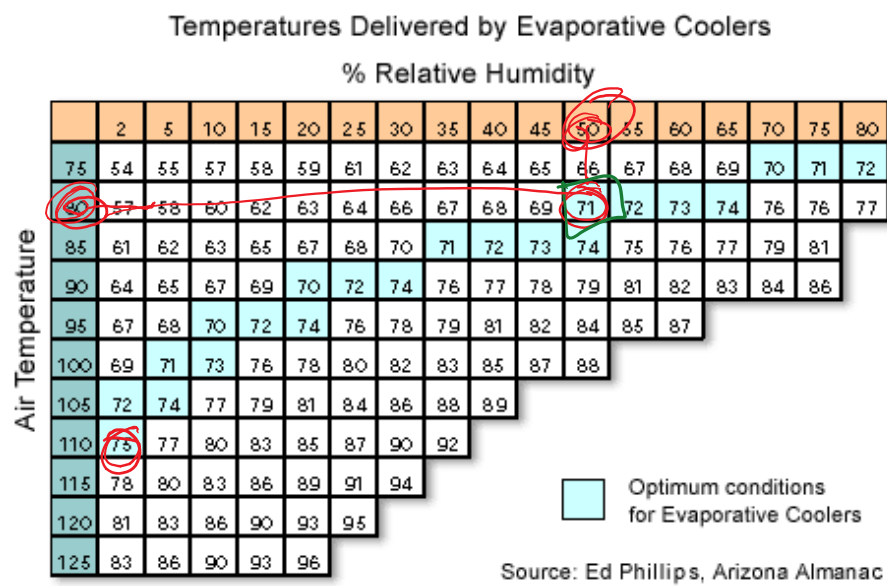


Sec 11.1 Functions of Several Variables

Functions can be represented in many ways.... Verbally, equations , charts and more. To get a sense for functions of more than one variable let's examine the chart below. It defines the temperature that a swamp cooler can blow given the outside air temperature and the % relative humidity.

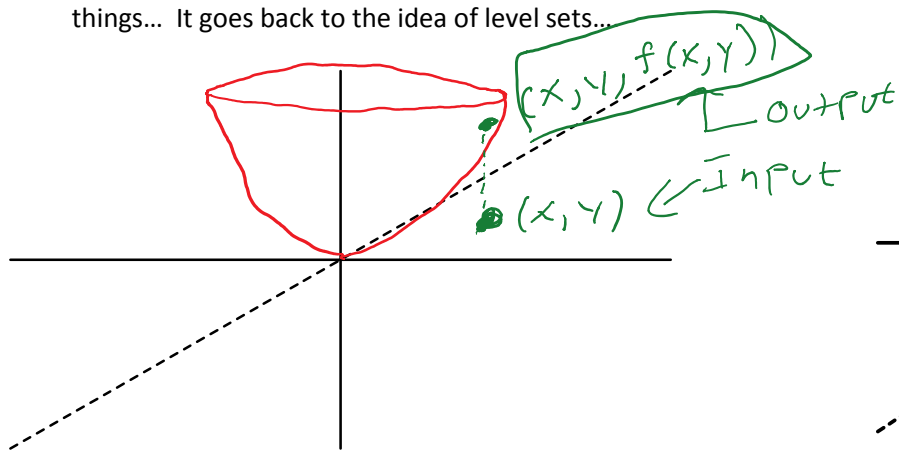
We have $f(T_{outside}, H)$...



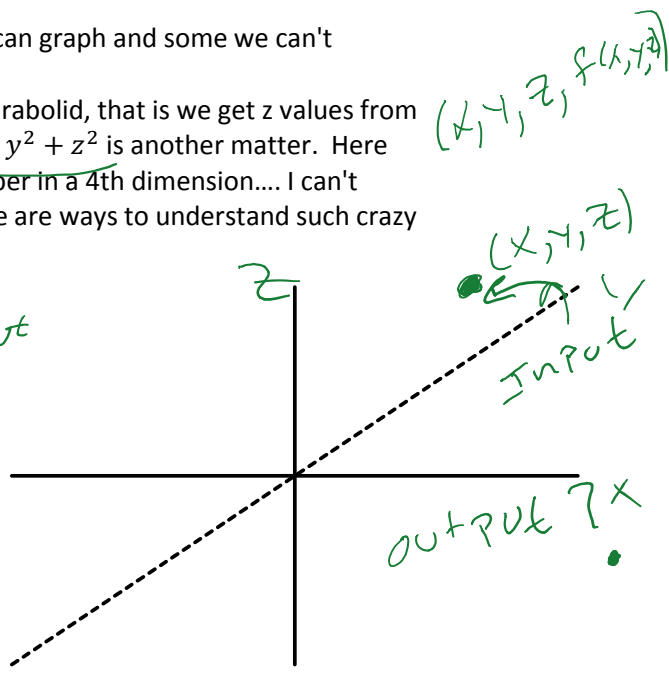
The upshot is that from an input of outside air temperature and % relative humidity we get one single value, the temperature of the air that will come out of the swamp cooler.

