

Sec 6.6 (Application of Integration)

Work: The energy required to move an object over a distance. The units are Joules = Newtons x Meters

Under a constant force Work = Force x Distance
where m is the mass and a the acceleration.

Force is given by Newton's second law $F=ma$

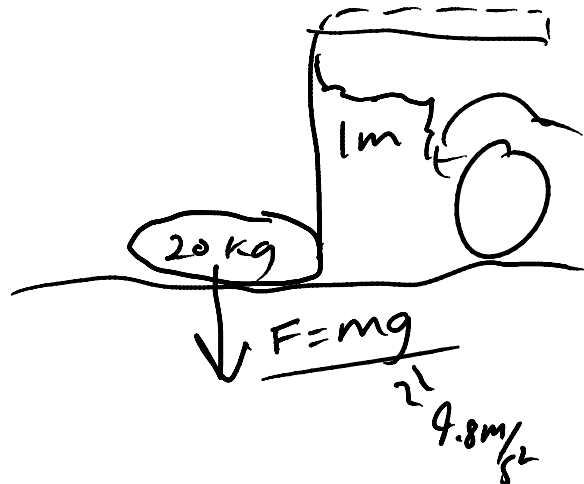
$$\frac{d^2x}{dt^2}$$

With a constant acceleration -> constant force work is defined as $W=Fd$

Example: Calculate the force required to lift a 20kg bag of cement 1m from the ground to the back of a pick up truck.

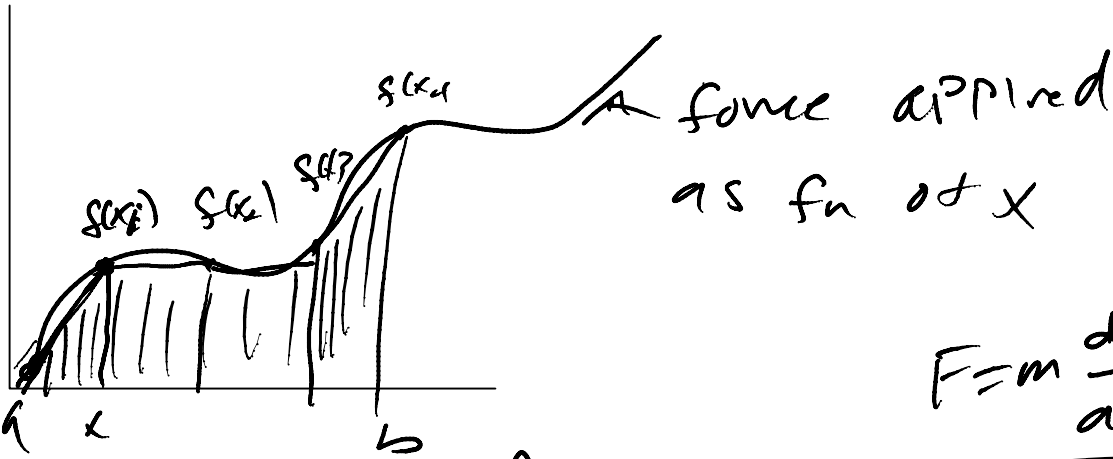
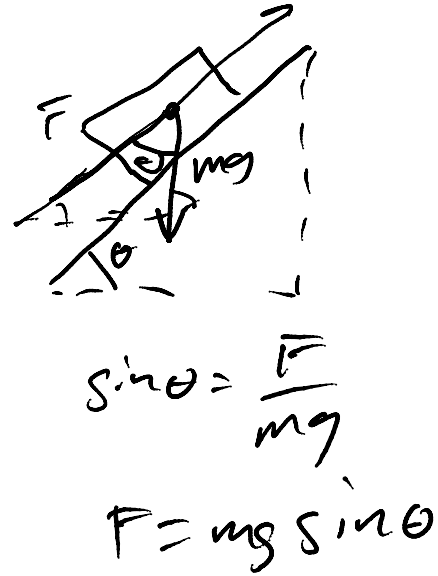
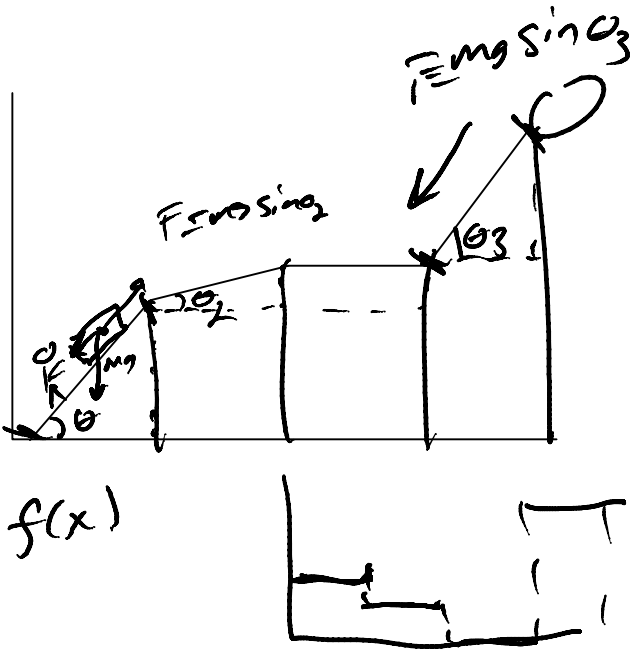
$$W = \overset{20(9.8)}{F} \times \overset{1m}{d}$$

