

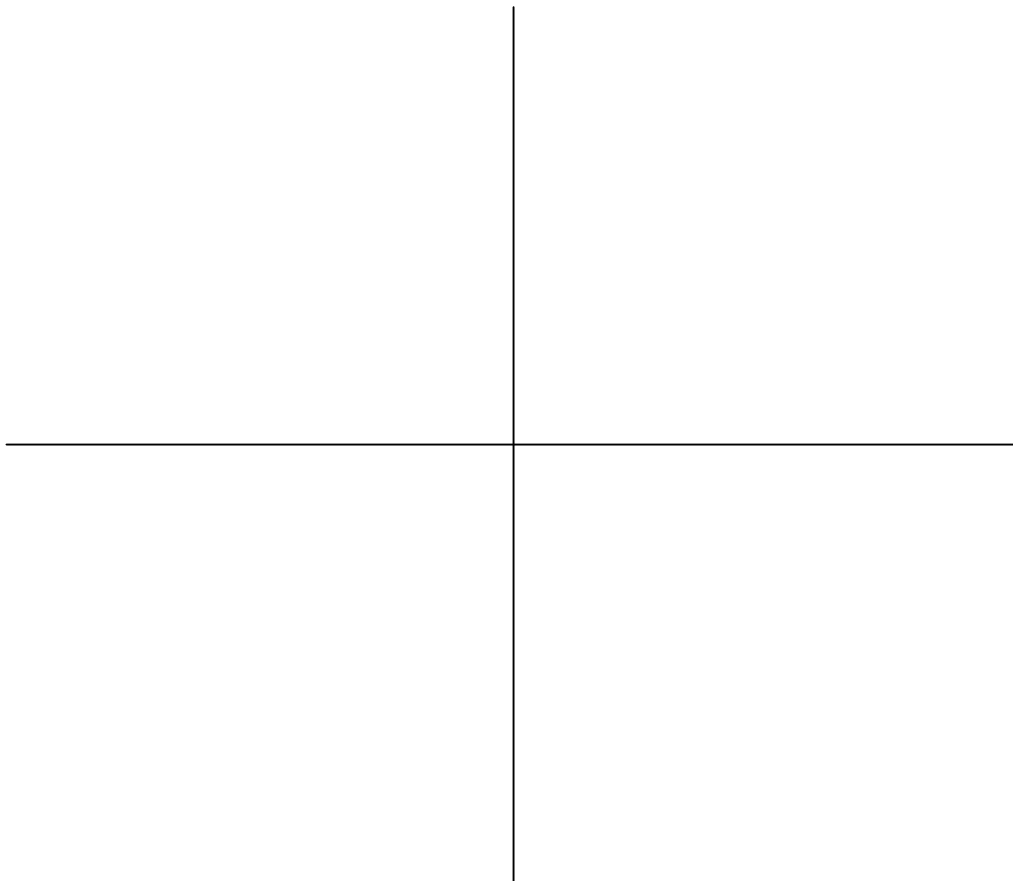
Sec 7.2 (Direction Fields and Euler's Method)

Suppose we have a differential equation for which we do not know the solution nor can we solve it easily. Is there a way we can gain an understanding of the dynamics of the system?

YES! We can look at the direction field.

Recall that the derivative of a function gives us the slope of the line tangent to the curve at a given point. For example consider

