

Sec 10.5 Parametric Surfaces

So far we have learned how to parameterize curves but can we do the same thing with a surface? Let's give it a shot. We know that from chapter 9 the equation of a cylinder of radius 2 is $x^2 + y^2 = 4$

How can we write this as a vector equation?

Let's first parameterize some level sets to get the idea. Choose

$z=2,4,6$

$z=2$

$r(t) = \langle 2\cos t, 2\sin t, 2 \rangle$

$z=4$

$r(t) = \langle 2\cos(t), 2\sin(t), 4 \rangle$

$z=6$

$r(t) = \langle 2\cos(t), 2\sin(t), 6 \rangle$

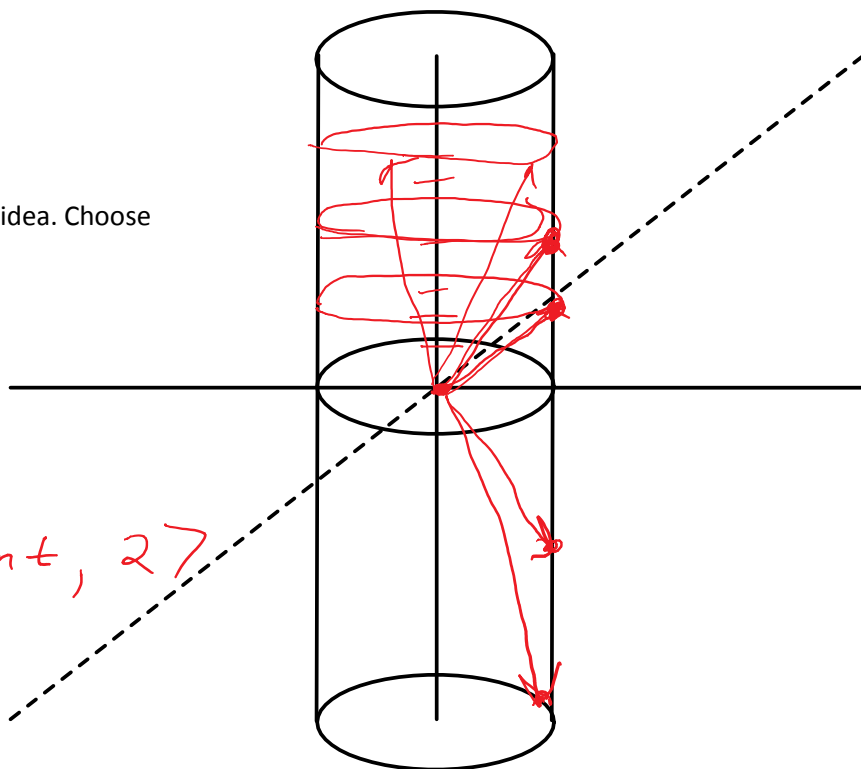
$r(t) = \langle 2\cos(t), 2\sin(t), t \rangle$

$r(u, v) = \langle 2\cos(u), 2\sin(u), v \rangle$

$r(\pi/4, 2) = \langle 2\sqrt{2}/2, 2\sqrt{2}/2, 2 \rangle$

So if we add a parameter we can have our vector equation for the cylinder!

$r(\pi/4, 4) = \langle 2\sqrt{2}/2, 2\sqrt{2}/2, 4 \rangle$



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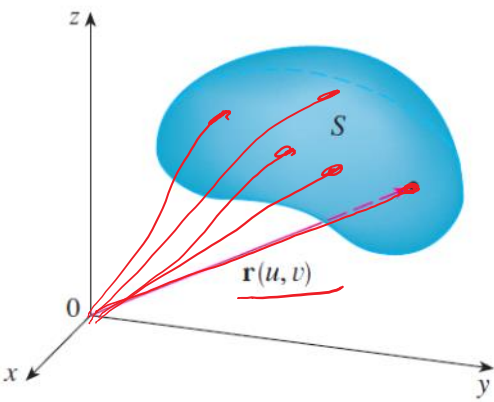
So in general the Vector and Parametric equations of surfaces are of the form.