$$W = 196(1) = 196 \text{ J}$$



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What if we don't have a constant force?



$$F = m \frac{dx^2}{dt^2}$$

$$W \approx \sum_{i=1}^n f(x_i) \Delta x$$

lim $n \rightarrow \infty$ ↓

$$W = \int_a^b f(x) dx$$

force as a function of x

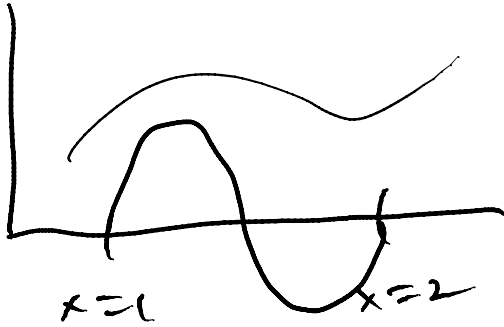
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Work done from moving an object from a to b:

$$W = \int_a^b f(x) dx$$

Where $f(x)$ is the force

A particle is located a distance x meters from the origin, a force of $f(x) = \cos\left(\frac{\pi x}{3}\right)$ acts on it. How much work is done in moving the particle from $x=1$ to $x=2$.



$$F'(x) = f(x)$$

$$\int f(ax) = \frac{1}{a} F(ax)$$

$$\int_1^2 \cos\left(\frac{\pi x}{3}\right) dx$$

$$\begin{cases} u = \frac{\pi}{3}x \\ du = \frac{\pi}{3} dx \\ dx = \left(\frac{3}{\pi}\right) du \end{cases}$$

$$= -\frac{3}{\pi} \sin\left(\frac{\pi}{3}x\right) \Big|_1^2 = -\frac{3}{\pi} \sin\left(\frac{2\pi}{3}\right) + \frac{3}{\pi} \sin\left(\frac{\pi}{3}\right)$$

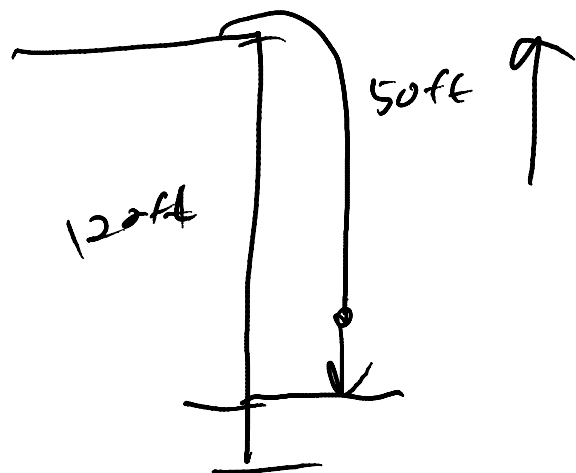
$$= -\frac{3}{\pi} \frac{\sqrt{3}}{2} + \frac{3}{\pi} \frac{\sqrt{3}}{2} = 0$$