Functions can depend on as many variables as we want. Some we can graph and some we can't

For example $f(x, y) = x^2 + y^2$ We know to be the equation of a paraboloid, that is we get z values from x and y inputs. However graphing something like $f(x, y, z) = x^2 + y^2 + z^2$ is another matter. Here the input is a point in 3 space (x, y, z) but the output is another number in a 4th dimension.... I can't picture that because my brain evolved in a 3d world. However there are ways to understand such crazy things... It goes back to the idea of level sets...



$f(x, y) = x^2 + y^2$



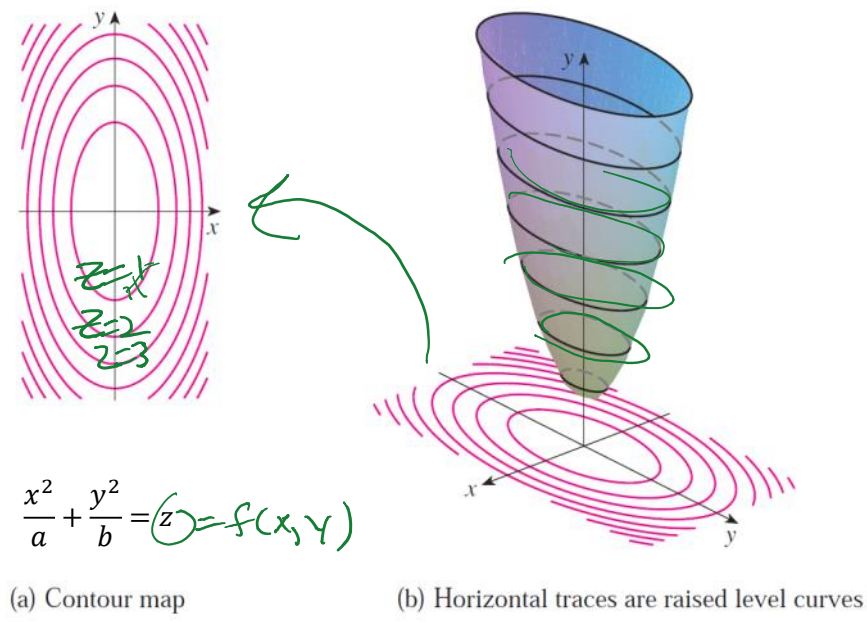
$f(x, y, z) = x^2 + y^2 + z^2$

Sec 11.1 Functions of Several Variables

To get a better sense let's stick with what we can picture first then abstract the idea.

Definition The **level curves** of a function f of two variables are the curves with equations $f(x, y) = k$, where k is a constant (in the range of f).

Let's consider the level sets of an elliptic paraboloid.



$$\frac{x^2}{a} + \frac{y^2}{b} = (z) = f(x, y)$$

In fact you have used level curves and contour plots before if you have ever studied a map!

