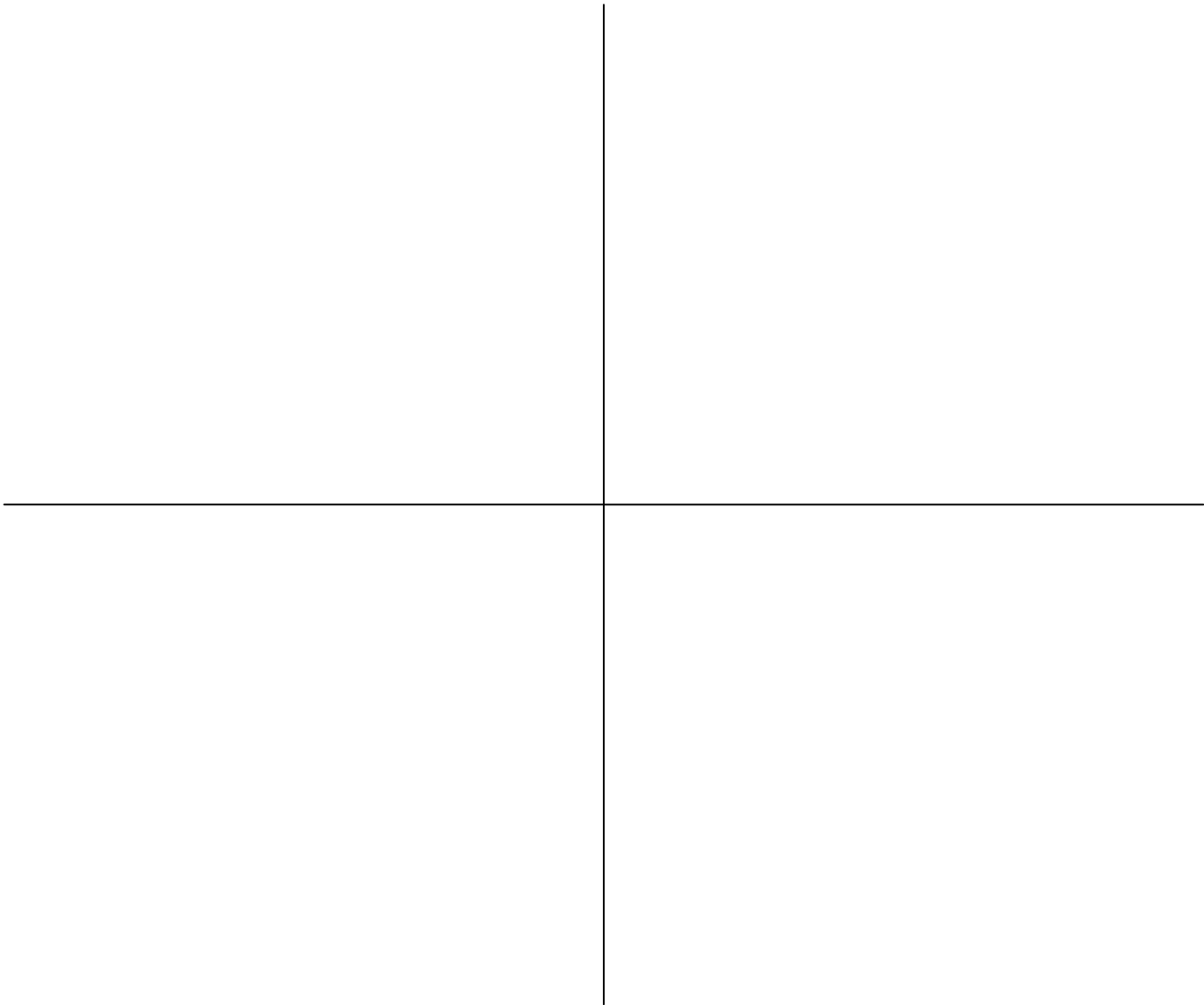


Sec 9.2 Vectors

A vector is a kind of number that has both magnitude *and* direction. This kind of number is particularly useful in science. For example a train can travel at 90 MPH but if we don't which direction , say east or west, it is moving we won't know where it ends up.