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$$g = 32 \frac{ft}{s^2} \quad \frac{1}{32} \frac{5105}{71+1} \quad 1260 +$$

~~$\frac{46}{ft}$~~ in earth

A heavy rope 50 ft long weighs .5 lb/ft and hangs over the edge of a building 120 ft high. How much work is done pulling the rope to the top of the building?



$$50 \left(\frac{1}{2} \right) = \boxed{25 \text{ lb}}$$

$$40 \left(\frac{1}{2} \right) = 20 \text{ lb}$$

$$(50 - x) \frac{1}{2} = F(x)$$

$$\int_0^{50} \frac{1}{2} (50 - x) dx = \frac{1}{2} \int_0^{50} \frac{50 - x}{1} dx$$

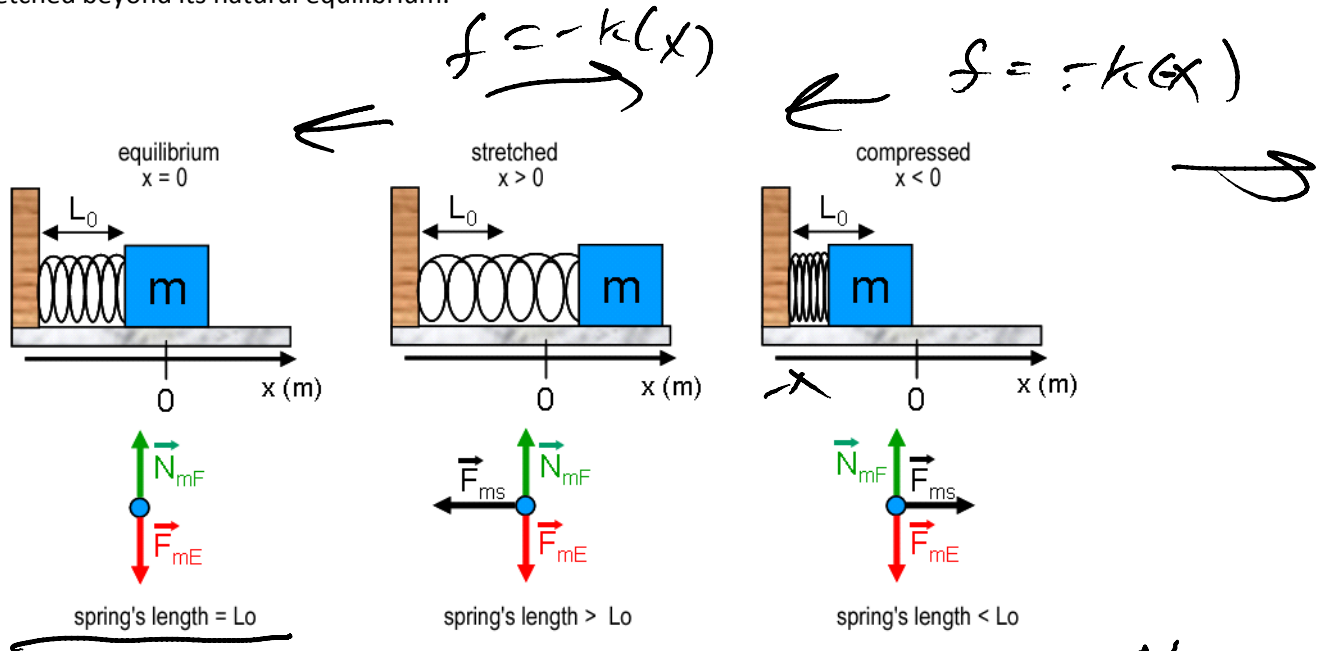
$$\frac{1}{2} \left(50x - \frac{x^2}{2} \right) \Big|_0^{50}$$

$$25x - \frac{x^2}{4} \Big|_0^{50}$$

$$25(50) - \frac{(50)^2}{4} - (0) = \boxed{625 \text{ ft} \cdot \text{lb}}$$

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Hooke's law: The restoring force of a spring is given by $f = -kx$ where k is the spring constant (a property of the spring) and x the distance it is stretched beyond its natural equilibrium.