$$y = x^2 \Rightarrow \frac{dy}{dx} = 2x$$

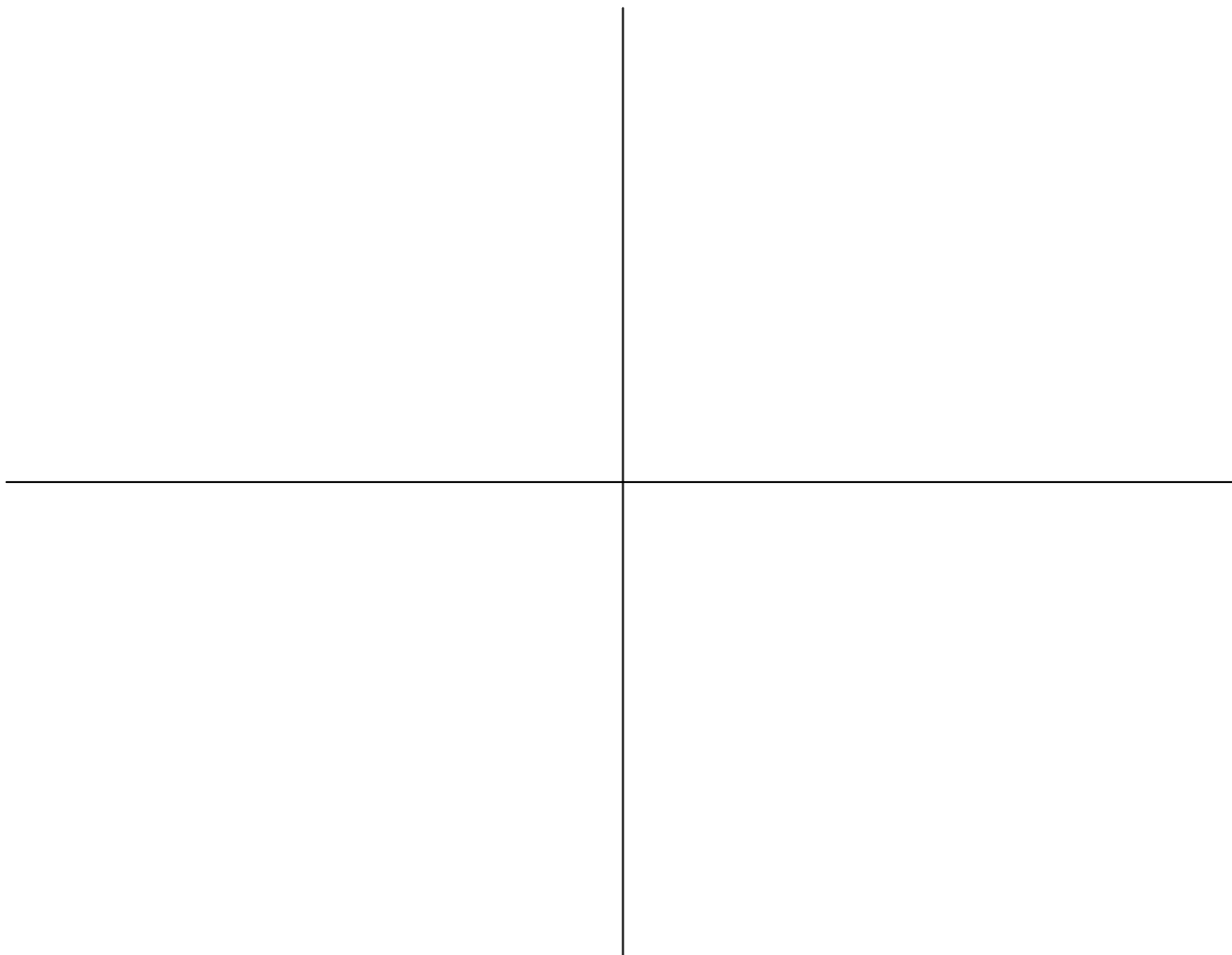


So derivatives of equations can give us information about the function itself! Differential equations involve derivatives and so we should be able to learn about the solution through them.

Sec 7.2 (Direction Fields and Euler's Method)

Consider the differential equation :

$\frac{dy}{dx} = x + y$ Lets sketch some of the slopes of the tangent lines at various (x,y) pairs in the x,y plane.



Suppose $y(0)=1$ What would the solution look like?

