x - y = \underline{(-1 + x + 2y)} \frac{dy}{dx}$$

$$\frac{2x - y}{-1 + x + 2y} = \frac{dy}{dx}$$

If a snow ball melts so that its surface area decreases at a rate of $1 \text{ cm}^2/\text{min}$ find the rate at which the diameter decreases when the diameter is 10 cm



$$SA = \frac{4}{3} \pi r^3$$

$$d = 2r$$

$$r = \frac{d}{2}$$

$$SA = \frac{4}{3} \pi \left(\frac{d}{2}\right)^3$$

$$\frac{d}{dt}(SA) = \frac{d}{dt} \frac{4}{3} \pi \frac{d^3}{8}$$

$$\frac{d(SA)}{dt} = \frac{4}{3} \pi \frac{1}{8} \frac{d}{dt}(d^3)$$

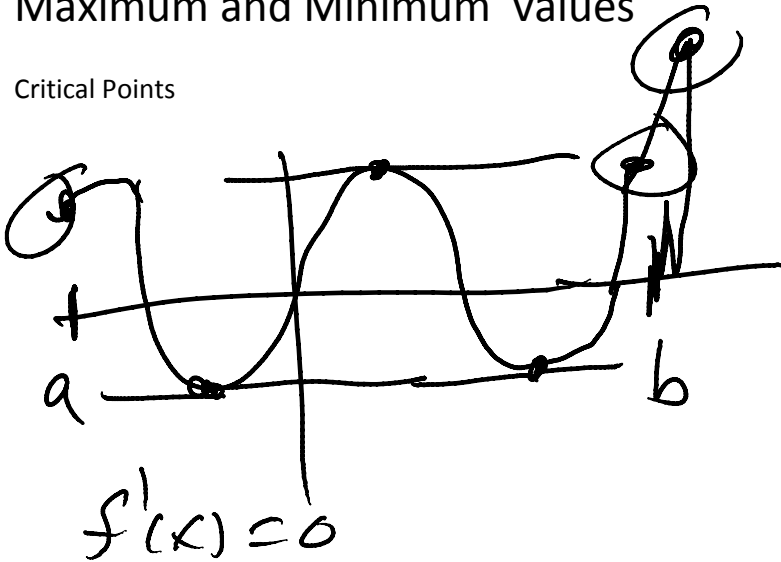
$$\rightarrow \frac{dSA}{dt} = \frac{4}{24} \pi 3 d^2 \cdot \left(\frac{dd}{dt}\right)$$

$$1 = \frac{1}{6} \pi 3 (10^2) \frac{dd}{dt}$$

$$\frac{dd}{dt} = \frac{1}{\frac{1}{6} \pi 3 (100)}$$

Maximum and Minimum values

Critical Points



at critical points

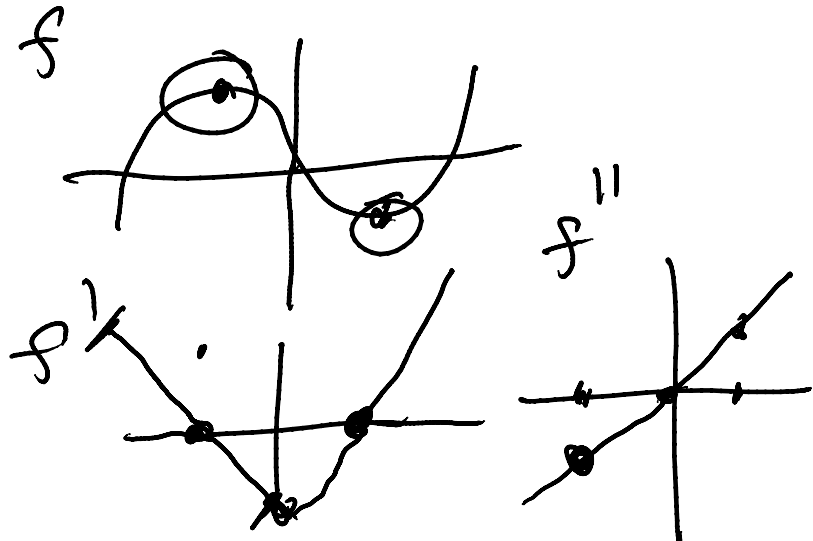
Where the derivative
 $= 0$ (max mins)

where f is not diff

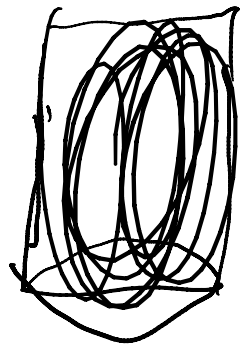
Second Derivative Test

$$x_c \quad \text{min} \\ f''(x_c) > 0$$

$$f''(x_c) < 0 \quad \text{max}$$



Find the minimal surface area required for a cylindrical can to contain 355 cm^3 (12 fl oz) of liquid.



$$S = 2\pi r h + 2\pi r^2$$

$$V = \pi r^2 h$$

$$355 = \pi r^2 h$$

$$\frac{355}{\pi r^2} = h$$

$$S = 2\pi \frac{355}{\pi r^2} r + 2\pi r^2$$

$$S = \frac{710}{r} + 2\pi r^2$$

$$\frac{dS}{dr} = -\frac{710}{r^2} + 4\pi r = 0$$

$$\rightarrow -710 + 4\pi r^3 = 0$$

$$4\pi r^3 = 710$$

$$r^3 = \frac{710}{4\pi}$$

$$r = \left(\frac{710}{4\pi} \right)^{1/3}$$

$$h = \frac{355}{\pi r^2}$$

$$\left| \frac{710}{r} = 710 \left(\frac{1}{r} \right) = 710 r^{-1} = 710 (-1) r^{-2} \right.$$

$$\left| \frac{710}{r} = 710 \left(\frac{1}{r} \right) = 710 r^{-1} = 710 (-1) r^{-2} \right.$$