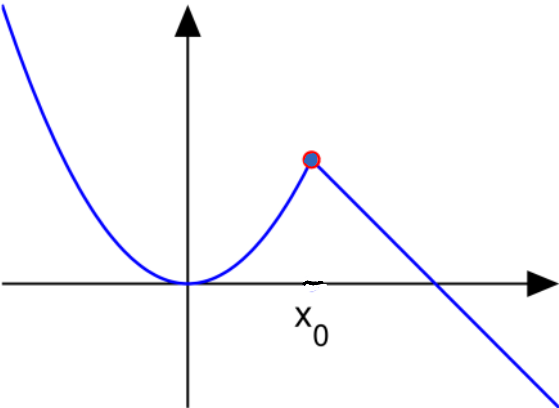
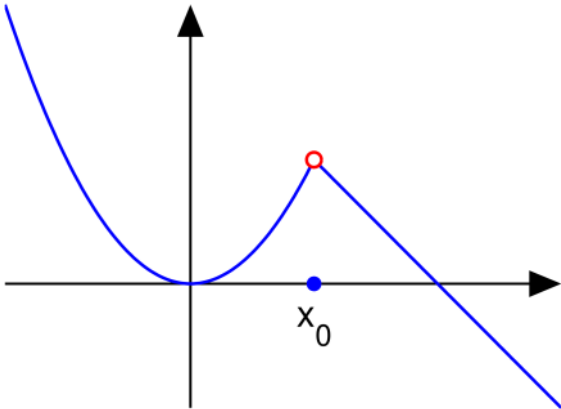
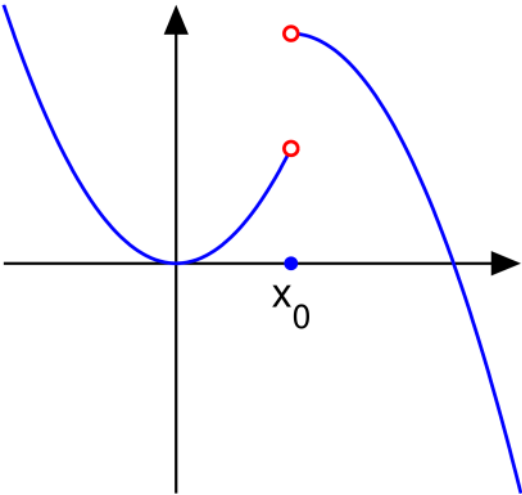


Sec 11.2 Limits and Continuity

Let's recall what it means for a function of one variable to be continuous.

Definition: $f(x)$ is continuous at $x=a$ if $\lim_{x \rightarrow x_0} f(x)$ exists (that is the right and left limits are the same) and $\lim_{x \rightarrow x_0} f(x) = f(x_0)$.

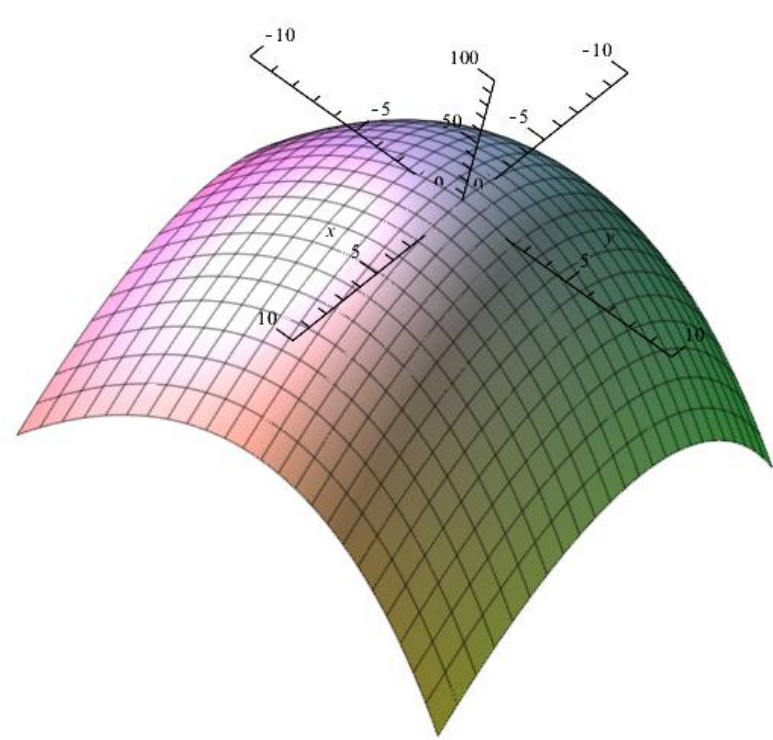
Let's look at some examples



Sec 11.2 Limits and Continuity

For functions of one variable we only needed to worry two directions, left and right for functions of more than one variable things are a bit more complex...

