

Sec 6.2 (Volumes)

Finding volumes uses the same idea as the Riemann Sum to find areas under curves. We approximate the area with smaller and smaller rectangles then take the limit as their width goes to zero to give us the true area under the curve.

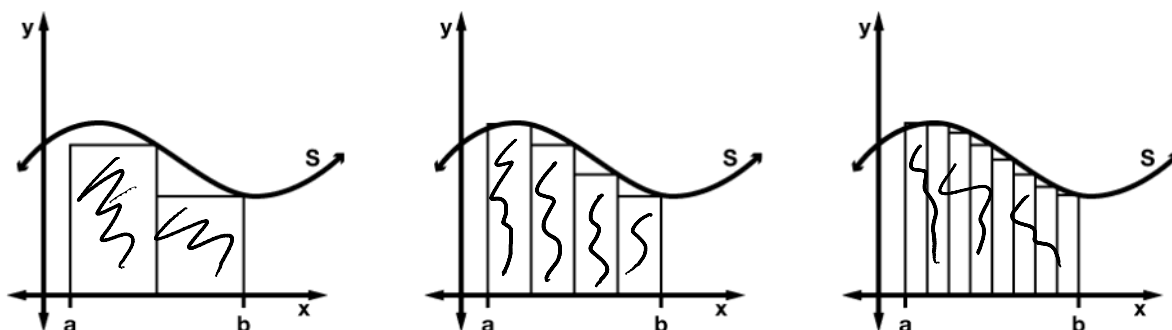
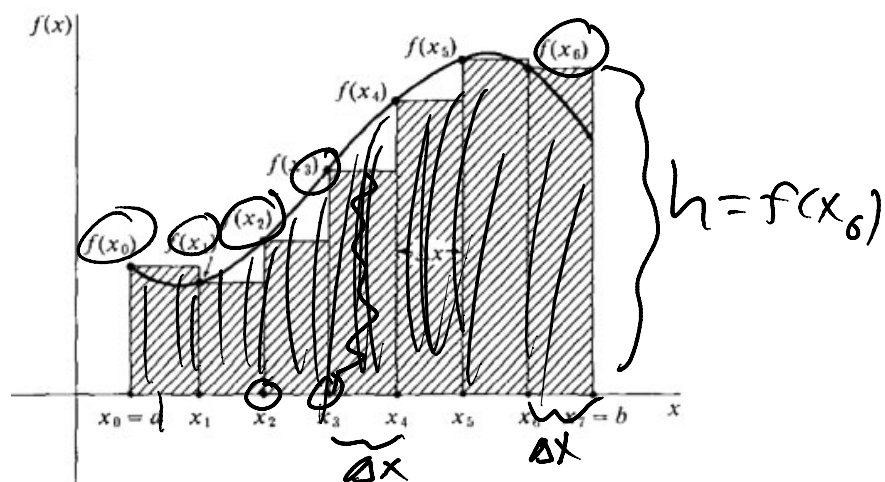


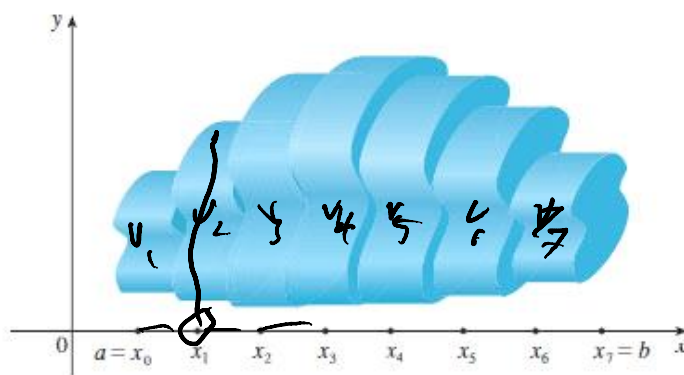
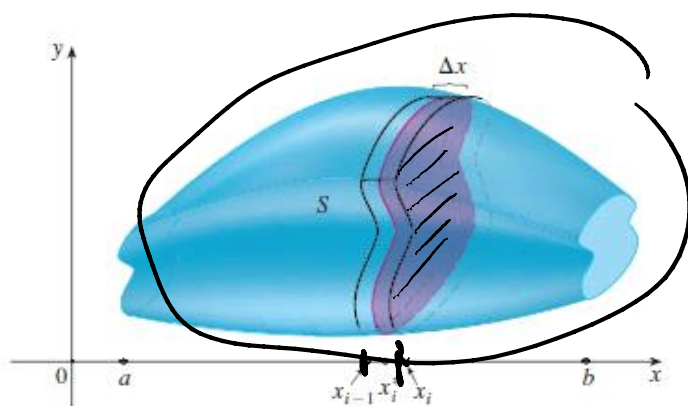
figure 1



$$A \approx \sum_{i=1}^n f(x_i) \Delta x_i$$

$$\lim_{\Delta x_i \rightarrow 0} \sum_{i=1}^n f(x_i) \Delta x_i = \int_a^b f(x) dx$$

