

Sec 8.5 (Power Series)

A Power Series is just what it sounds like. A series involving powers of some quantity. Something like:

$$\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + x^4 + x^5 + \dots$$

Of course all of the coefficients of the powers don't have to be 1 or even the same... Formally a power series is a series of the form :

$$\sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 + c_5 x^5 + \dots$$

We will see that many functions can be represented by power series. Thus one very important question is when will a power series converge?

Consider

$$\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + x^4 + x^5 + \dots$$

When will this series converge?

We consider the series above to be centered at 0 but we can center our series where ever we like. A power series centered at x=a is defined by

$$\sum_{n=0}^{\infty} c_n (x - a)^n = c_0 + c_1 (x - a) + c_2 (x - a)^2 + c_3 (x - a)^3 + c_4 (x - a)^4 + c_5 (x - a)^5 + \dots$$

When suppose $c_n=1$ for all n , will this series converge?

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How can we determine when a power series will converge when $c_n \neq 1$ or just some other constant.

Ex Find the values for x for which the series below converges

$$\sum_{i=1}^{\infty} \frac{n(x-2)^n}{n+1}$$

Let's Draw a picture of this.

We call the distance away from the center that a power series will converge the Radius of convergence!

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3 Theorem For a given power series $\sum_{n=0}^{\infty} c_n(x - a)^n$ there are only three possibilities:

- (i) The series converges only when $x = a$.
- (ii) The series converges for all x .
- (iii) There is a positive number R such that the series converges if $|x - a| < R$ and diverges if $|x - a| > R$.

A Power series that converges only at its center

$$\sum_{i=1}^{\infty} n! x^n$$

A Power Series that converges for all x

$$\sum_{i=1}^{\infty} \frac{x^n}{n!}$$

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A Power series which converges for some values of x

$$\sum_{i=1}^{\infty} \frac{(-2)^n(x-1)^n}{n}$$

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Theorem For a given power series $\sum_{n=0}^{\infty} c_n(x-a)^n$ there are only three possibilities:
(i) The series converges only when $x = a$.
(ii) The series converges for all x .
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The Interval of convergence is the interval over which a power series converges.

