

Sec 10.1 Vector Functions

We can define a vector function by putting parameterizations for x , y , z into a vector

That is if $x=f(t)$ $y= g(t)$ $z=h(t)$ we can define a vector function as:

$$\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle$$

We call $f(t)$, $g(t)$, $h(t)$ the component functions.

The domain of a vector function: This will be all of the t values we can plug in with no "mathematical issues" and which produce points that we can plot in 3d space.

Ex Find the domain of:

$$r(t) = \langle \sqrt{t+2}, t^2 + 3, \frac{1}{t} \rangle$$

$$\sqrt{t+2}$$

$t^2 + 3$

$\frac{1}{t}$

$t \neq 0$

$$t \geq -2$$

$$t+2 \geq 0$$

$$t \geq -2$$

$t \neq 0$

$$[-2, 0) \cup (0, \infty)$$

Range of a vector function. For a vector function we get vectors as outputs.

Ex Sketch $r(t)=\langle t, 2t, 3t \rangle$

$$r(t) = \langle 0, 0, 0 \rangle$$

$t \approx 1$

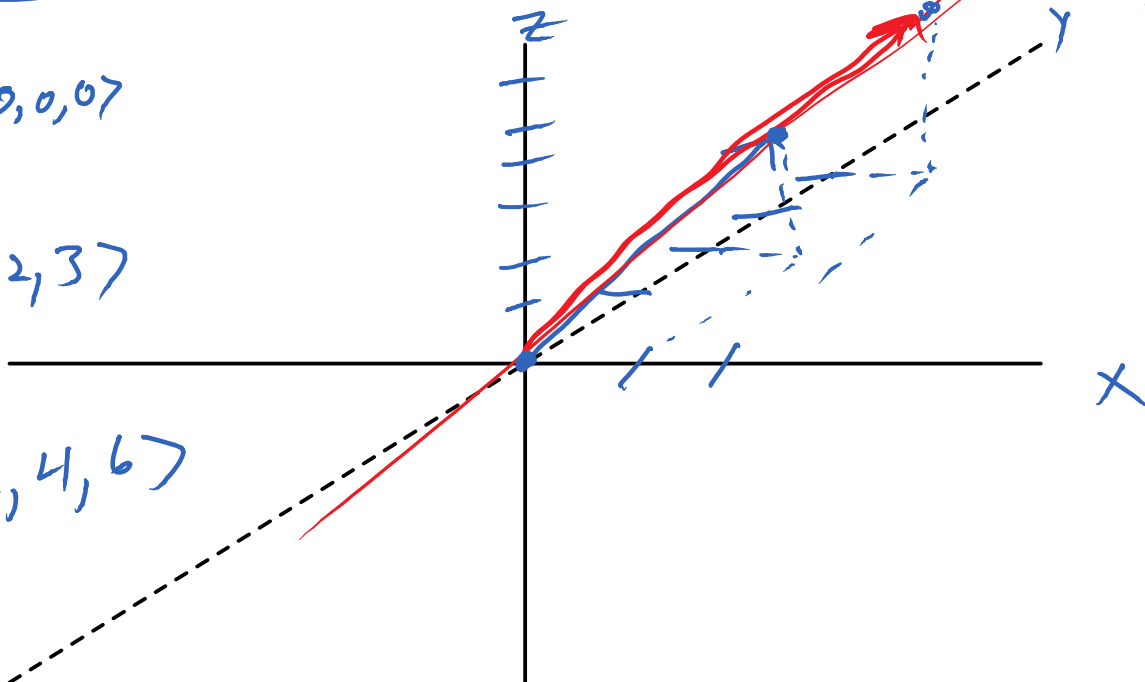
$$r(1) = \langle 1, 2, 3 \rangle$$

$$r(2) = \{2, 4, 6\}$$

$t = 0$

$t = 1$

$$L = 2$$





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The limit of a vector function $\mathbf{r}$ is defined by taking the limits of its component functions as follows.