Sec 6.2 Volumes

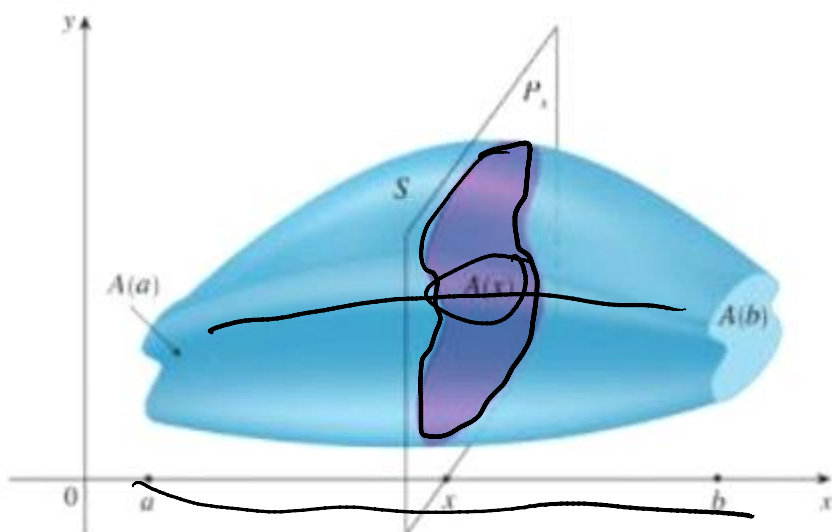


$$\sum_{i=1}^n V_i = V$$

$$V \approx \sum_{i=1}^n A(x_i) \Delta x$$

$$\Delta x \rightarrow 0$$

SECTION 6.2 VOLUME

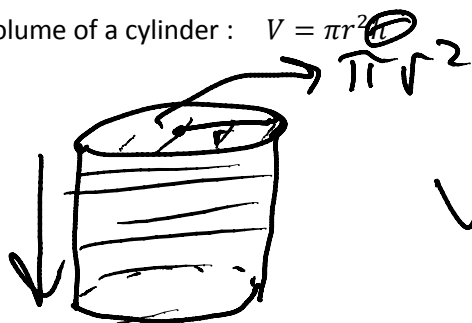


$$V = \lim_{\Delta x \rightarrow 0} \sum A(x) \Delta x$$

Sec 6.2 Volumes

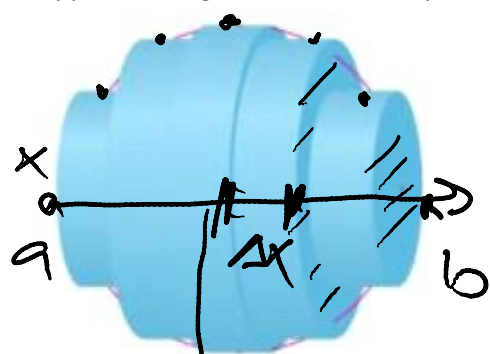
To Approximate the volume of an object we need to know the volume of the objects we are using to approximate it. A commonly useful volume is that of a cylinder which gives us the method of disks.

Volume of a cylinder : $V = \pi r^2 h$

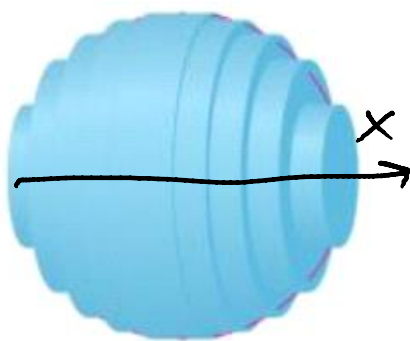


$$V = \pi r^2 h$$

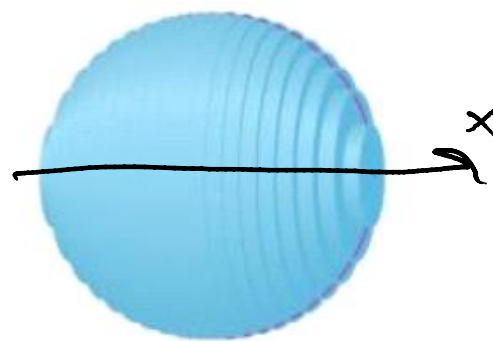
Approximating the volume of a sphere with cylinders (method of disks)