$$f(x,y) = -x^2 - y^2 + 5$$



1 Definition We write

$$\lim_{(x,y) \rightarrow (a,b)} f(x,y) = L$$

and we say that the **limit of $f(x,y)$ as (x,y) approaches (a,b)** is L if we can make the values of $f(x,y)$ as close to L as we like by taking the point (x,y) sufficiently close to the point (a,b) , but not equal to (a,b) .

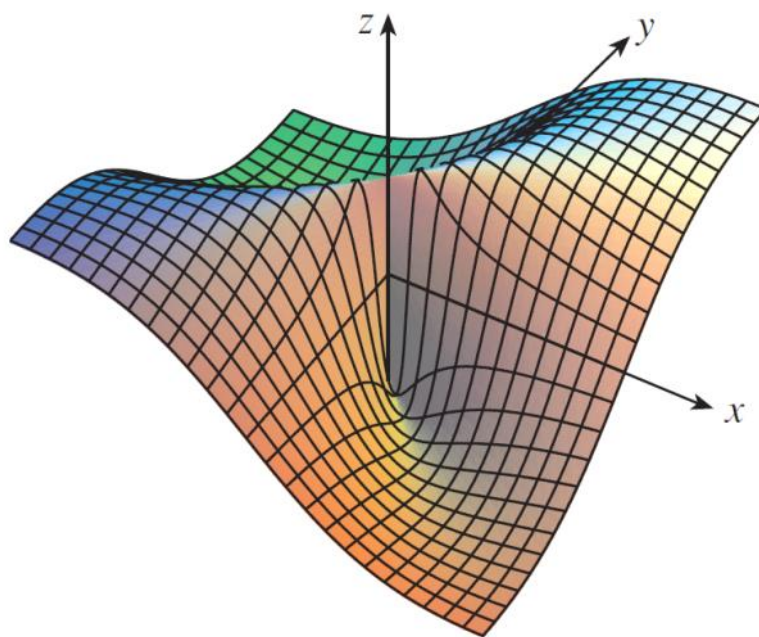
If $f(x,y) \rightarrow L_1$ as $(x,y) \rightarrow (a,b)$ along a path C_1 and $f(x,y) \rightarrow L_2$ as $(x,y) \rightarrow (a,b)$ along a path C_2 , where $L_1 \neq L_2$, then $\lim_{(x,y) \rightarrow (a,b)} f(x,y)$ does not exist.

Sec 11.2 Limits and Continuity

Showing a limit does not exist...

Show that $\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2+y^2}$ does not exist.

What we need to do is find at least two curves on the surface that we can follow to $(0,0)$ and obtain different limits. Let's try $x=0$, $y=0$ and $y=x$



Sec 11.2 Limits and Continuity

With functions of more than one variable it is easier to show a limit does not exist by picking curves to follow, but how do we find a limit and know that it exists when there are infinitely many curves we would have to check?

HW example Some are easy...

5–20 Find the limit, if it exists, or show that the limit does not exist.

5. $\lim_{(x,y) \rightarrow (1,2)} (5x^3 - x^2y^2)$

Some are not so easy...

Find $\lim_{(x,y) \rightarrow (0,0)} \frac{3x^2y}{x^2 + y^2}$ if it exists.

1 Definition We write

$$\lim_{(x,y) \rightarrow (a,b)} f(x,y) = L$$

and we say that the **limit of $f(x,y)$ as (x,y) approaches (a,b)** is L if we can make the values of $f(x,y)$ as close to L as we like by taking the point (x,y) sufficiently close to the point (a,b) , but not equal to (a,b) .

Sec 11.2 Limits and Continuity

When do we say a function of more than one variable is continuous?

3 Definition A function f of two variables is called **continuous at** (a, b) if

$$\lim_{(x, y) \rightarrow (a, b)} f(x, y) = f(a, b)$$

We say f is **continuous on** D if f is continuous at every point (a, b) in D .

$$f(x, y) = \begin{cases} \frac{3x^2y}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

Sec 11.2 Limits and Continuity

HW example

27–34 Determine the set of points at which the function is continuous.

33.
$$f(x, y) = \begin{cases} \frac{x^2 y^3}{2x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 1 & \text{if } (x, y) = (0, 0) \end{cases}$$