The speed of the train would be the magnitude of the velocity vector and east or west it's direction

A vector can be defined using two points. As it has direction it also has a starting point called the tip and an ending point called the tail. For example suppose a particle moves from the point (1,2) to the point (3,5). What would be the associated displacement vector? What the vector look like if it moved from (3,5) to (1,2)? What would be it's Length?



Sec 9.2 Vectors

Components of a vector. In order to make a vector an algebraic quantity we need a way to represent it with numbers instead of just an arrow with a length. Lets visit the particle that moved from (1,2) to (3,5).

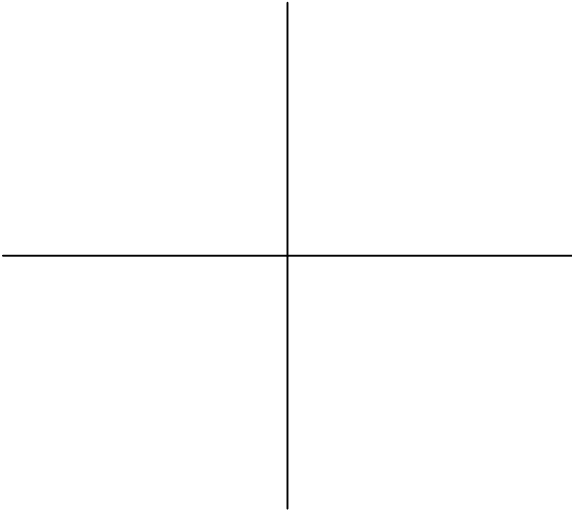
We saw that the x change in distance was +2 and the y change in distance was +3 We can then define our vector to be <2,3> that is the amount in the x direction we moved and the amount in the y direction that we moved.

If the particle moves from (3,5) to (1,2) then the x change is $1-3 = -2$ and the y change is $2-5 = -3$ so our vector would be <-2,-3>

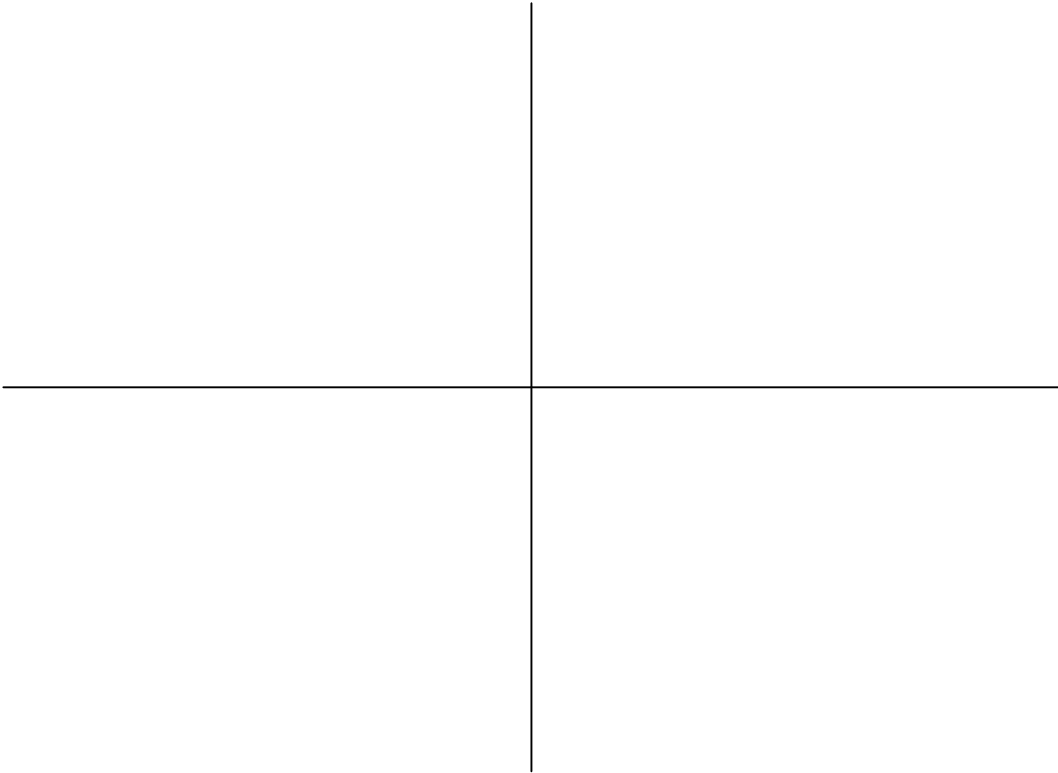
That is it tells us to go -2 in the x direction then -3 in the y direction. We call each change amount a component of the vector.

