

Sec 7.1 Modeling With Differential Equations

Differential Equations are equations which relate a desired function and the rates it changes with it's independent variable.

Rates of change are given in terms of derivatives thus we use differential equations.

Example: How might population change with time? Sit back and think about it.



What does the solution look like?

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Did we miss anything with the model $\frac{dP}{dt} = kP$? What if the population is too high to be supported by it's environment?

We can model this using the idea of a carrying capacity. The carrying capacity is the largest population which can be supported by its environment, lets call it M.

When the population is small we would expect it to grow proportional to itself

$$\frac{dP}{dt} = kP$$

When it gets above it's carrying capacity (M) we would expect the population to decrease ie

$$\frac{dP}{dt} < 0 . \text{ How could we model this?}$$



What do the solutions look like?

$$\frac{dP}{dt} = kP \left(1 - \frac{P}{m} \right)$$

The Logistic equation

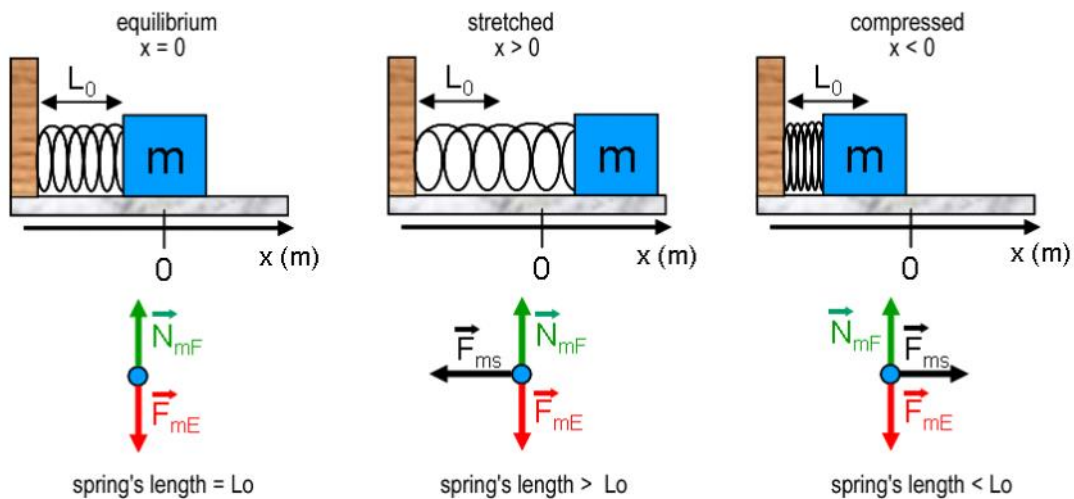
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Newton's Law and differential equations.

When modeling physical systems one relies on the laws of physics for guidance. One of the most powerful is Newton's second law

$$F = m \frac{d^2x}{dt^2}$$

Lets apply Newton's second law to model the motion of a spring. Recall that the restoring force of a stretch or compressed spring is $F=-kx$ where k is a material property of the spring.



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Ok So We can write down all sorts of Differential Equations . What Next? Well Lets define the solution to a differential equation.

The Solution to a differential equation is a function that satisfies the differential equation. I'll show you what I mean.

Consider the differential equation (DE) $\frac{d^2x}{dt^2} = -4x$ I claim a solution to this DE is $x=\sin(2t)$ Lets prove it.

Are there any other solutions?

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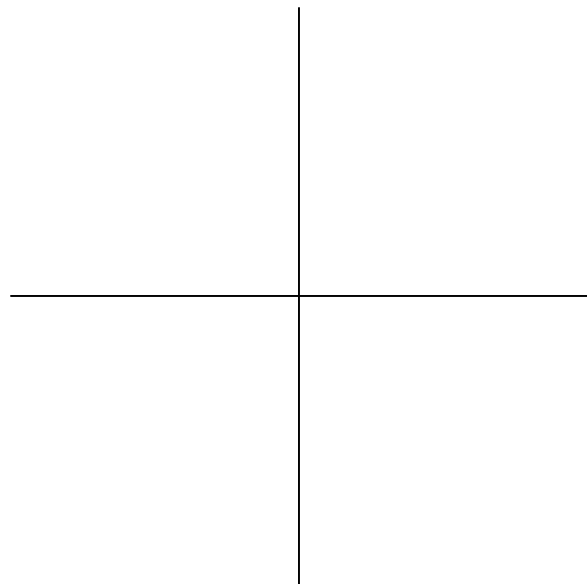
HW Example Problem 1 sec 7.1

Show that $y = \frac{2}{3}e^x + e^{-2x}$ is a solution to the DE $y' + 2y = e^{2x}$

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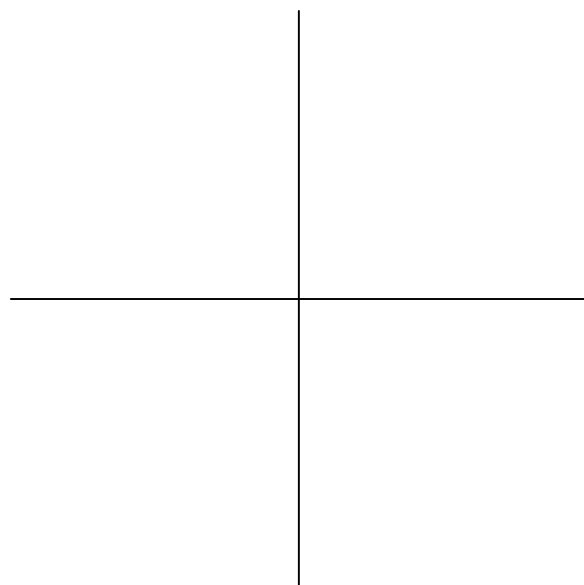
How do we find solutions to differential equations? Sometimes we flat out guess but other times we have methods.

Consider the differential equation $y' = x$ This reads as $\frac{dy}{dx} = x$ That is to say it is implied that y is a function of x



Initial Value Problem: An initial value problem is a DE with an initial condition which allows us to find unknown constants.

Find the solution to the IVP $y' = x^2$, $y(0) = 2$



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A Special Solution of a DE the equilibrium solution. Things are considered to be in equilibrium when they are no longer changing. This would mean that our derivatives would be zero.

Example: What are the equilibrium solutions of the logistic equation $\frac{dP}{dt} = kP \left(1 - \frac{P}{m}\right)$?

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Bonus question for your quiz. Consider the DE

$\frac{dx}{dt} = x^2 + a$ For what values of (a) are there equilibrium solutions?