Example Calculate the work required to stretch a spring with spring constant $k=4$ a distance of 10m from equilibrium.

$$W = \int_a^b f(x) dx$$

$$f(x) = -kx$$

$$W = \int_0^{10} -kx dx = - \int_0^{10} kx dx$$

$$= - \frac{kx^2}{2} \Big|_0^{10} = - \frac{4(10)^2}{2} + \frac{4(0)^2}{2}$$

$$= - \frac{400}{2} = -200$$

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Hydro static pressure.

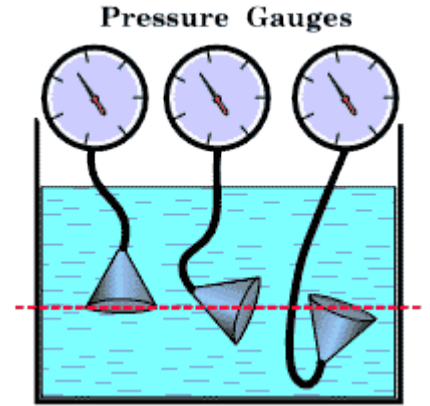
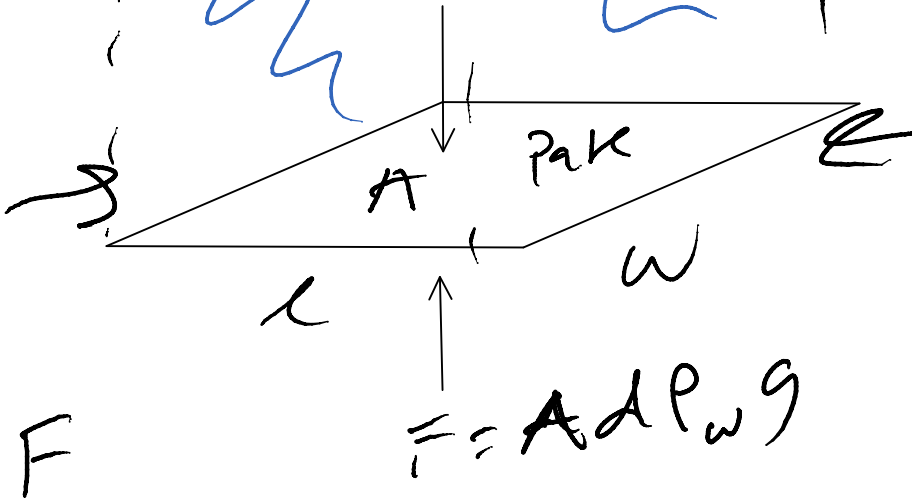
$$F=mg \quad P=F/A$$

$$V = \underbrace{LWD}$$

~~$\frac{K_9}{m^3}$~~

$$m = A d \rho_w$$

$$F = (A \wedge P_w \mathcal{G})$$



$$P = \frac{A \rho_w g}{A}$$

$$P = F/A$$

$$P = \frac{F}{A} = \rho_w g d$$

If we use 1 square meter for our area then the hydrostatic pressure at a depth d is $P = \rho g d$

$$P_n = \rho_w g d$$

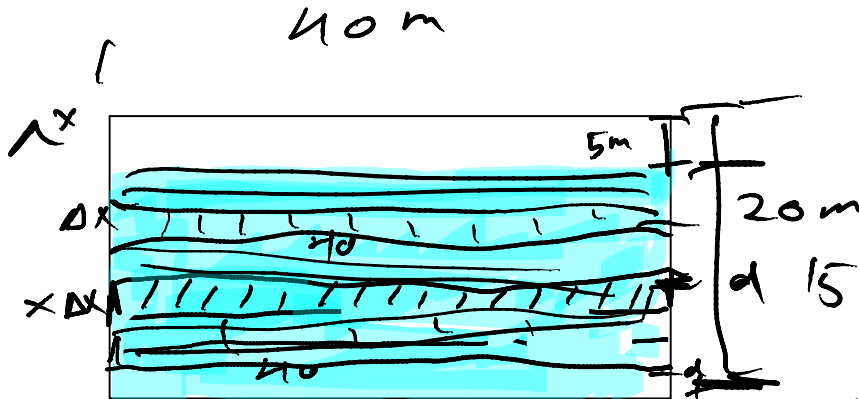
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$$P = \frac{F}{A}$$