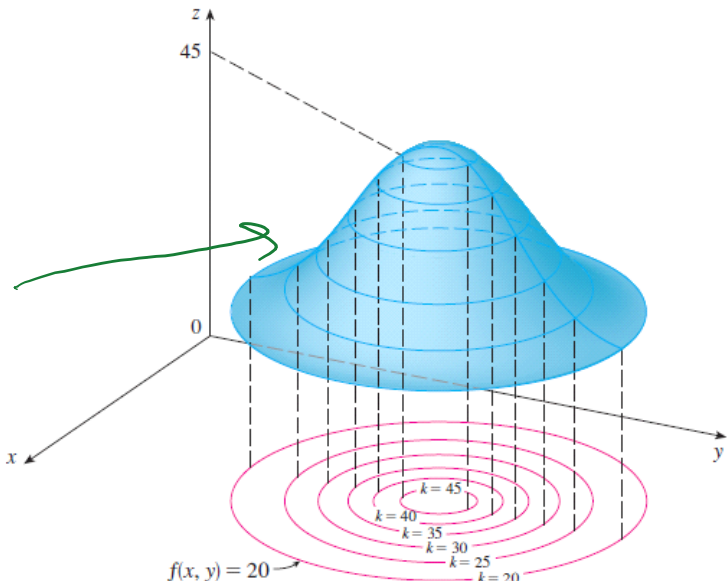


FIGURE 4

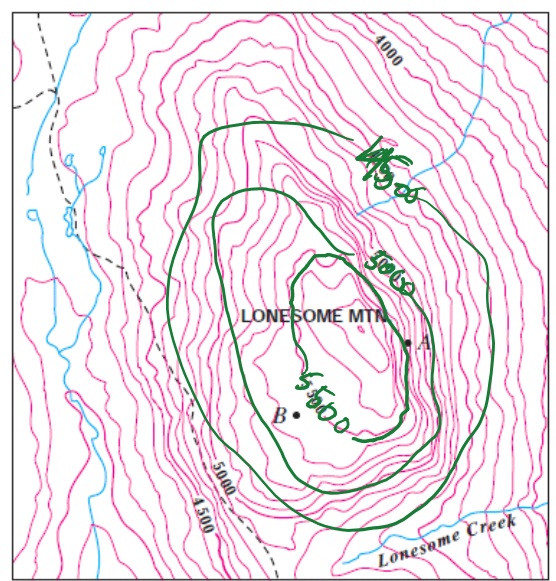
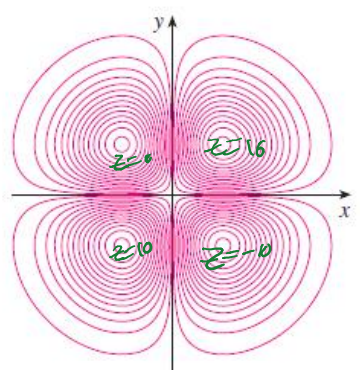


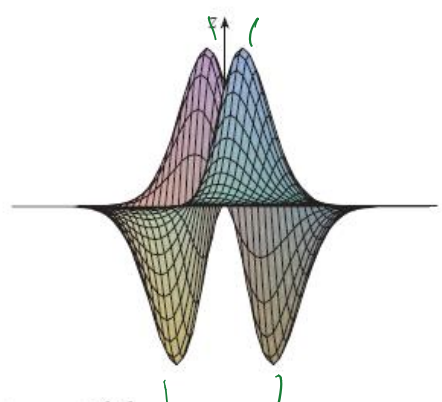
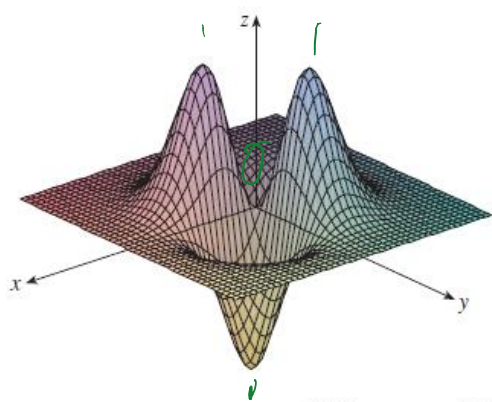
FIGURE 5

Sec 11.1 Functions of Several Variables

Other Examples of Contour plots of 3d surfaces...



(a) Level curves of $f(x, y) = -xye^{-x^2-y^2}$



(b) Two views of $f(x, y) = -xye^{-x^2-y^2}$

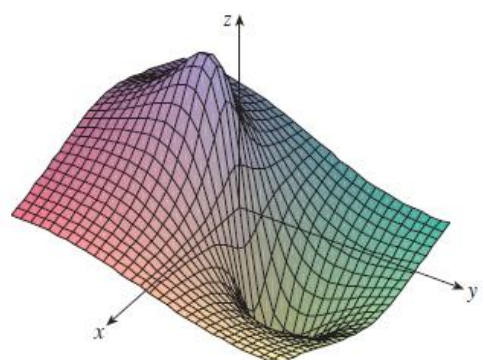
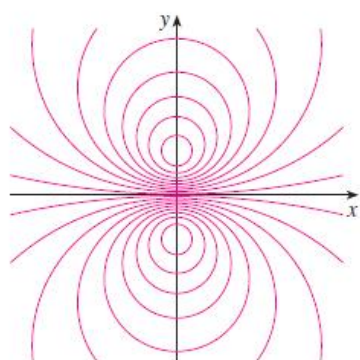