To draw these graphically we draw the arrows emanating from the origin.

In General if we are given two points we can find the vector between them by taking the tip minus the tail.

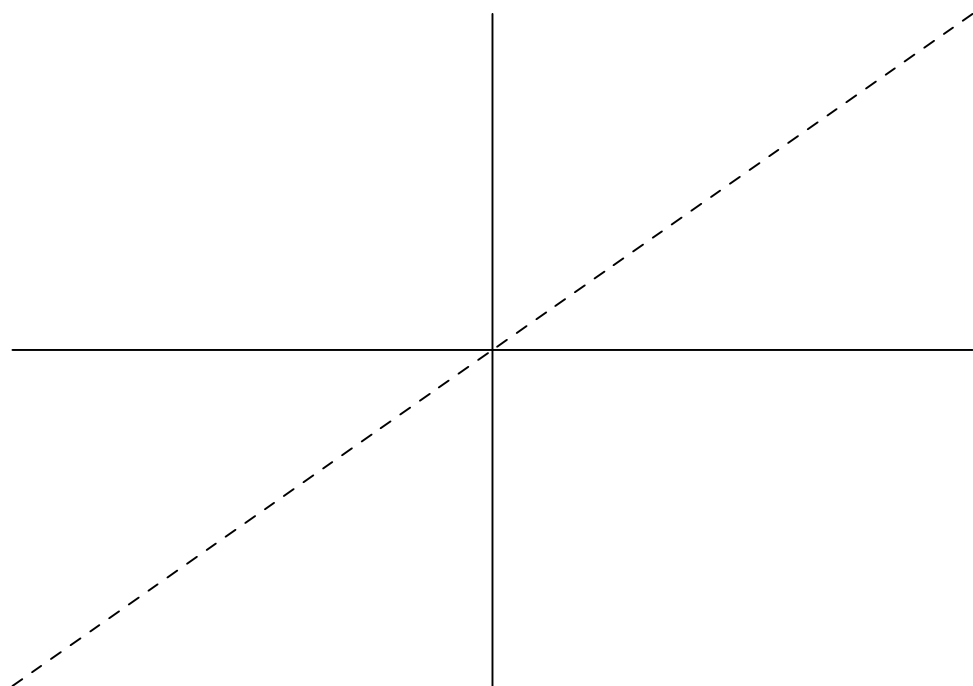


Ex. Define a vector for a particle that moves the point (-3,4) to the point (5,6) and draw it in the x-y plane.



Sec 9.2 Vectors

Finding a three dimensional vector. Define the vector between the 3-d points (1,2,3) and (4,5,6) and draw it in 3 space. How long is the vector?



1 Given the points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$, the vector $\mathbf{a}$ with representation $\overrightarrow{AB}$ is

$$\mathbf{a} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$$

The length of the two-dimensional vector $\mathbf{a} = \langle a_1, a_2 \rangle$ is

$$|\mathbf{a}| = \sqrt{a_1^2 + a_2^2}$$

The length of the three-dimensional vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ is

$$|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

Sec 9.2 Vectors

Adding vectors. To add two vectors we simply add the like components. For example add the two dimensional vectors $u=\langle 1,2\rangle$ $v=\langle 3,4\rangle$ and add the 3-d vectors $u=\langle 1,-2,3\rangle$ and $v=\langle 4,1,6\rangle$

Geometrically adding vectors amounts to traveling along one of them first the traveling along the other from where you left off on the first..... Ill draw it. Let $u=\langle 1,1\rangle$ $v=\langle 3,4\rangle$ find and draw $u+v$



If $\mathbf{a} = \langle a_1, a_2 \rangle$ and $\mathbf{b} = \langle b_1, b_2 \rangle$, then

$$\mathbf{a} + \mathbf{b} = \langle a_1 + b_1, a_2 + b_2 \rangle \qquad \mathbf{a} - \mathbf{b} = \langle a_1 - b_1, a_2 - b_2 \rangle$$
$$c\mathbf{a} = \langle ca_1, ca_2 \rangle$$