1

$$\mathbf{r}(u, v) = x(u, v) \mathbf{i} + y(u, v) \mathbf{j} + z(u, v) \mathbf{k}$$

2

$$x = x(u, v) \quad y = y(u, v) \quad z = z(u, v)$$



HW example

3-6 Identify the surface with the given vector equation.

3. $\mathbf{r}(u, v) = (u + v) \mathbf{i} + (3 - v) \mathbf{j} + (1 + 4u + 5v) \mathbf{k}$

$\mathbf{r}(u, v) = \langle u + v, 3 - v, 1 + 4u + 5v \rangle$

$u = 0 \quad \langle x_0 + a\cancel{u}, y_0 + b\cancel{v}, z_0 + c\cancel{v} \rangle$

$\mathbf{r}(0, v) = \langle v, 3 - v, 1 + 5v \rangle$

$x_0 = 0 \quad y_0 = 3, \quad z_0 = 1$

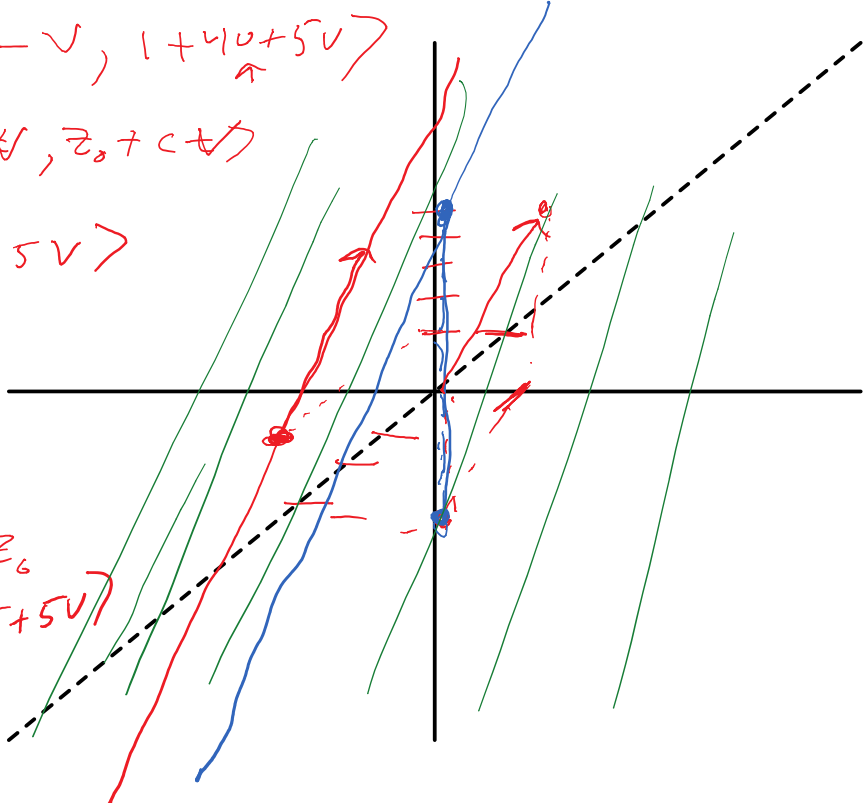
$\mathbf{r}'(v) = \langle 1, -1, 5 \rangle$

$u = 1 \quad x_0 \quad y_0 \quad z_0$

$\mathbf{r}(1, v) = \langle 1 + v, 3 - v, 5 + 5v \rangle$

$\mathbf{r}'(1, v) = \langle 1, -1, 5 \rangle$

To understand how to do this we can fix parameter values see what curves we get then see how varying the other parameter moves those curves to trace out a surface. Let's fix some values for u...



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Parametric Equation of a Plane: We can define the parametric equation of a plane in general if we have two vectors which are not parallel and which lie in the plane.

