

# Practice Final

Math 1320 001 Summer 2014

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Name and Unid:

by writing my name I swear by the honor code

**Read all of the following information before starting the exam:**

**Note this is the practice exam ignore point values on problems there are extras but the exam will look similar**

- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Justify your answers algebraically whenever possible to ensure full credit. Sketch all relevant graphs and explain all relevant mathematics.
- Circle or otherwise indicate your final answers.
- You MAY NOT use a calculator or notes. You may use the formulas provided.
- This test has 11 problems and one bonus problem and is worth 100 points, plus some extra credit at the end. It is your responsibility to make sure that you have all of the pages!
- Good luck!

Good Luck!

Here are some Formulas and rules for you.

**1.** (10 points)

Find the volume of the solid obtained by rotating the region bound by the curves  $y = \cos(x)$  ,  $y = 0$  when rotated around the y-axis.

**2.** (10 points) The force on a particle moving along the x-axis from  $x = 0$  to  $x = 1$  is given by  $F(x) = x\sqrt{x^2 + 1}$  find the work done on the particle.

**3.** (10 points) Verify that  $y = \cos(kx)$  is a solution to the differential equation  $y'' + k^2y = 0$   
Note don't try to solve just verify.

**4.** (15 points)

**a.** (5 pts) Solve the initial value problem for the differential equation  $\frac{dy}{dx} = x^2$  with  $y(0)=1$

**b.** (10 pts) Find the general solution to the differential equation  $\cos(x)h' = \sin(x)$

**5.** (*20 points*) A population of bacteria grows exponentially and starts with 2 cells. After 10 minutes there are 8 cells.

**a.** (*5 pts*) Write down the equation that gives the population as a function of time

**b.** (*5 pts*) When will the population reach 75?

**6.** (10 points) Determine whether the given series is convergent or divergent. If it is convergent find its sum

**a.** (5 pts)  $\sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n + \frac{1}{n} - \frac{1}{n+1}$

**b.** (5 pts)  $\sum_{n=1}^{\infty} \frac{n^2+3n^6}{n^3+n^6}$

**c.** (5 pts)  $\sum_{n=0}^{\infty} \frac{2^n}{n!}$

**7.** (10 points) Determine whether or not the given series is absolutely convergent, conditionally convergent, or divergent.  $\sum_{n=1}^{\infty} (-1)^n \frac{1}{3n}$

**8.** (10 points) Find the radius of convergence and interval of convergence for the following series.

**a.** (5 pts)  $\sum_{n=1}^{\infty} (x-2)^n$

**b.** (5 pts)  $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n!}$

**c.** (5 pts)  $\sum_{n=1}^{\infty} \frac{(-2)^n (x-2)^n}{n}$

- 9.** (10 points) Find a power series for the following function using the geometric series.  
 $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$  when  $|x| < 1$   
**a.** (5 pts)  $f(x) = \arctan(x)$

**10.** (10 points)

Find Taylor series out to 4 terms for the following functions at the given centers

- a.** (5 pts)  $f(x) = e^x$  centered at  $x = 1$

- b.** (5 pts)  $f(x) = \sin(x)$  centered at  $x = \pi$

**11.** (10 points)

Calculate the indefinite integral  $\int \frac{\tan(x)}{x} dx$  using power series.

**12.** (10 points)

Find the distance between the points  $(1, 0, 6)$  and  $(3, -2, 5)$

**13.** (10 points)

Show that the equation represents a sphere, find its center and radius.  $x^2 + y^2 + z^2 + 7x - 8y - 6z = 0$

**14.** (10 points) Find  $\vec{a} + \vec{b}$ ,  $\vec{a} - \vec{b}$ ,  $|\vec{a}|$ ,  $|\vec{a} - \vec{b}|$  for the vectors  $\vec{a} = \langle 1, 2, 3 \rangle$ ,  $\vec{b} = \langle 4, -2, -1 \rangle$

**15.** (5 points) Find the cross product between  $\vec{a} = \langle 1, -2, 0 \rangle$  and  $\vec{b} = \langle 0, 1, 3 \rangle$  and verify it is orthogonal to both  $\vec{a}$  and  $\vec{b}$

**16.** (15 points)

**a.** (5 pts) Find the symmetric, parametric, and vector equations of the line between the points  $(1, -2, 3)$  and  $(2, 4, 2)$

**b.** (10 pts) Find the equation of the plane parallel to the plane  $2x + 3y - 4z = 2$  going through the point  $(1, 2, 3)$

**17.** (10 points) Find the distance from the point  $(4, 1, -2)$  to the plane  $2x + 3y + 4z = 1$

**18.** (15 points) Do the following coordinate changes and plots

**a.** (5 pts) Change  $(1, \sqrt{2}/2, \sqrt{2}/2)$  from Rectangular to spherical coordinates.

**b.** (5 pts) Plot the point given in cylindrical coordinates as  $(2, \frac{\pi}{4}, 2)$  Then convert it to rectangular coordinates.

**c.** (5 pts) Plot the point in spherical coordinates given by  $(2, \frac{\pi}{3}, \frac{\pi}{6})$  Then convert it to rectangular coordinates.

**19.** (10 points) Find the velocity, acceleration and speed of a particle whose position is given by  $\vec{r}(t) = \langle 1 + t, t^2, \cos(\pi t) \rangle$  at  $t = 1$

**20.** (10 points) A particle is traveling in the x,y plane with velocity  $\vec{v}(t) = \langle -2, 2 \rangle$  how fast is the particle moving? How far did the particle move from time  $t = 1$  to  $t = 3$ ?

- 21.** (10 points) Find the following vectors using the vector function  $\vec{r}(t) = \langle \sin(t), 1, \cos(t) \rangle$
- a.** (5 pts) Find the unit tangent vector  $\vec{T}(t)$

- b.** (5 pts) Find the unit Normal vector  $\vec{N}(t)$

- c.** (5 pts) Find the curvature  $\kappa$

**22.** (10 points)

Suppose  $f(x, y, z) = 2xy^4 + 3xyz$  and  $x = st^2$ ,  $y = sy^2$ ,  $z = t$

**a.** (5 pts) Find  $\frac{\partial f}{\partial t}$

**b.** (5 pts) Find  $\frac{\partial f}{\partial s}$

**c.** (5 pts) Find  $\frac{\partial z}{\partial x}$

**23.** (10 points)

Find the equation of the tangent plane to the surface  $f(x, y) = x^2y + y^2x + 2x$  at the point  $(1, 1, 2)$

**24.** (10 points) Find the the relative maximums, minimums, or saddle points for the function  $f(x, y) = x^2 - 2xy + y^2$

**25.** (*10 points*) Find the extreme values of the function  $f(x, y, z) = x + 2y^2 + y + 3z^2 - 2z$  Subject to the constraint  $g(x, y, z) = x + y + z = 2$