

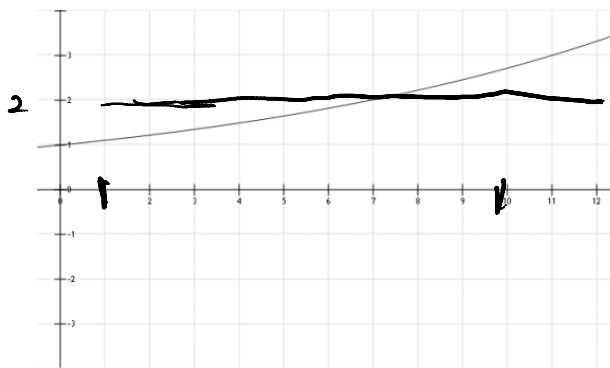
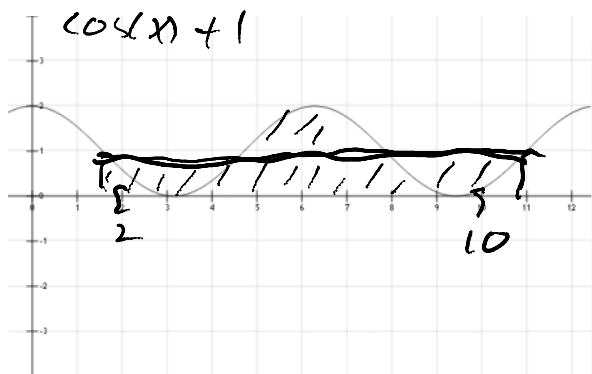
Sec 6.5 (Average Value of A Function)

Average of a finite set of numbers: $\{y_0, y_1, y_2, \dots, y_n\}$ $\leftarrow$

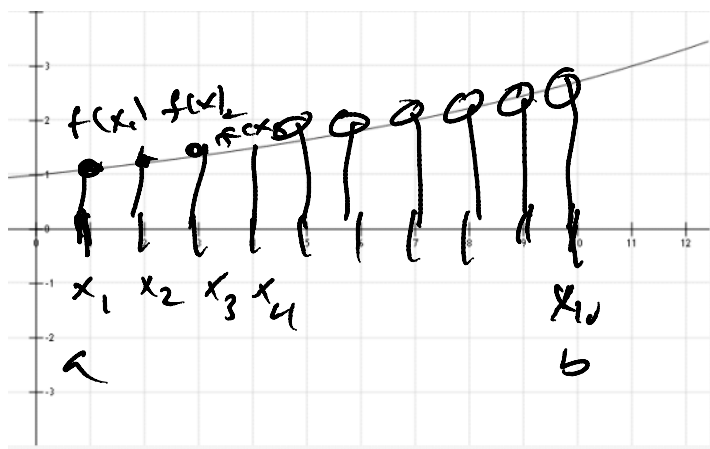
$$A = \frac{y_0 + y_1 + y_2 + \dots + y_n}{n}$$

What about the average value of a function on an interval $[a, b]$?

$e^{1/10} x$



We could approximate by taking a partition of our interval....



$$\Delta x = \frac{b-a}{n}$$

$$n = \frac{b-a}{\Delta x}$$

$$\sum_{i=1}^{10} f(x_i)$$

(10)

Riemann
Sum
for f

$$\sum_{i=1}^n f(x_i) \Delta x = \frac{1}{b-a} \sum_{i=1}^n f(x_i) \Delta x$$

Sec 6.5 (Average Value of A Function)

avg value

$$f_{\text{avg}} \approx \frac{1}{b-a} \sum_{i=1}^n f(x_i) \Delta x_i$$

q

$$\lim_{\substack{\Delta x \rightarrow 0 \\ n \rightarrow \infty}} \frac{1}{b-a} \sum_{i=1}^n f(x_i) \Delta x_i =$$

$$\frac{1}{b-a} \int_a^b f(x) dx$$

Ex. Find the average value of $f(x) = e^{\frac{1}{10}x}$ on $(1,10)$ $\rightarrow a=1, b=10$

$$V = \int_a^b f(x) dx$$

$$\frac{1}{10-1} \int_1^{10} e^{\frac{1}{10}x} dx$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}$$

$$\frac{1}{9} \int_1^{10} e^{\frac{1}{10}x} dx = \frac{1}{9} 10 e^{\frac{1}{10}x} \Big|_1^{10}$$

$$= \frac{10}{9} e^{\frac{1}{10} \cdot 10} - \frac{10}{9} e^{\frac{1}{10} \cdot 1} = \boxed{\frac{10}{9} e - \frac{10}{9} e^{\frac{1}{10}}}$$

Sec 6.5 (Average Value of A Function)

HW Example. Find the average value of $h(x) = \cos^4(x) \sin(x)$ on $[0, \pi]$

$$\frac{1}{b-a} \int_a^b f(x) dx$$

$$a = 0$$

$$b = \pi$$

$$\cos^4(x) = (\cos(x))^4$$

$$\frac{1}{\pi} \int_0^{\pi} \cos^4(x) \sin(x) dx$$

$$\begin{aligned} u &= \cos(x) \\ \frac{du}{dx} &= -\sin(x) \end{aligned}$$

$$\frac{du}{-\sin(x)} = dx$$

$$\frac{1}{\pi} \int_0^{\pi} u^4 \sin(x) \frac{du}{-\sin(x)}$$

$$-\int_a^b f = \int_b^a f$$

$$\begin{aligned} x &= 0 \\ u &= \cos(0) = 1 \end{aligned}$$

$$\begin{aligned} x &= \pi \\ u &= \cos(\pi) = -1 \end{aligned}$$

$$-\frac{1}{\pi} \int_1^{-1} u^4 du = \frac{1}{\pi} \int_{-1}^1 u^4 du$$

$$\frac{1}{\pi} \left. \frac{u^5}{5} \right|_{-1}^1 = \frac{1}{\pi} \left(\frac{1^5}{5} \right) - \frac{1}{\pi} \left(\frac{(-1)^5}{5} \right)$$