(a) Using 5 disks, $V \approx 4.2726$



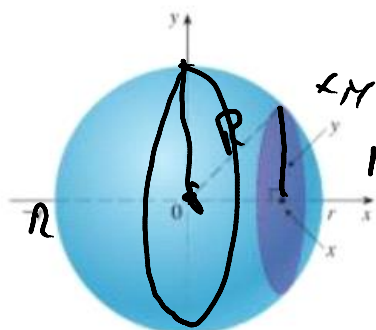
(b) Using 10 disks, $V \approx 4.2097$



(c) Using 20 disks, $V \approx 4.1940$

FIGURE 5 Approximating the volume of a sphere with radius 1

$$V \approx \sum A(x) \Delta x = \sum \pi r^2 \Delta x \xrightarrow{\Delta x \rightarrow 0} \boxed{\int_a^b \pi r^2 dx}$$



$$\int \pi y^2 dx = \int \pi y^2 dx$$

$$= \int_{-R}^R \pi (\sqrt{R^2 - x^2})^2 dx$$

$$= \int_{-R}^R \pi (R^2 - x^2) dx = \pi R^2 x - \pi \frac{x^3}{3} \Big|_{-R}^R$$

$$= \pi R^2 R - \pi \frac{R^3}{3} - \left(\pi R^2 (-R) - \pi \frac{(-R)^3}{3} \right)$$

$$x^2 + y^2 = R^2$$

$$y = \sqrt{R^2 - x^2}$$

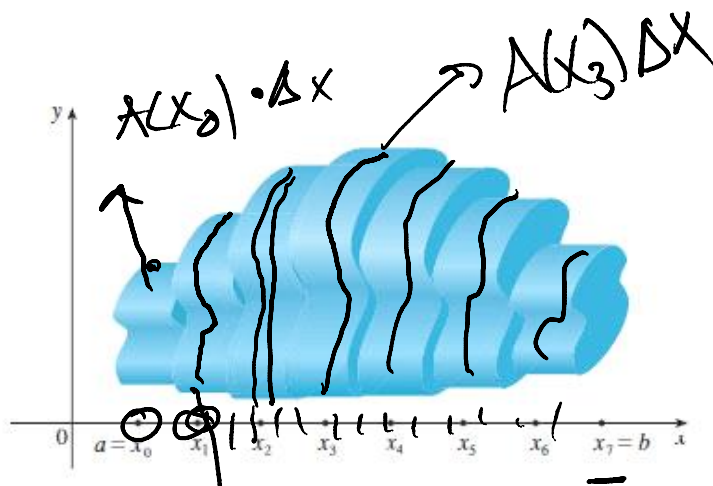
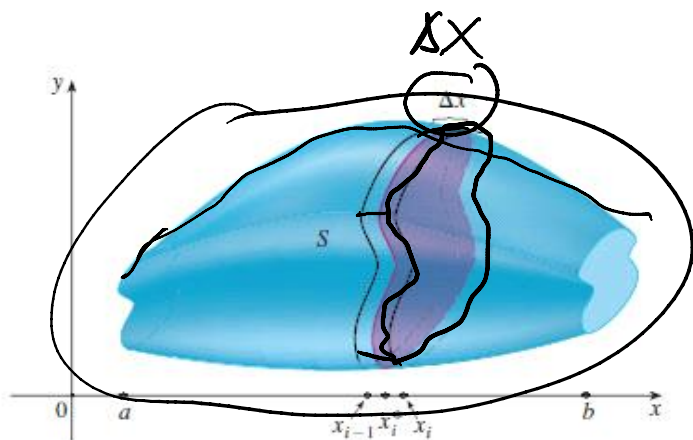
$$= \int_{-R}^R \pi (R^2 - x^2) dx = \pi R x - \frac{\pi x^3}{3} \Big|_{-R}^R$$

$$= \pi R^2 R - \pi \frac{R^3}{3} - \left(\pi R^2 (-R) - \frac{\pi (-R)^3}{3} \right)$$

$$\frac{\pi R^3}{3} - \frac{\pi R^3}{3} = \left(-\frac{\pi R^3}{3} + \frac{\pi R^3}{3} \right)$$