Similarly, for three-dimensional vectors,

$$\langle a_1, a_2, a_3 \rangle + \langle b_1, b_2, b_3 \rangle = \langle a_1 + b_1, a_2 + b_2, a_3 + b_3 \rangle$$
$$\langle a_1, a_2, a_3 \rangle - \langle b_1, b_2, b_3 \rangle = \langle a_1 - b_1, a_2 - b_2, a_3 - b_3 \rangle$$
$$c\langle a_1, a_2, a_3 \rangle = \langle ca_1, ca_2, ca_3 \rangle$$

Sec 9.2 Vectors

If $\mathbf{a} = \langle a_1, a_2 \rangle$ and $\mathbf{b} = \langle b_1, b_2 \rangle$, then

$$\mathbf{a} + \mathbf{b} = \langle a_1 + b_1, a_2 + b_2 \rangle \qquad \mathbf{a} - \mathbf{b} = \langle a_1 - b_1, a_2 - b_2 \rangle$$
$$c\mathbf{a} = \langle ca_1, ca_2 \rangle$$

Scalar multiplication .

Similarly, for three-dimensional vectors,

$$\langle a_1, a_2, a_3 \rangle + \langle b_1, b_2, b_3 \rangle = \langle a_1 + b_1, a_2 + b_2, a_3 + b_3 \rangle$$
$$\langle a_1, a_2, a_3 \rangle - \langle b_1, b_2, b_3 \rangle = \langle a_1 - b_1, a_2 - b_2, a_3 - b_3 \rangle$$
$$c\langle a_1, a_2, a_3 \rangle = \langle ca_1, ca_2, ca_3 \rangle$$

How do we calculate something like $4\langle 1, 2 \rangle$? Well

$$4\langle 1, 2 \rangle = \langle 1, 2 \rangle + \langle 1, 2 \rangle + \langle 1, 2 \rangle + \langle 1, 2 \rangle$$

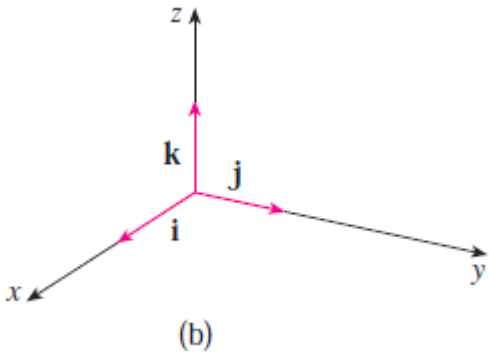
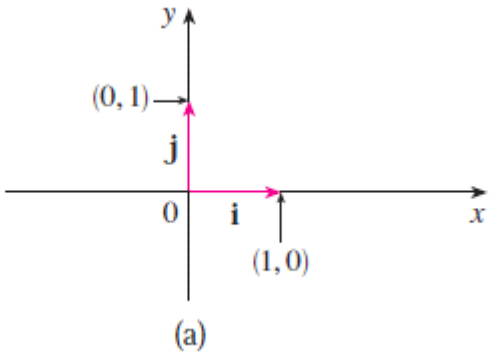
Properties of Vectors If $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ are vectors in V_n and c and d are scalars, then

- | | |
|---|--|
| 1. $\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$ | 2. $\mathbf{a} + (\mathbf{b} + \mathbf{c}) = (\mathbf{a} + \mathbf{b}) + \mathbf{c}$ |
| 3. $\mathbf{a} + \mathbf{0} = \mathbf{a}$ | 4. $\mathbf{a} + (-\mathbf{a}) = \mathbf{0}$ |
| 5. $c(\mathbf{a} + \mathbf{b}) = c\mathbf{a} + c\mathbf{b}$ | 6. $(c + d)\mathbf{a} = c\mathbf{a} + d\mathbf{a}$ |
| 7. $(cd)\mathbf{a} = c(d\mathbf{a})$ | 8. $1\mathbf{a} = \mathbf{a}$ |

Sec 9.2 Vectors

Another notation for vectors. We can represent vectors as sums of multiples of what we call basis vectors. While there are many sets of basis vectors the standard one is defined by

$$\mathbf{i} = \langle 1, 0, 0 \rangle \qquad \mathbf{j} = \langle 0, 1, 0 \rangle \qquad \mathbf{k} = \langle 0, 0, 1 \rangle$$



Show that the vector $\mathbf{v} = \langle 1, 2, 3 \rangle$ is equivalent to $\mathbf{v} = 1\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ Then add $\mathbf{v}$ to the vector $\mathbf{u} = 2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$

Sec 9.2 Vectors

Unit vectors: A unit vector is simply a vector with length one. We can turn any vector in to a unit vector by dividing by it's length.