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Example from your Lab 2 :

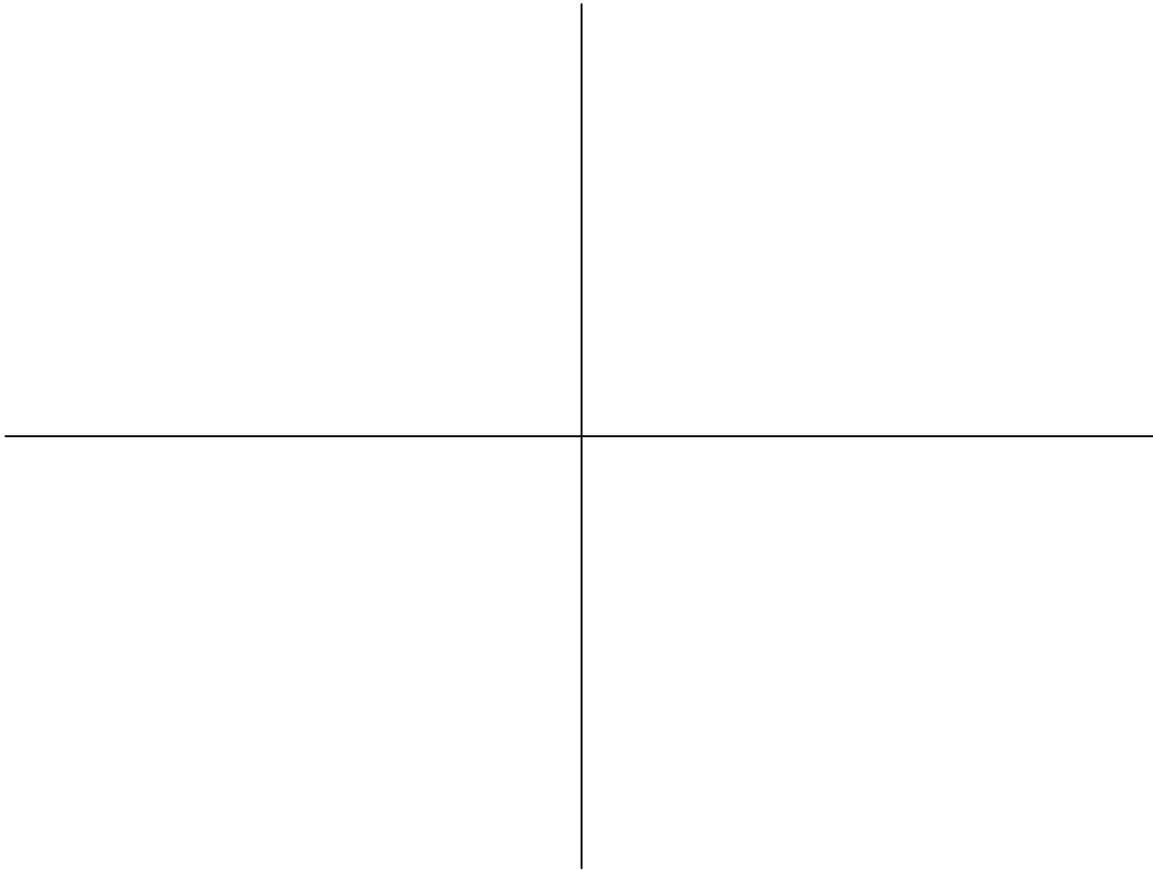
A flow meter is a device used to measure fluid flow velocities. Suppose we use a flow meter in to measure the fluid velocities on the surface of a lake. It is found that the x velocity is given by :

$$\frac{dx}{dt} = \frac{-y}{x^2 + y^2}$$

While the y velocity is given by :