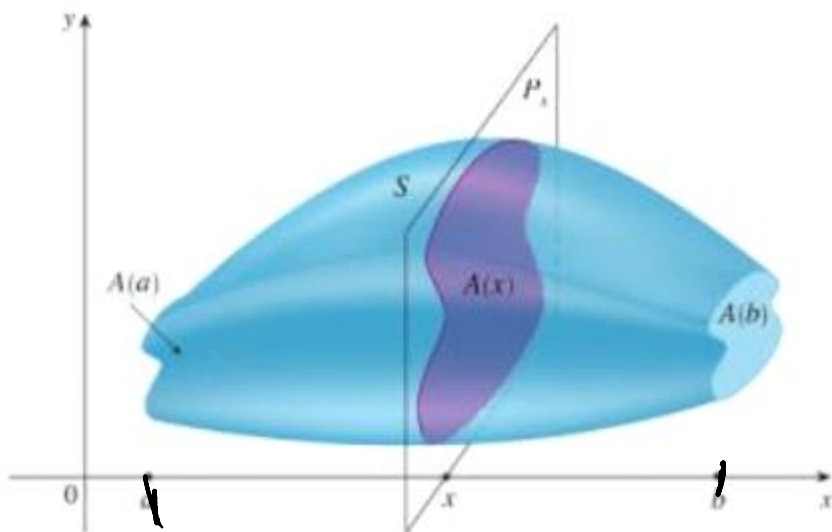
$$= \left[\frac{4}{3} \pi r^3 \right]$$

Sec 6.2 Volumes



$$A \approx \sum_{n=0}^7 A(x_n) \Delta x = A = \sum_{n=0}^7 A(x_n) \Delta x$$

SECTION 6.2 VOLUME



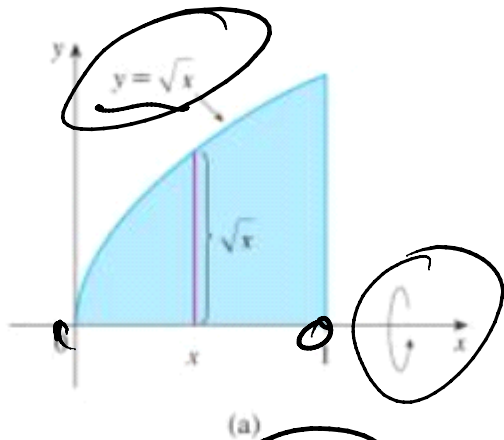
$$A \approx \sum_{n=0}^N A(x_n) \Delta x \xrightarrow[\lim_{\Delta x \rightarrow 0}]{\lim_{N \rightarrow \infty}} \int_a^b A(x) dx$$

Sec 6.2 Volumes (VIVA LA REVOLUTION!)

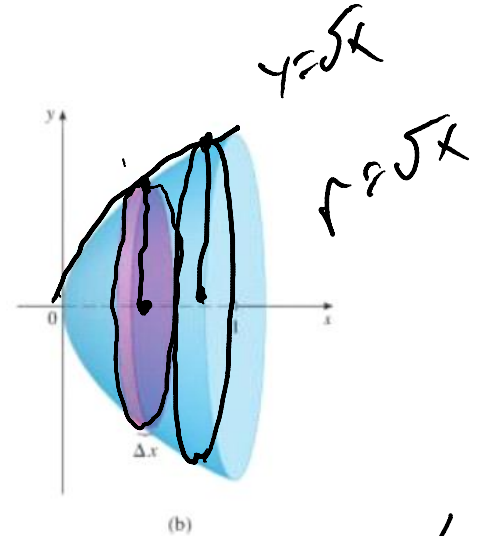
$$\int_a^b f(x) dx = f(b) - f(a)$$

<https://www.youtube.com/watch?v=WbFQv6a4k6U&feature=youtu.be>

Often times we can create shapes by rotating a curve about an axis or a line.



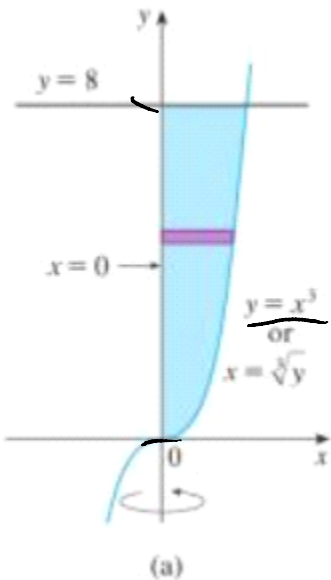
Rotate about the x axis to make a volume.....



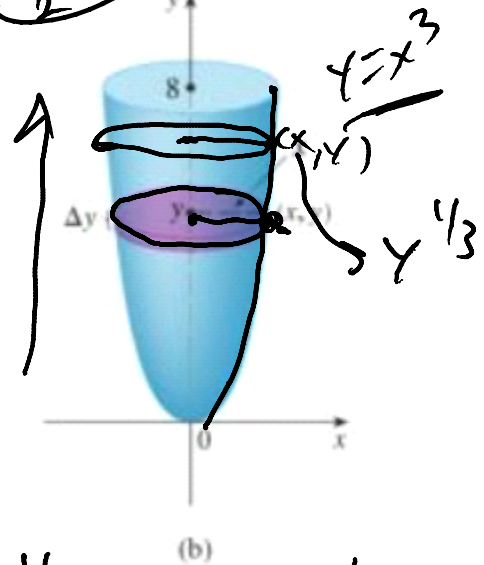
$$8^{5/3} = (8^{1/3})^5 = 2^5 = 32$$

$$\int_0^1 \pi (\sqrt{x})^2 dx = \int_0^1 \pi x dx = \pi \left[\frac{x^2}{2} \right]_0^1 = \frac{\pi}{2}$$