$$F = PA$$

$$P = \rho g d \quad F = PA$$

Suppose we have a rectangular dam which is 40 m wide and 20 m high is filled to the point where the water is 5 m from the top. Find the hydrostatic force on the dam. Is there a design that might be better than a rectangle?



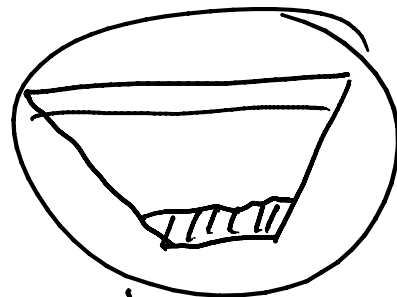
$$F \approx PA = \rho g d 40 \Delta x$$

$$F \approx \sum_{i=1}^n \underbrace{\rho g (15 - x)}_d 40 \Delta x$$

$$F = \int_0^{15} \rho g (15 - x) 40 dx$$

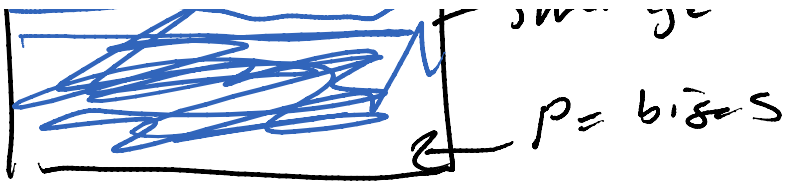
$$= 40 \rho g \int_0^{15} (15 - x) dx$$

$$40 \rho g \left(15x - \frac{x^2}{2} \right) \Big|_0^{15}$$



smallest

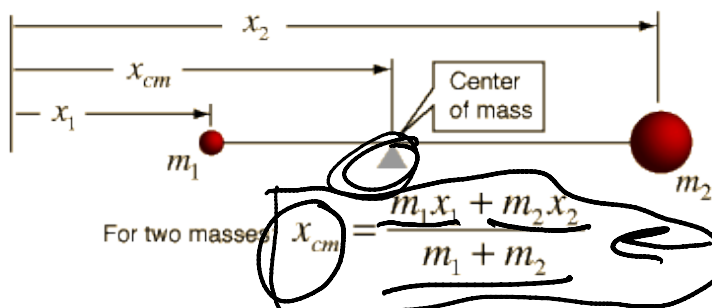
$$F = P A$$



$$F = P(A)$$

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Moments and centers of mass



For two masses on a "sea saw" They balance when $m_1 x_1 = m_2 x_2$
That is when the moments are equal.

Handwritten calculation for the center of mass of three masses:

$$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$$

Masses and positions: $m_1 = 2 \text{ kg}$ at $x_1 = -4$, $m_2 = 1 \text{ kg}$ at $x_2 = 1$, $m_3 = 1 \text{ kg}$ at $x_3 = 3$.

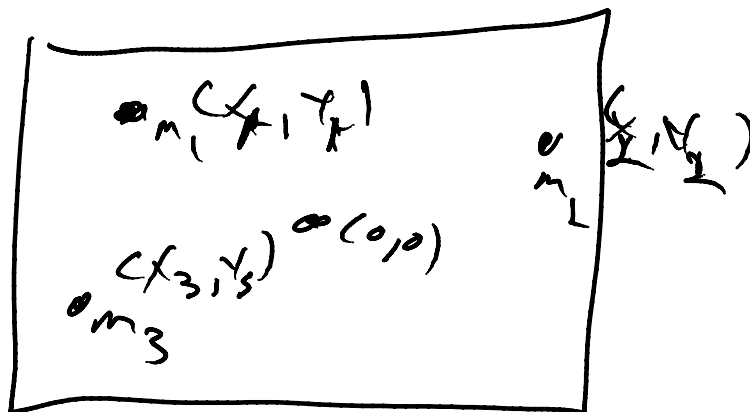
$$x_{cm} = \frac{(1)(1) + (1)(3) + 2(-4)}{2 + 1 + 1} = \frac{1 + 3 - 8}{4} = \frac{-4}{4} = -1$$

moment

Center of mass in a plane:

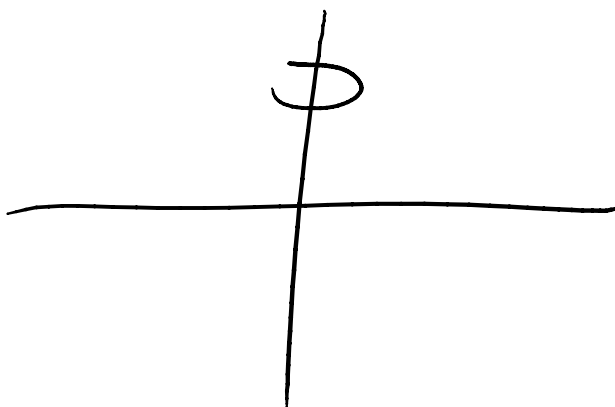
$$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$$

$$y_{cm} = \frac{\sum m_i y_i}{\sum m_i}$$



Moments about an axis

$$\left\{ \begin{array}{l} \underline{M_y} = \sum \underline{m_i x_i} \\ \underline{M_x} = \sum \underline{m_i y_i} \end{array} \right.$$

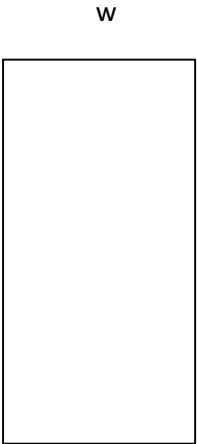


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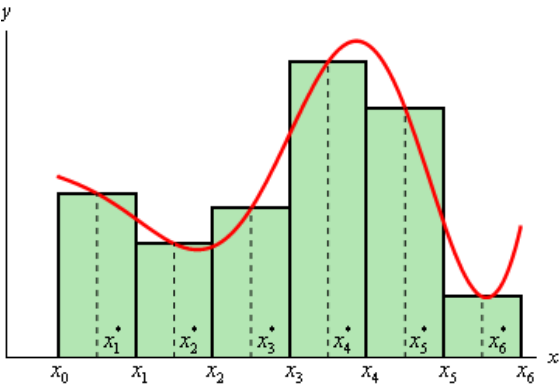
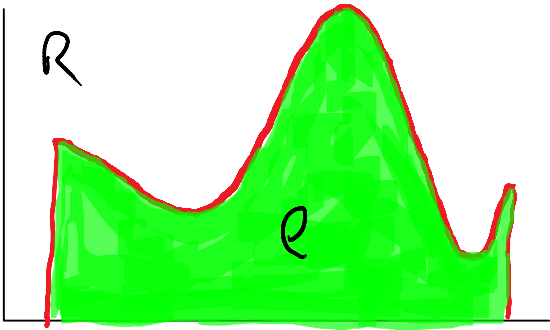
Centroid: The center of mass of a plane or 2d object.

Until you learn about integrals over a plane we will need to use some physical principles to find the Centroid of a 2d region R. They are:

- 1.The symmetry principle says that if R is symmetric about a line l , then the centroid of R lies on l.
- 2.If R is reflected about the line l then R remains the same so its centroid remains fixed.



$m = \rho A , x_{cm} = \frac{w}{2} , y_{cm} = \frac{L}{2}$



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$$M_y = \rho \int_a^b x f(x) dx$$

$$M_x = \rho \int_a^b \frac{1}{2} (f(x))^2 dx$$

$$m = \rho \int_a^b f(x) dx$$

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Find the Centroid of the region bounded by the curves $y = 4 - x^2$ and $y = 0$