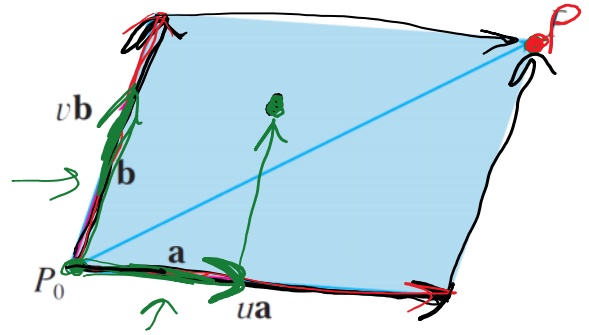


FIGURE 6

$$\vec{P_0P} = u\vec{a} + v\vec{b}$$

$$\vec{P_0} = \langle x_0, y_0, z_0 \rangle$$

$$\vec{P_0} + u\vec{a} + v\vec{b} = \vec{P}$$

$$\langle x_0, y_0, z_0 \rangle + u\langle a_1, a_2, a_3 \rangle + v\langle b_1, b_2, b_3 \rangle$$

$$\langle x_0, y_0, z_0 \rangle + \langle ua_1, ua_2, ua_3 \rangle + \langle vb_1, vb_2, vb_3 \rangle$$

$$\vec{r}(u, v) = \langle x_0 + ua_1 + vb_1, y_0 + ua_2 + vb_2, z_0 + ua_3 + vb_3 \rangle$$

Parametric equation of a plane

$$x = x_0 + ua_1 + vb_1 \quad y = y_0 + ua_2 + vb_2 \quad z = z_0 + ua_3 + vb_3$$

Where (x_0, y_0, z_0) is a point in the plane and vectors $\vec{a} = \langle a_1, a_2, a_3 \rangle$, $\vec{b} = \langle b_1, b_2, b_3 \rangle$ are any two non parallel vectors which lie in the plane.

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HW Example

Find the parametric equations for the surface defined by

23. The part of the sphere $x^2 + y^2 + z^2 = 4$ that lies above the cone $z = \sqrt{x^2 + y^2}$

$$x^2 + y^2 + x^2 + y^2 = 4$$

$$2x^2 + 2y^2 = 4 \rightarrow x^2 + y^2 = 2$$

Here we can simply parameterize the sphere
Then restrict u, v so that we only get points above the cone.

$$z = \sqrt{x^2 + y^2} \quad z = \sqrt{2}$$

$$r(u, v) \quad \phi, \theta$$

$$x = \rho \sin \phi \cos \theta$$

$$y = \rho \sin \phi \sin \theta$$

$$z = \rho \cos \phi$$

$$\rho = 2$$

$$r(u, v) = (2 \sin(u) \cos(v), 2 \sin(u) \sin(v), 2 \cos(u))$$

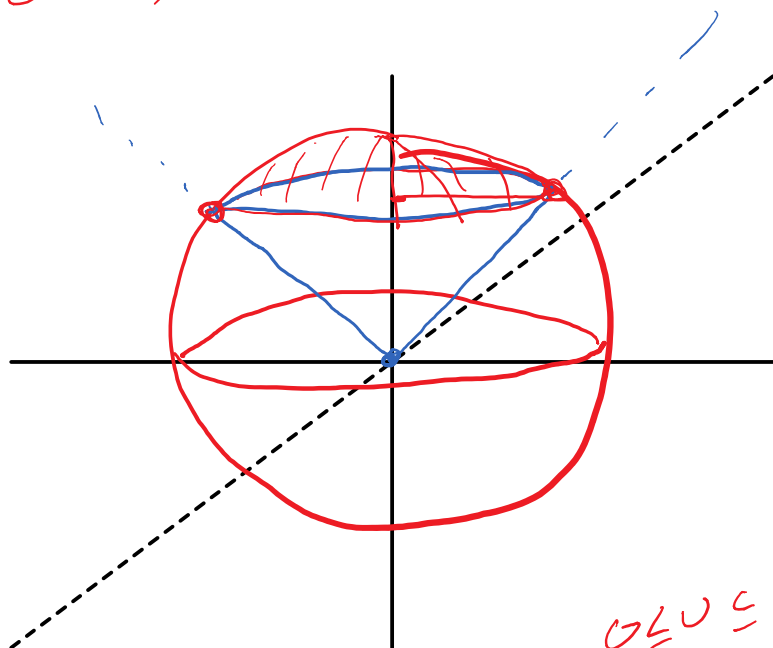
$$z = \sqrt{2} \quad \rho$$

$$\sqrt{2} = 2 \cos(u)$$

$$\frac{\sqrt{2}}{2} = \cos(u)$$

$$u = \pi/4$$

$$0 \leq u \leq \pi/4$$



$$0 \leq v \leq \pi/4$$