$$A(x) = \pi r^2 = \pi (\sqrt{x})^2$$



Rotate about the y axis to make a volume



$$A = \pi r^2$$

$$r = x \quad y = x^3$$

$$x = \sqrt[3]{y} = y^{1/3} = r$$

$$A(y) = \pi (y^{1/3})^2 = \pi y^{2/3}$$

$$(x^a)^b = x^{a \cdot b}$$

$$\sqrt[5]{x^2} = x^{2/5}$$

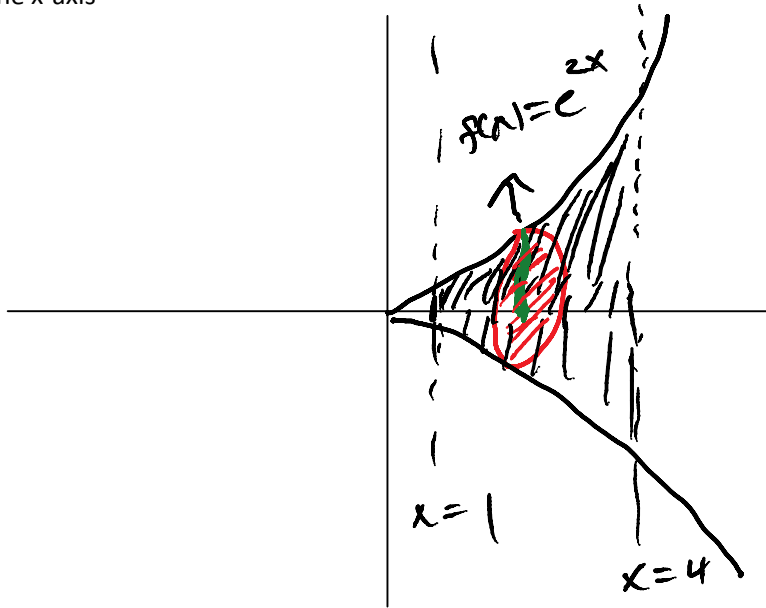
$$A = \int_0^8 A(y) dy = \int_0^8 \pi(y^{4/3})^2 dy = \int_0^8 \pi y^{8/3} dy = \pi y^{5/3} \Big|_0^8 = \pi 8^{5/3} \cdot \frac{3}{5}$$

Sec 6.2 Volumes (Method of Disks)

Saturday, May 10, 2014
12:35 AM

<https://www.youtube.com/watch?v=gPFIM1BqoU&feature=youtu.be>

Ex. Find the volume of the solid obtained by rotating the region bounded by $y = e^{2x}$, $y=0$, $x=1$, $x=4$ about the x-axis



$$A = \pi r^2$$

$$r = e^{2x}$$

$$A(x) = \pi (e^{2x})^2$$

$$A(x) = \pi e^{4x}$$

$$\int_a^b A(x) dx = \int_1^4 \pi e^{4x} dx = V$$

$$\int e^u du = e^u$$

$$\int_1^4 \pi e^{4x} dx = \pi \int_1^4 e^{4x} dx$$

$$u = 4x$$

$$\frac{du}{dx} = 4$$

$$dx = \frac{du}{4}$$

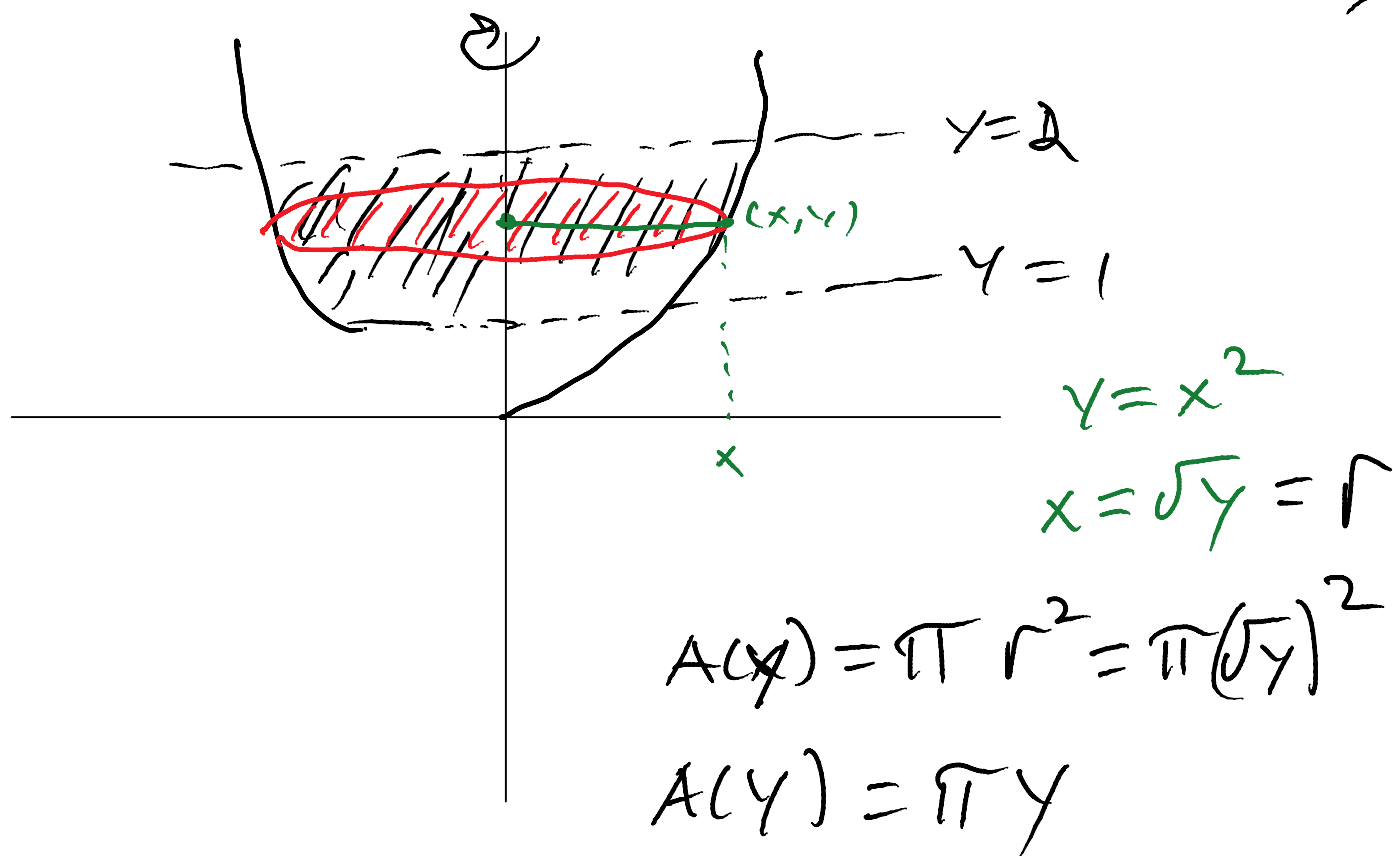
$$= \pi \int e^u \frac{du}{4} = \frac{\pi}{4} \int e^u du = \frac{\pi}{4} e^u = \frac{\pi}{4} e^{4x} \Big|_1^4$$

