

Sec8.4(More Convergence Tests)

When we finally get to Taylor series we will want to know when and for what values they converge. As a preliminary step we are going to arm you with some useful convergence tests.

Def: An Alternating Series is a series whose terms alternate between positive and negative signs.

$$\sum_{n=1}^{\infty} (-1)^n \frac{1}{n} =$$

Alternating Series are nice in the sense that checking their convergence only comes down to taking the limit of the absolute value of the sequence.

The Alternating Series Test If the alternating series

$$\sum_{n=1}^{\infty} (-1)^{n-1} b_n = b_1 - b_2 + b_3 - b_4 + b_5 - b_6 + \dots \quad (b_n > 0)$$

satisfies

$$b_{n+1} \leq b_n \quad \text{for all } n$$
$$\lim_{n \rightarrow \infty} b_n = 0$$

then the series is convergent.

Ex. Show that the series $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n}$ converges

Sec8.4(More Convergence Tests)

Ex. Show that the alternating series below converges.

$$\sum_{n=0}^{\infty} (-1)^n \frac{n}{n^2 + 1}$$

The Alternating Series Test If the alternating series

$$\sum_{n=1}^{\infty} (-1)^{n-1} b_n = b_1 - b_2 + b_3 - b_4 + b_5 - b_6 + \dots \quad (b_n > 0)$$

satisfies

$$b_{n+1} \leq b_n \quad \text{for all } n$$

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then the series is convergent.

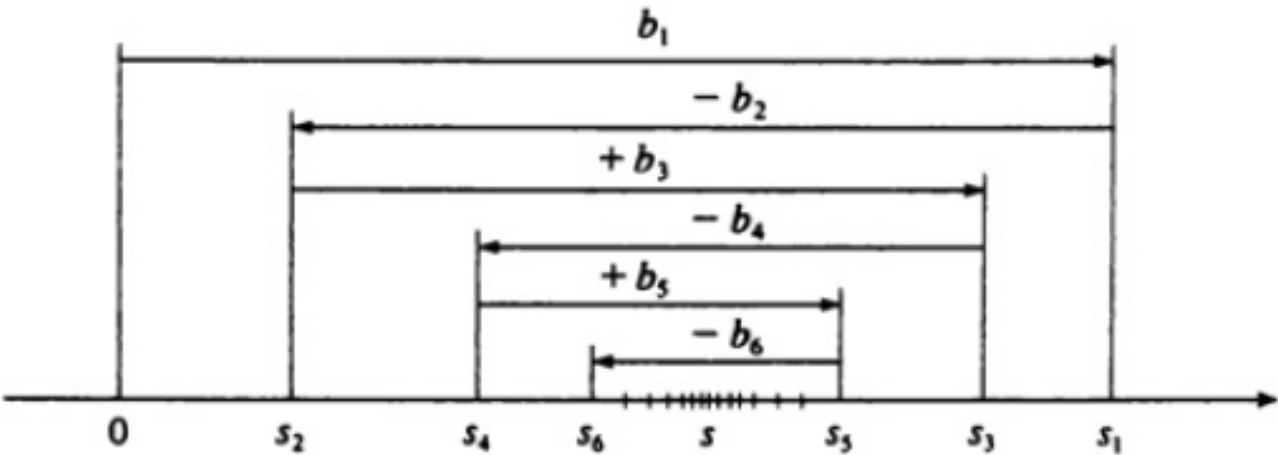
Sec8.4(More Convergence Tests)

Estimating the Error of an alternating series.

Alternating Series Estimation Theorem If $s = \sum (-1)^{n-1} b_n$ is the sum of an alternating series that satisfies

(i) $b_{n+1} \leq b_n$ and (ii) $\lim_{n \rightarrow \infty} b_n = 0$

then $|R_n| = |s - s_n| \leq b_{n+1}$



Sec8.4(More Convergence Tests)

Estimate the error in sum of the alternating series. Using the 5th partial sum. How many terms do you need to take to ensure the error is less than 0.0001 ?

$$\sum_{n=0}^{\infty} (-1)^n \frac{1}{n^2 + 1}$$

Sec8.4(More Convergence Tests)

Absolute Convergence.

Definition A series $\sum a_n$ is called **absolutely convergent** if the series of absolute values $\sum |a_n|$ is convergent.

An absolutely convergent Series

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n^2}$$

A conditionally convergent series

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n}$$

1 Theorem If a series $\sum a_n$ is absolutely convergent, then it is convergent.

The Ratio Test

- (i) If $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = L < 1$, then the series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent (and therefore convergent).
- (ii) If $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = L > 1$ or $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \infty$, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.
- (iii) If $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = 1$, the Ratio Test is inconclusive; that is, no conclusion can be drawn about the convergence or divergence of $\sum a_n$.

Ex Determine whether the series below converges or not.

$$\sum_{n=0}^{\infty} \frac{n^2}{n!}$$

Sec8.4(More Convergence Tests)

Determine the convergence of the sequence

$$\sum_{n=1}^{\infty} \frac{(-2)^n}{(2n)!}$$