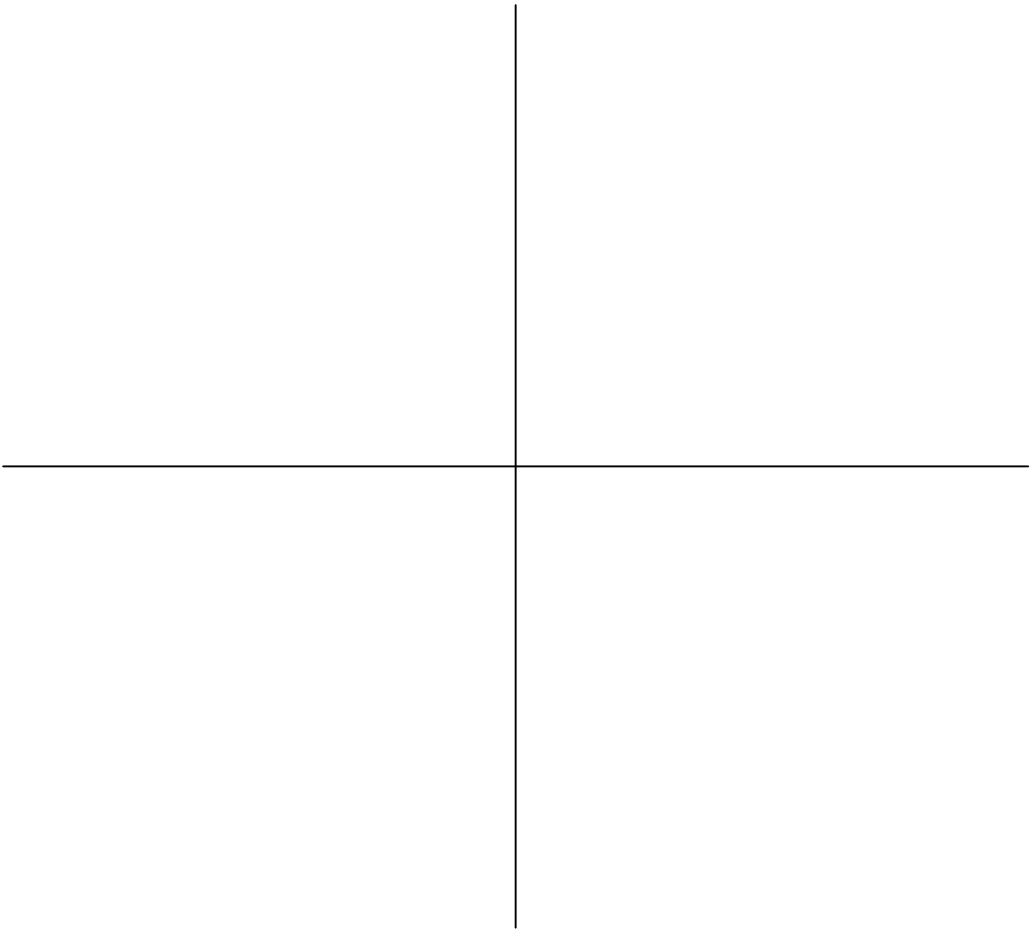
If $\mathbf{a}$ is a vector then one can find a unit vector in the same direction as $\mathbf{a}$ by calculating:

$$\mathbf{u} = \frac{1}{|\mathbf{a}|} \mathbf{a} = \frac{\mathbf{a}}{|\mathbf{a}|}$$

Example Find unit vectors in the directions of the vectors $\langle 3, 4 \rangle$ and $\langle 2, 4, 6 \rangle$

Sec 9.2 Vectors

Vectors defined with angles: Instead of components we can define a vector with an angle and a length.



Add the two vectors shown and find the angle the resultant makes with the x-axis.

27.