FIGURE 12

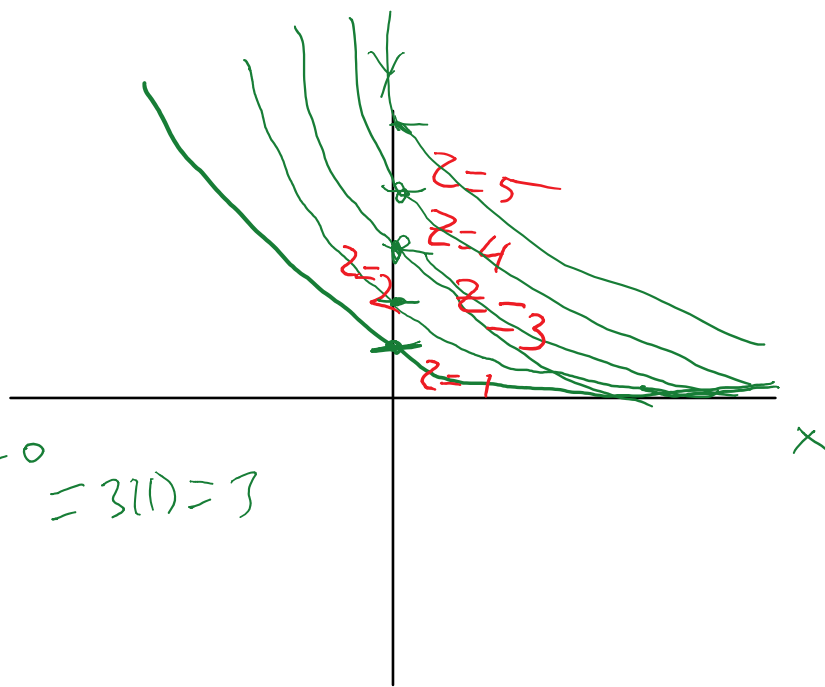
(c) Level curves of $f(x, y) = \frac{-3y}{x^2 + y^2 + 1}$

(d) $f(x, y) = \frac{-3y}{x^2 + y^2 + 1}$

HW Example Draw a contour map of the function

23. $f(x, y) = ye^x$

$f(x, y) = 1, 2, 3, 4, 5$
 $1 = ye^x \rightarrow y = \frac{1}{e^x} = e^{-x}$
 $2 = ye^x \rightarrow y = \frac{2}{e^x} = 2e^{-x}$
 $3 = ye^x \rightarrow y = \frac{3}{e^x} = 3e^{-x}$
 $4 = ye^x \rightarrow y = \frac{4}{e^x} = 4e^{-x}$
 $5 = ye^x \rightarrow y = \frac{5}{e^x} = 5e^{-x}$
 $y = 3e^{-0} = 3(1) = 3$



Sec 11.1 Functions of Several Variables

Ok how does this work for functions of 3 variables? Or more.... Let's think of the function $f(x,y,z) = x^2 + y^2 + z^2$ Our Level sets will be will come from fixing the function value...