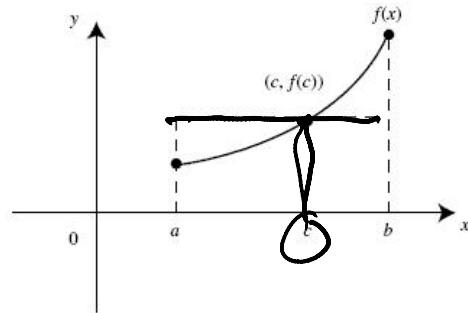
$$\left(\frac{2}{5\pi} \right)$$

Sec 6.5 (Average Value of A Function)

If f is continuous on $[a, b]$ then there exists a number c such that:

$$f(c) = \frac{1}{b-a} \int_a^b f(x) dx$$

That is; $\int_a^b f(x) dx = f(c)(b-a)$



To prove this recall the mean value theorem for derivatives $f'(c) = \frac{f(b)-f(a)}{b-a}$

Let's apply it to the function $F(x) = \int_a^x f(t) dt$

There exists

$\exists c$ between $[a, b]$ such that

$$F'(c) = \frac{F(b) - F(a)}{b - a}$$

↓

$$f(c) = \frac{\int_a^b f(t) dt - 0}{b - a}$$

$$F(2) = \int_a^2 f(t) dt$$

$$F'(x) = \frac{d}{dx} \int_a^x f(t) dt$$

$$= f(x)$$

$$F'(c) = f(c)$$

$$F(a) = \int_a^a f(t) dt = 0$$

$$F(b) = \int_a^b f(t) dt$$

$$f(c) = \frac{1}{b-a} \int_a^b f(t) dt$$

→ avg of f

Sec 6.5 (Average Value of A Function)

Example: Find C such that $f_{avg} = f(c)$ for $f(x) = \ln(x)$ on $[1,3]$

$$f(c) = \frac{1}{b-a} \int_a^b f(x) dx$$

↓

$$\ln(c) = \frac{1}{3-1} \int_1^3 \ln(x) dx$$

$$\int u dv = u \cdot v - \int v du$$

$$\begin{matrix} \int u dv \\ \downarrow \\ u \\ \downarrow \\ dv \end{matrix}$$

$$u = \ln(x) \quad dv = dx$$

$$du = \frac{1}{x} dx \quad v = x$$

$$\ln(c) = \frac{1}{2} \left[x \ln(x) - \int x \cdot \frac{1}{x} dx \right]$$

$$= \frac{1}{2} \left[x \ln(x) \Big|_1^3 - \int_1^3 dx \right]$$

$$= \frac{1}{2} \left[x \ln(x) \Big|_1^3 - x \Big|_1^3 \right]$$

$$= \frac{1}{2} \left[(3 \ln(3) - 3) - (\cancel{1 \ln(1)} - 1) \right]$$

$$\ln(c) = \left(\frac{1}{2} (3 \ln(3) - 3) + 1 \right)$$

$$x^a x^b = x^{a+b}$$

$$(x^a)^b = x^{a \cdot b}$$

e

$$e^{\frac{3}{2} \ln(3) - \frac{2}{2}} = e^{\frac{3}{2} \ln(3) - 1}$$

$$c = e^{\frac{3}{2} \ln(3) - 1}$$

$$= \frac{(3^3)^{1/2}}{e}$$

$$\frac{\sqrt{27}}{e}$$

$$\frac{x^a}{x^b} = x^{a-b}$$

Sec 6.5 (Average Value of A Function)

Ex. In a certain city the temperature in degrees F is t hours after 9 am is modeled by the function

$$T(t) = 50 + 14 \sin\left(\frac{\pi t}{12}\right)$$

$t=0$ 9 am

Find the average temperature from 9 am to 9 pm

$t=12$ 9 pm

$$\frac{1}{12-0} \int_0^{12} 50 + 14 \sin\left(\frac{\pi t}{12}\right) dt$$

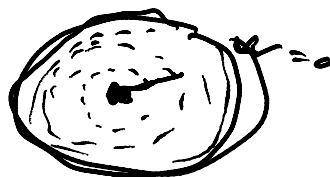
$$\frac{1}{12} \left(50x + 14 \left(\frac{12}{\pi} \right) \left(-\cos\left(\frac{\pi t}{12}\right) \right) \right) \Big|_0^{12}$$

$\int \sin(at) = -\frac{1}{a} \cos(at)$

Sec 6.5 (Average Value of A Function)

The velocity of blood that flows in a blood vessel with radius R and length l at a distance r from the central axis is

$$v(r) = \frac{P}{4\eta l} (R^2 - r^2)$$



Where P is the pressure difference between the ends of the vessel and η is the viscosity of the blood. Find the average velocity (with respect to r) on the interval $0 \leq r \leq R$



$$v_{\text{avg}} = \int_0^R \frac{P}{4\eta l} (R^2 - r^2) dr$$