

Limits and Continuity

Find the following Limits:

$$\lim_{x \rightarrow \infty} \frac{3x^2 + 2x}{x^2 + 4}$$

$$\lim_{x \rightarrow 0} x \ln(x)$$

$$\lim_{x \rightarrow -3} \frac{x + 3}{x^2 + 5x + 6}$$

$$\lim_{x \rightarrow 0} x^+$$

Continuity:

A Function $f(x)$ is continuous at $x=a$ if the limit L of f at a exists and $f(a)=L$

Which of the following functions are continuous at the specified value?

$$f(x) = \begin{cases} x^2 & x < 1 \\ \sqrt{x} & x \geq 1 \end{cases}$$

$$f(x) = \begin{cases} 2x+1 & x < 2 \\ -x & x \geq 2 \end{cases}$$

Derivatives!

A Derivative gives the rate of change of a function

Rules for Derivatives

Constant Multiple Rule

$$\frac{d}{dx}(c f(x)) = c \frac{d}{dx} f(x)$$

Power Rule

$$\frac{d}{dx} x^n = n x^{n-1}$$

Ex

$$\frac{d}{dx} x^5 =$$

$$\frac{d}{dx} x^{-2} =$$

$$\frac{d}{dx} x =$$

Sum Rule

$$\frac{d}{dx} (f(x) + g(x)) = \frac{d}{dx} f(x) + \frac{d}{dx} g(x)$$

Ex

$$\frac{d}{dx} x^2 + x^4 - 3x^2 + x^{-1/2} =$$

Product Rule

$$\frac{d}{dx} f(x)g(x) = f(x)\frac{d}{dx} g(x) + g(x)\frac{d}{dx} f(x)$$

Ex

$$\frac{d}{dx} (x^2 \cos(x)) =$$

when $g(x) \neq 0$

Quotient Rule

$$\frac{d}{dx} \frac{f(x)}{g(x)} = \frac{g(x) \frac{d}{dx} f(x) - f(x) \frac{d}{dx} g(x)}{(g(x))^2}$$

Ex

$$\frac{d}{dx} \left(\frac{x^3}{\sin(x)} \right) =$$

Chain Rule

$$\frac{d}{dx} f(g(x)) = \frac{d}{dx} (f(g(x))) \cdot \frac{d}{dx} g(x)$$

Ex

$$\frac{d}{dx} (\cos^2(2x+1))$$

Integrals

An integral is the area under a curve and can be found using anti-derivatives.

Methods of Integration

Fundamental Theorem of Calculus

Anti-Derivative Rule

$$\int x^n = \frac{x^{n+1}}{n+1} + C$$

Ex

$$\int x^3 dx =$$

$$\int \frac{1}{\sqrt{x}} dx =$$

Sum Rule

$$\int f(x) + g(x) dx = \int f(x) dx + \int g(x) dx$$

Ex

$$\int x^2 + 2x + x^{-1} dx =$$

Integration by parts

$$\int v du = u \cdot v - \int u dv$$

Ex

$$\int x \sin(x) dx =$$

u-substitution

$$\int f(g(x)) g'(x) dx = \int f(u) du \quad \text{when } u = g(x)$$

Ex

$$\int x e^{x^2} dx =$$

Implicit Differentiation and Relating Rates:

Find $\frac{dy}{dx}$ If $y + x^2 = xy + y^2$

If a snow ball melts so that its surface area decreases at a rate of $1\text{cm}^2/\text{min}$ find the rate at which the diameter decreases when the diameter is 10 cm

Maximum and Minimum values

Critical Points

Second Derivative Test

Find the minimal surface area required for a cylindrical can to contain 355 cm^3 (12 fl oz) of liquid .