1

If $\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle$, then

$$\lim_{t \rightarrow a} \mathbf{r}(t) = \left\langle \lim_{t \rightarrow a} f(t), \lim_{t \rightarrow a} g(t), \lim_{t \rightarrow a} h(t) \right\rangle$$

provided the limits of the component functions exist.

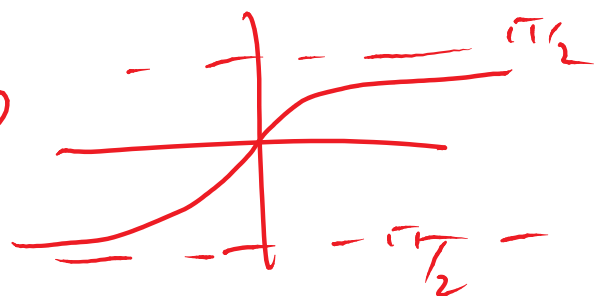
3-4 Find the limit.

3. $\lim_{t \rightarrow \infty} \left\langle \frac{1+t^2}{1-t^2}, \tan^{-1} t, \frac{1-e^{-2t}}{t} \right\rangle = \left\langle \lim_{t \rightarrow \infty} \frac{1+t^2}{1-t^2}, \lim_{t \rightarrow \infty} \tan^{-1}(t), \lim_{t \rightarrow \infty} \frac{1-e^{-2t}}{t} \right\rangle$

$\lim_{t \rightarrow \infty} \frac{1+t^2}{1-t^2} = -1$

$\lim_{t \rightarrow \infty} \tan^{-1}(t) = \pi/2$

$\lim_{t \rightarrow \infty} \frac{1-e^{-2t}}{t} = \frac{1}{\infty} = 0$



$= \langle -1, \pi/2, 0 \rangle$

Sec 10.1 Vector Functions

Finding the vector equation of a line.

$$y - y_0 = m(x - x_0)$$

↑ ↑ ↓

Hw example

15-18 Find a vector equation and parametric equations for the line segment that joins P to Q .

15. $P(0, 0, 0)$, $Q(1, 2, 3)$

$$\vec{r}_0 = \langle 1, 2, 3 \rangle$$

$$\vec{v} = (0, 0, 0) - (1, 2, 3)$$

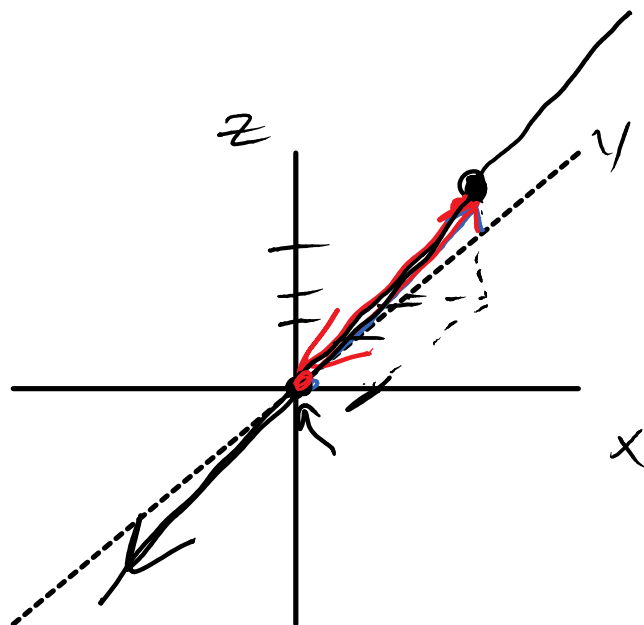
$$\begin{aligned}\vec{v} &= \langle 0-1, 0-2, 0-3 \rangle \\ &= \langle -1, -2, -3 \rangle\end{aligned}$$

$$\vec{r}(t) = \vec{r}_0 + \vec{v}t = \langle 1, 2, 3 \rangle + t\langle -1, -2, -3 \rangle$$

$$\vec{r}(t) = \langle 1, 2, 3 \rangle + \langle -t, -2t, -3t \rangle$$

$$\boxed{\vec{r}(t) = \langle 1-t, 2-2t, 3-3t \rangle} \leftarrow$$

$$\left\{ \begin{aligned}\vec{r}(t) &= \langle 0, 0, 0 \rangle + t\langle -1, -2, -3 \rangle \\ &= \langle -t, -2t, -3t \rangle\end{aligned} \right. \leftarrow$$



$$\begin{aligned}\vec{r}(t) &= \langle 0, 0, 0 \rangle + t \langle -1, -2, -3 \rangle \\ &= \langle -t, -2t, -3t \rangle\end{aligned}$$