$$= \left(\frac{\pi}{4} e^{4(4)} - \frac{\pi}{4} e^{4(1)} \right) = V$$

$$\int \sin(2x) = -\frac{1}{2} \cos(2x)$$

Section 6.2 Volumes (Method of Disks)

Ex. Find the volume of the solid obtained by rotating the region bounded by $y = x^2$, $x = 0$, $y = 1$, $y = 2$ about the y-axis



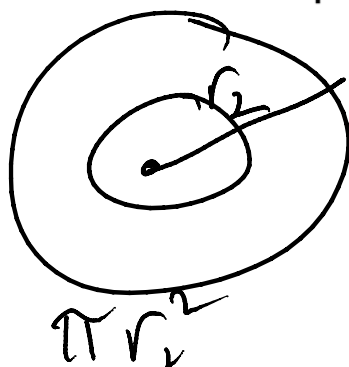
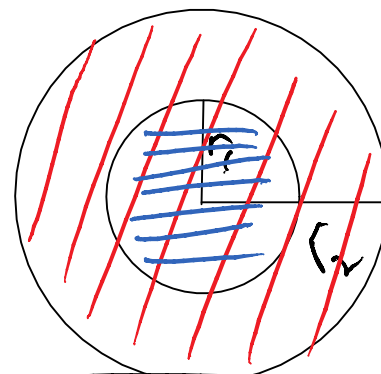
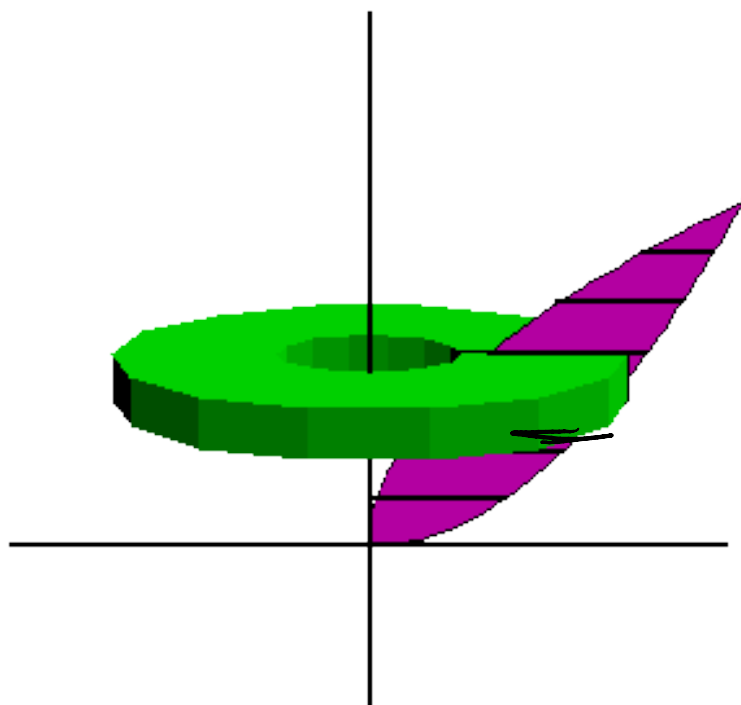
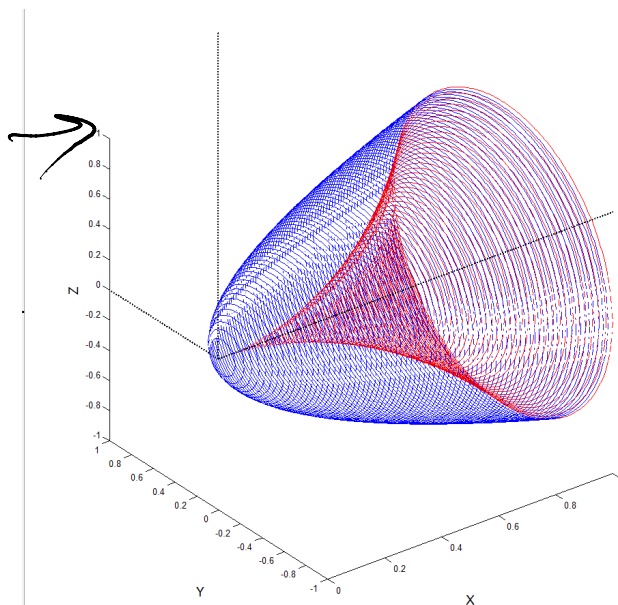
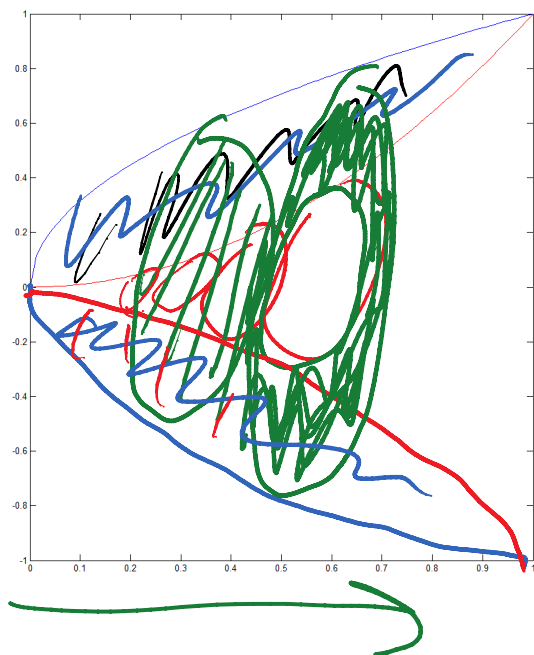
$$\int_a^b A(y) dy = \int_1^2 \pi y dy = \frac{\pi y^2}{2} \Big|_1^2$$