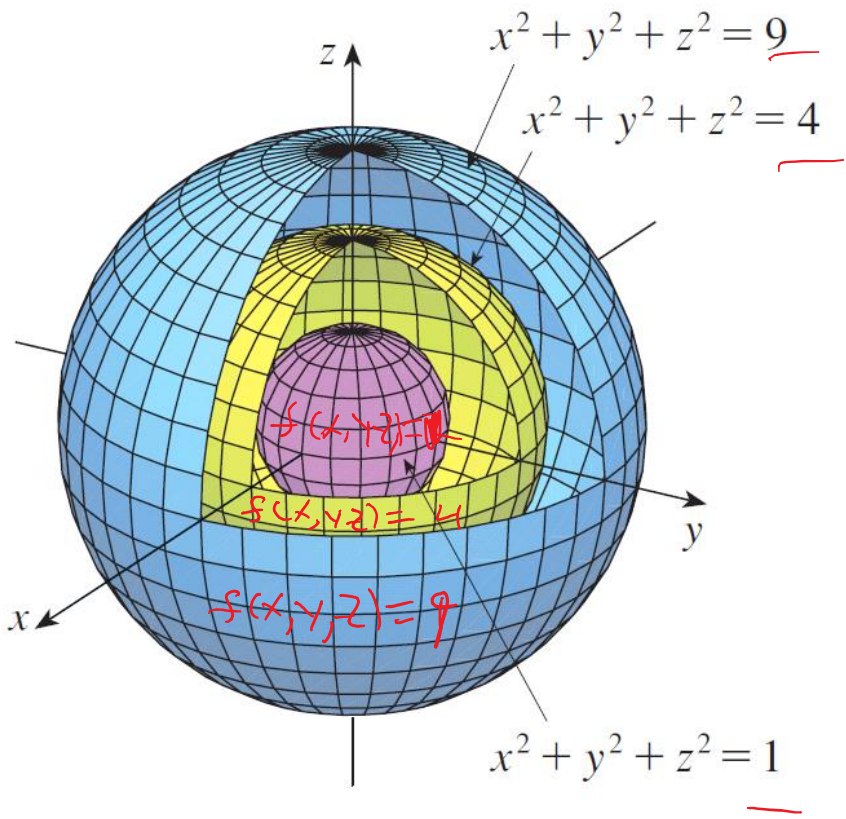
$f(x,y,z) = k$

$k = 2, 4, 9$

$2 = x^2 + y^2 + z^2$

$4 = x^2 + y^2 + z^2$

$9 = x^2 + y^2 + z^2$

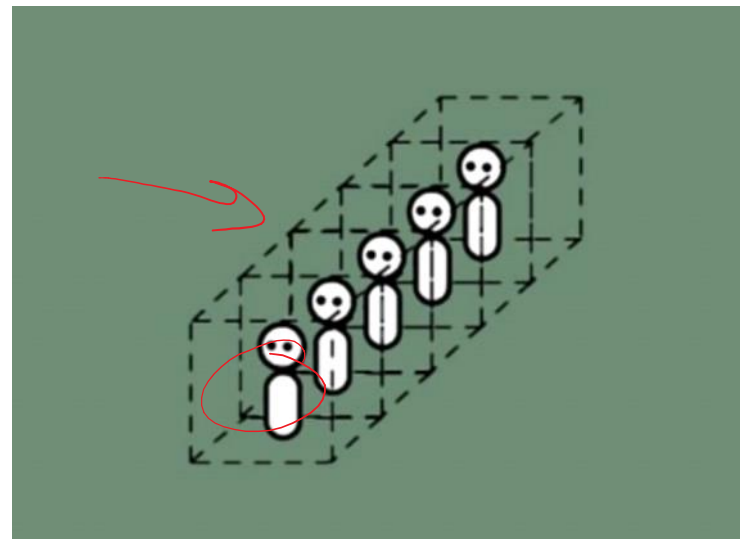
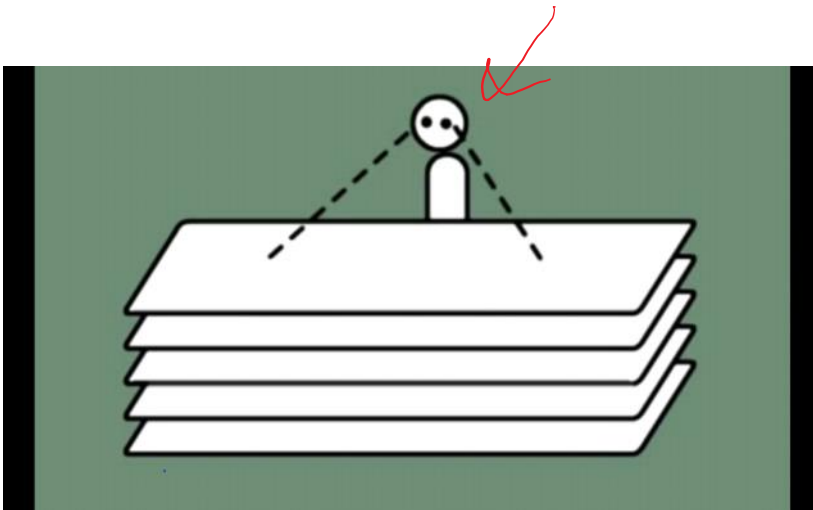


The analogue of a contour plot in 2-d is a surface plot in 3d. Here the color represents the $f(x,y,z)$ value

Sec 11.1 Functions of Several Variables

Carl Sagan on the matter...

<https://www.youtube.com/watch?v=UnUREICzGc0>



4d cube moving through 3 space

<https://www.youtube.com/watch?v=skgrLMUFazs>

More videos to boggle your mind....

<https://www.youtube.com/watch?v=uDaKzQNIMFw&list=PL1JiOkTkL2nSrQoEkTjWaHeisbooMC5T6>