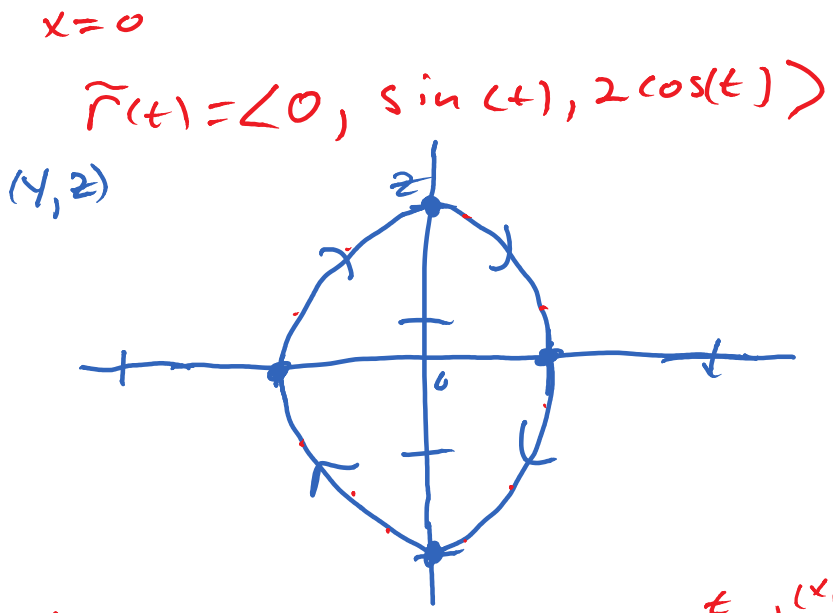
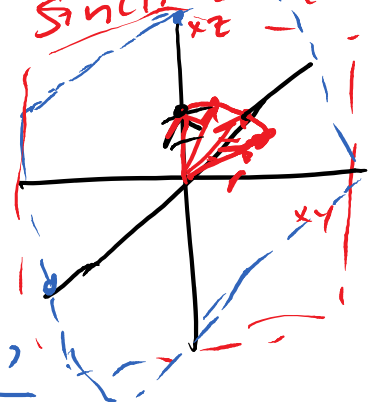
Sec 10.1 Vector Functions

Using projections on to the coordinate planes to sketch a curve.