$$\frac{\pi (2)^2}{2} - \frac{\pi (1)^2}{2} = \frac{4\pi}{2} - \frac{\pi}{2}$$

$$= \boxed{\frac{3\pi}{2}} = \text{Volume}$$

Sec 6.2 Volumes (Method of Washers)

<https://www.youtube.com/watch?v=3oAjcLD34kc&feature=youtu.be>

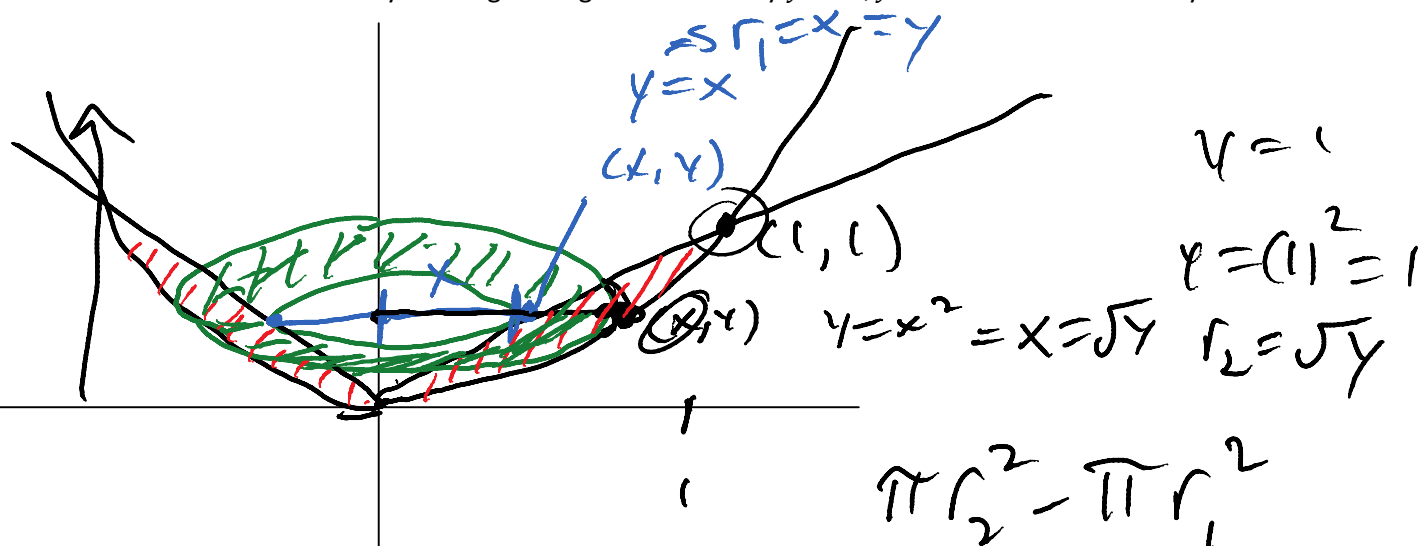


$$A(r) = \pi r_2^2 - \pi r_1^2$$

$$= \pi (r_2^2 - r_1^2)$$

Sec 6.2 Volumes (Method of Washers)