$$\frac{dy}{dt} = \frac{x}{x^2 + y^2}$$

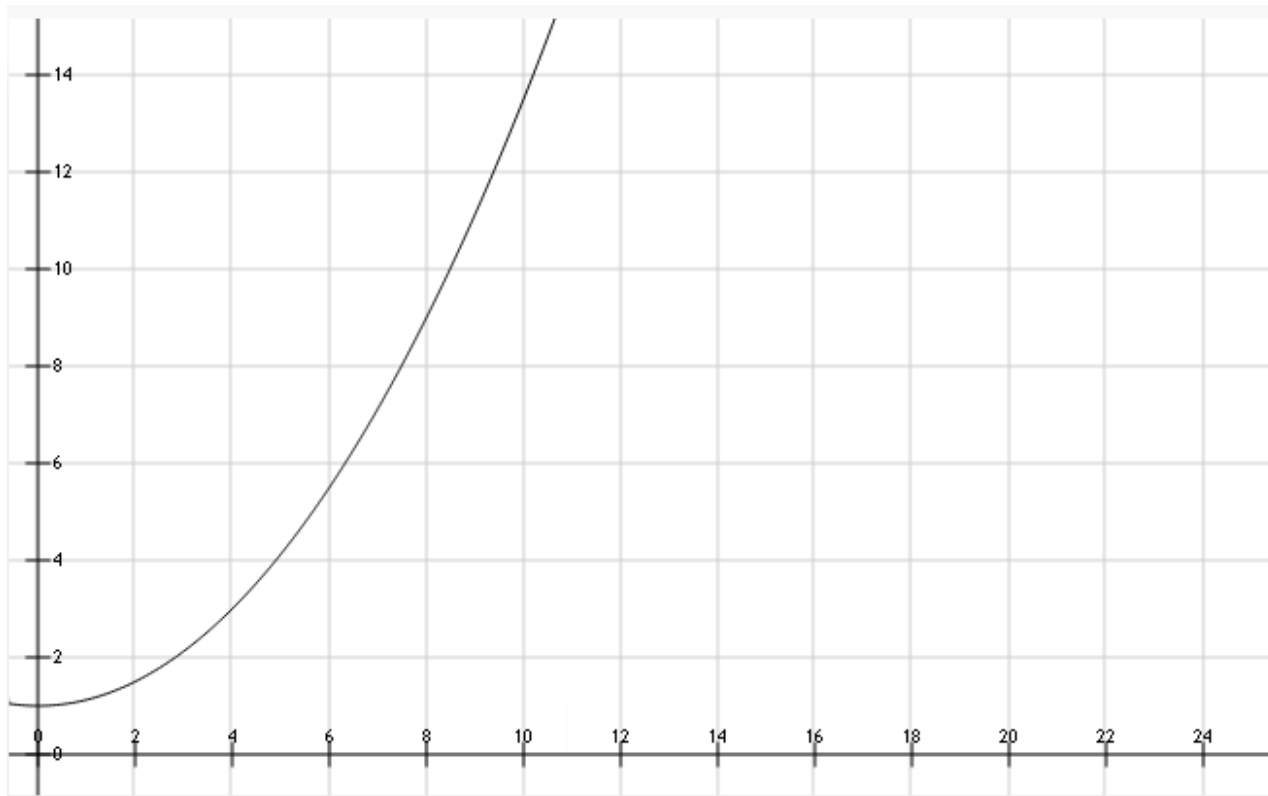
Find an expression for $\frac{dy}{dx}$ and sketch the direction field to understand how the water is moving.



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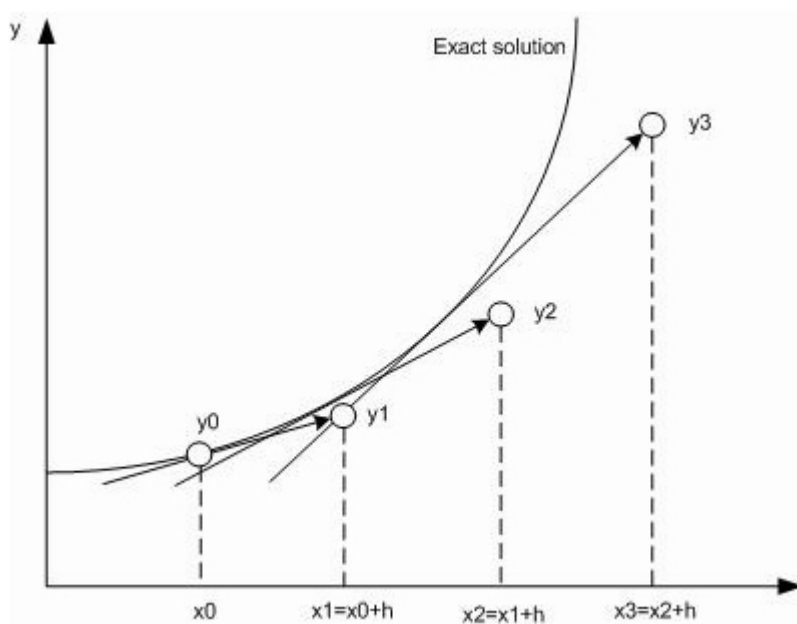
So we can get a sense of the solution to a differential equation from the direction field, this leads one to wonder if we can actually obtain the solution from it. The answer is that we can at least get close. Here is the idea: Below is the function