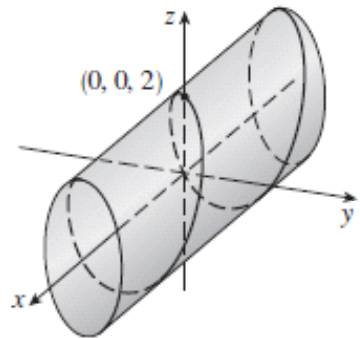
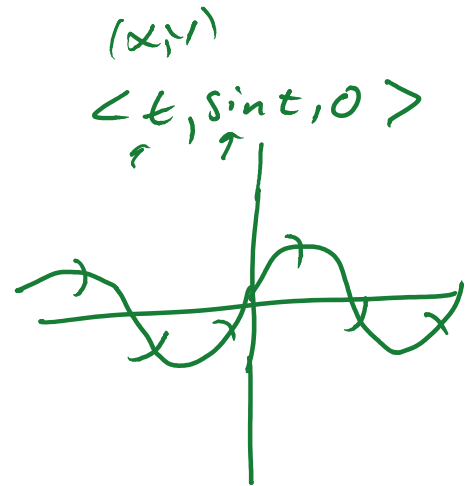
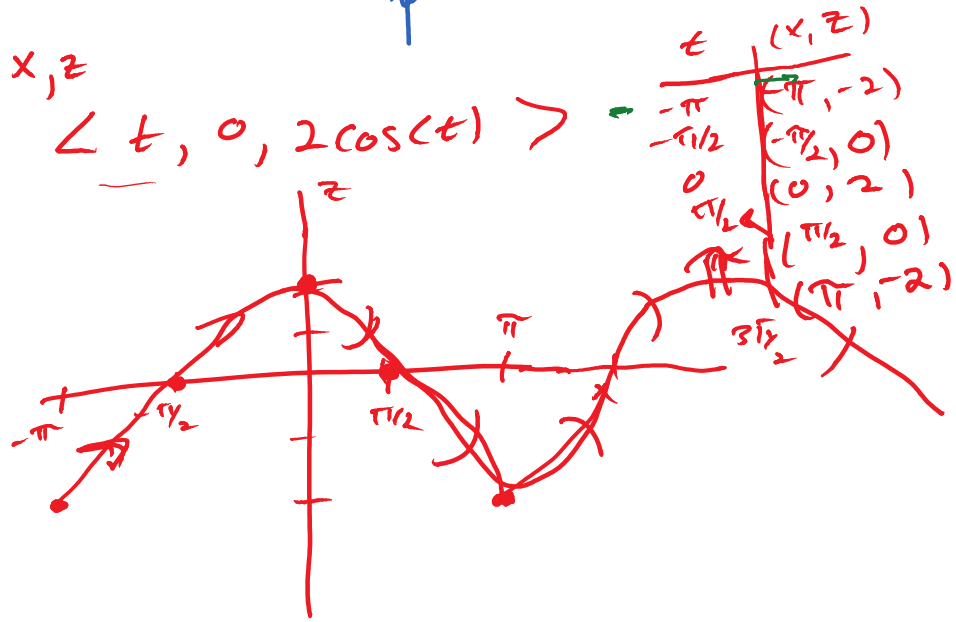
13-14 Draw the projections of the curve on the three coordinate planes. Use these projections to help sketch the curve.

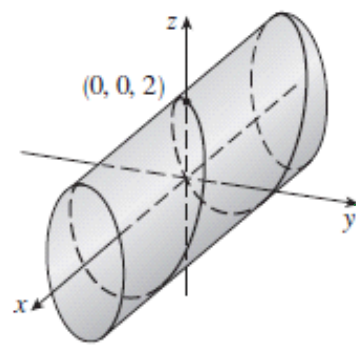
13. $\mathbf{r}(t) = \langle t, \sin t, 2 \cos t \rangle$

$\mathbf{r}(0) = \langle 0, \sin(0), 2 \cos(0) \rangle$
 $\langle 0, 0, 2 \rangle$
 $\langle 1, \sin(1), 2 \cos(1) \rangle$



t	(x, y)
$-\pi$	$(0, -2)$
$-\pi/2$	$(-1, 0)$
0	$(0, 2)$
$\pi/2$	$(1, 0)$
π	$(0, -2)$





$$\vec{r}(t) = \langle t \cos t, t \sin t, t \rangle$$

25. Show that the curve with parametric equations $x = t \cos t$, $y = t \sin t$, $z = t$ lies on the cone $z^2 = x^2 + y^2$, and use this fact to help sketch the curve.

$$z = t \quad x = t \cos t \quad y = t \sin t$$

$$z^2 = t^2 \quad x^2 = t^2 \cos^2 t \quad y^2 = t^2 \sin^2 t$$

$$z^2 = \underbrace{x^2} + \underbrace{y^2}$$

$$t^2 = t^2 \cos^2 t + t^2 \sin^2 t$$

$$t^2 = t^2 (\cos^2(t) + \sin^2(t))$$

$$t^2 = t^2 \quad \checkmark$$

$$16 = x^2 + y^2$$

$$4 = x^2 + y^2$$

$$(-2)^2 = x^2 + y^2$$

$$4 = x^2 + y^2$$