Find the volume of the solid obtained by rotating the region bounded by $y = x$, $y = x^2$ rotated about the y-axis



$$A(y) = \pi(\sqrt{y})^2 - \pi(y^2)$$

$$A(y) = \pi y - \pi y^2$$

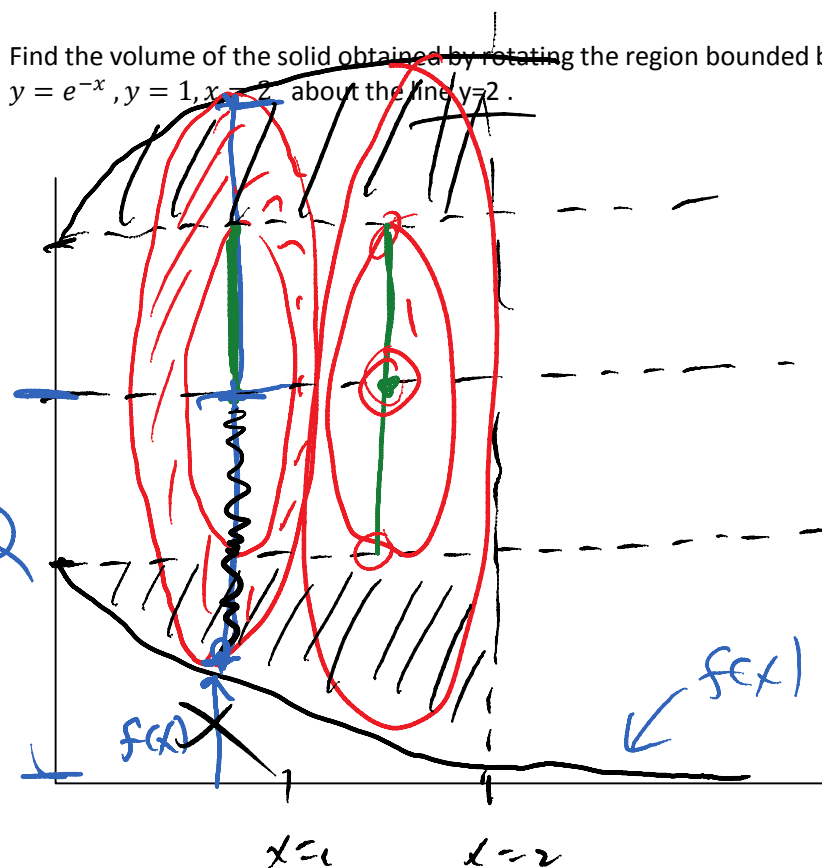
$$\int_0^1 \pi y - \pi y^2 dy = \frac{\pi y^2}{2} - \frac{\pi y^3}{3} \Big|_0^1$$

$$= \frac{\pi 1^2}{2} - \frac{\pi 1^3}{3} - 0 = \frac{3\pi}{3 \cdot 2} - \frac{\pi}{3 \cdot 2}$$

$$\frac{3\pi}{6} - \frac{2\pi}{6} = \frac{\pi}{6} = \text{volume}$$

Sec 6.2 (Method of Washers)

Find the volume of the solid obtained by rotating the region bounded by the curves $y = e^{-x}$, $y = 1$, $x = 2$ about the line $y = 2$.



$$2 - f(x)$$

$$r_2 = 2 - e^{-x}$$

$$r_1 = 1$$

$$y = 2$$

$$A(y) = \pi r_2^2 - \pi r_1^2$$

$$A(y) = \pi (2 - e^{-x})^2$$

$$- \pi (1)^2$$

$$(2 - e^{-x})(2 - e^{-x})$$

$$= 4 - 2e^{-x} - 2e^{-x}$$

$$+ (e^{-x})^2$$

$$\downarrow e^{-2x}$$

$$\int_0^2 \pi (2 - e^{-x})^2 - \pi dx$$

$$\int_0^2 \pi (4 - 4e^{-x} + e^{-2x}) - \pi dx$$

$$\pi \int_0^2 4 - 4e^{-x} + e^{-2x} - 1 dx$$

$$= \pi \left(4x + 4e^{-x} + \left(-\frac{1}{2}\right)e^{-2x} - x \right) \Big|_0^2$$

$$= \pi (8 + 4e^{-2} - \frac{1}{2}e^{-4} - 2) - \pi (4e^0 - \frac{1}{2})$$

Sec 6.2 Volumes

Find the volume of the solid obtained by rotating the region bounded by the curves $y = x$, $x = \sqrt{x}$ about the line $x=2$