$$y = \frac{1}{8}x^2$$

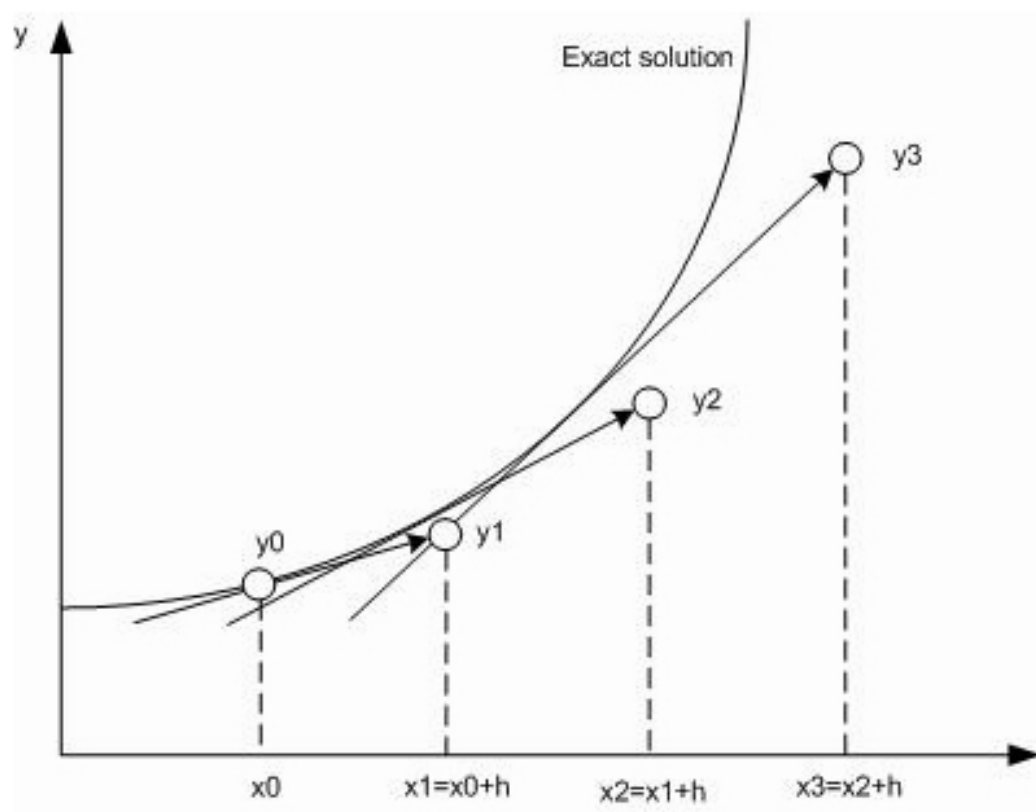


In Steps what we will do is:

1. Calculate the slope of the tangent line using the DE at the given initial value.
2. Using that slope we follow the tangent line for a small distance and see where we end up.
3. At the place we end up we calculate that the slope of the tangent line for the function at that point.
4. We then repeat steps 2 and 3 until we have enough of the approximate curve for our purposes.



Sec 7.2 (Direction Fields and Euler's Method)



Euler's Method: Approximate values for the solution of the initial value problem $y' = F(x, y)$, $y(x_0) = y_0$ with step size h at $x_n = x_{n+1} + h$ can be found using the recursive relationship:

$$y_n = y_{n-1} + hF(x_{n-1}, y_{n-1}) \quad n = 1, 2, 3, \dots$$

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HW Example: #21 Use Euler's method with step size $h=.5$ to compute the approximate y -values y_1, y_2, y_3, y_4 Of the solution to the IVP $y'=y-2x$, $y(1)=0$

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