

Sec 7.3 (Separable DE's)

Sunday, May 25, 2014

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We so far have looked at what a differential equation is and what a solution to a DE is and solved very simple equations like $y' = 2x \rightarrow y = x^2 + c$. But there are differential equations that we can not just directly integrate.

Ex. $\frac{dy}{dx} = 4x^2 y^3$

$$\int \frac{dy}{dx} = \int 4x^2 y^3 dx \quad Y(x)$$
$$Y(x) = ?$$

How can we solve this equation. Well this equation is what we call a separable differential equation and we can solve it with the method of separation of variables.

Definition: Separable differential equation. A separable differential equation is one in which we can write our derivative as a product or quotient of two functions each only depending on one variable.

Ex. $\frac{dy}{dx} = \underbrace{g(x)} \underbrace{f(y)}$ or $\frac{dy}{dx} = \frac{g(x)}{f(y)}$ or $\frac{dy}{dx} = \frac{f(y)}{g(x)}$

Ex. $\frac{dy}{dx} = 4x^2 y^3 = g(x) f(y)$

$\downarrow \quad \downarrow$

$g(x) \quad f(y)$

$$g(x) = 4x^2$$
$$f(y) = y^3$$

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$$\frac{dy}{dx} = \frac{f(x)}{g(y)} = f(x) \left(\frac{1}{g(y)} \right)$$

Method of Separation of variables.

To solve a separable equation like $\frac{dy}{dx} = g(x)f(y)$ we write it in differential form

$$\frac{1}{f(y)} dy = g(x) dx$$

$$\frac{dy}{dx} = g(x)f(y) \quad dx \quad dy = g(x)f(y) dx$$

That is to say I separate all of the x stuff and y stuff on either side of the equation. I can then integrate both sides.

$$\frac{dy}{f(y)} = g(x) dx$$

Ex. Solve $\frac{dy}{dx} = 4x^2 y^3$

~~$y = x + 2$~~

$$\frac{dy}{dx} = 4x^2 y^3$$

$$dy = 4x^2 y^3 dx$$

$$\int \frac{dy}{y^3} = \int 4x^2 dx$$

$$\int y^{-3} dy =$$

$$\frac{y^{-2}}{-2} = \frac{4x^3}{3} + C$$

$$-\frac{1}{2y^2} = \frac{4x^3}{3} + C$$

$$-2y^2 = \frac{1}{\frac{4x^3}{3} + C}$$

$$y^2 = \frac{1}{-2(\frac{4x^3}{3} + C)}$$

$$y = \sqrt{\frac{1}{-2(\frac{4x^3}{3} + C)}}$$

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Separate and solve the following DE's With the given initial condition.

$$y \frac{dy}{dx} = \frac{\ln(x)}{xy} \quad y(1) = 2$$

$$\frac{dx}{dx} y = \frac{\ln(x)}{x}$$

$$\int y dy = \int \frac{\ln(x)}{x} dx = \int \ln(x) \frac{1}{x} dx \quad \text{let } u = \ln(x)$$

$$\frac{du}{dx} = \frac{1}{x}$$

$$du = \frac{1}{x} dx$$

$$\frac{y^2}{2} = \int u du$$

$$\frac{y^2}{2} = \frac{u^2}{2} + C$$

$$\frac{y^2}{2} = \frac{(\ln(x))^2}{2} + C$$

$$y^2 = (\ln(x))^2 + \tilde{C}$$

$$y^2 = (\ln(x))^2 + 4$$

$$y = \sqrt{(\ln(x))^2 + 4}$$

$$u = \ln(x)$$

$$y(1) = 2$$

$$2^2 = (\ln(1))^2 + \tilde{C}$$

$$4 = \tilde{C}$$

$$\frac{y^2}{2} = \frac{(\ln(x))^2}{2} + C$$

$$\frac{2^2}{2} = \frac{(\ln(1))^2}{2} + C$$

$$\frac{4}{2} = C$$

$$C = 2$$

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$$\frac{du}{dt} = (2t + \sec^2 t) \frac{1}{2u} \quad u(0) = -5$$

$$du = (2t + \sec^2 t) \frac{1}{2u} dt$$

$$\int 2u du = \int (2t + \sec^2 t) dt$$

$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$

$$\cancel{\frac{2}{2}} u^2 = \int 2t dt + \int \sec^2(t) dt$$

$$u^2 = \cancel{\frac{2}{2}} t^2 + \tan(t) + C \quad u(0) = -5$$

$$(-5)^2 = \cancel{\frac{2}{2}} (0)^2 + \cancel{\tan(0)} + C$$

$$25 = C$$

$$u^2 = t^2 + \tan(t) + 25$$

$$u(t) = \sqrt{t^2 + \tan(t) + 25}$$

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$$x \ln(x) = y(1 + \sqrt{3 + y^2}) y'$$

Trick
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$$x \ln(x) = y(1 + \sqrt{3 + y^2}) \frac{dy}{dx}$$

$$\int x \ln(x) dx = \int y(1 + \sqrt{3 + y^2}) dy$$

$$u = 3 + y^2$$

$$du = 2y dy$$

$$dy = \frac{du}{2y}$$

↓

$$u = \ln(x) \quad du = \frac{1}{x} dx$$

$$du = \frac{1}{x} dx \quad u = \frac{x^2}{2}$$

$$\int u du = u \cdot u - \int u du$$

$$\frac{x^2}{2} \ln(x) - \int \frac{x^2}{2} \cdot \frac{1}{x} dx$$

$$\frac{x^2}{2} \ln(x) - \frac{1}{2} \int x dx$$

$$\frac{x^2}{2} \ln(x) - \frac{1}{2} \frac{x^2}{2}$$

$$\frac{x^2}{2} \left(\ln(x) - \frac{1}{2} \right)$$

$$\int y(1 + \sqrt{u}) \frac{du}{2y}$$

$$\frac{1}{2} \int (1 + \sqrt{u}) du$$

$$= \frac{1}{2} \int 1 du + \frac{1}{2} \int u^{1/2} du$$

$$= \frac{1}{2} u + \frac{1}{2} \frac{u^{3/2}}{3/2}$$

$$= \frac{1}{2} u + \frac{1}{2} \cdot \frac{2}{3} u^{3/2}$$

$$= \frac{1}{2} u + \frac{1}{3} u^{3/2}$$

$$= \frac{1}{2} (3 + y^2) + \frac{1}{3} (3 + y^2)^{3/2}$$

$$\boxed{\frac{x^2}{2} \left(\ln(x) - \frac{1}{2} \right) = \frac{1}{2} (3 + y^2) + \frac{1}{3} (3 + y^2)^{3/2} + C}$$