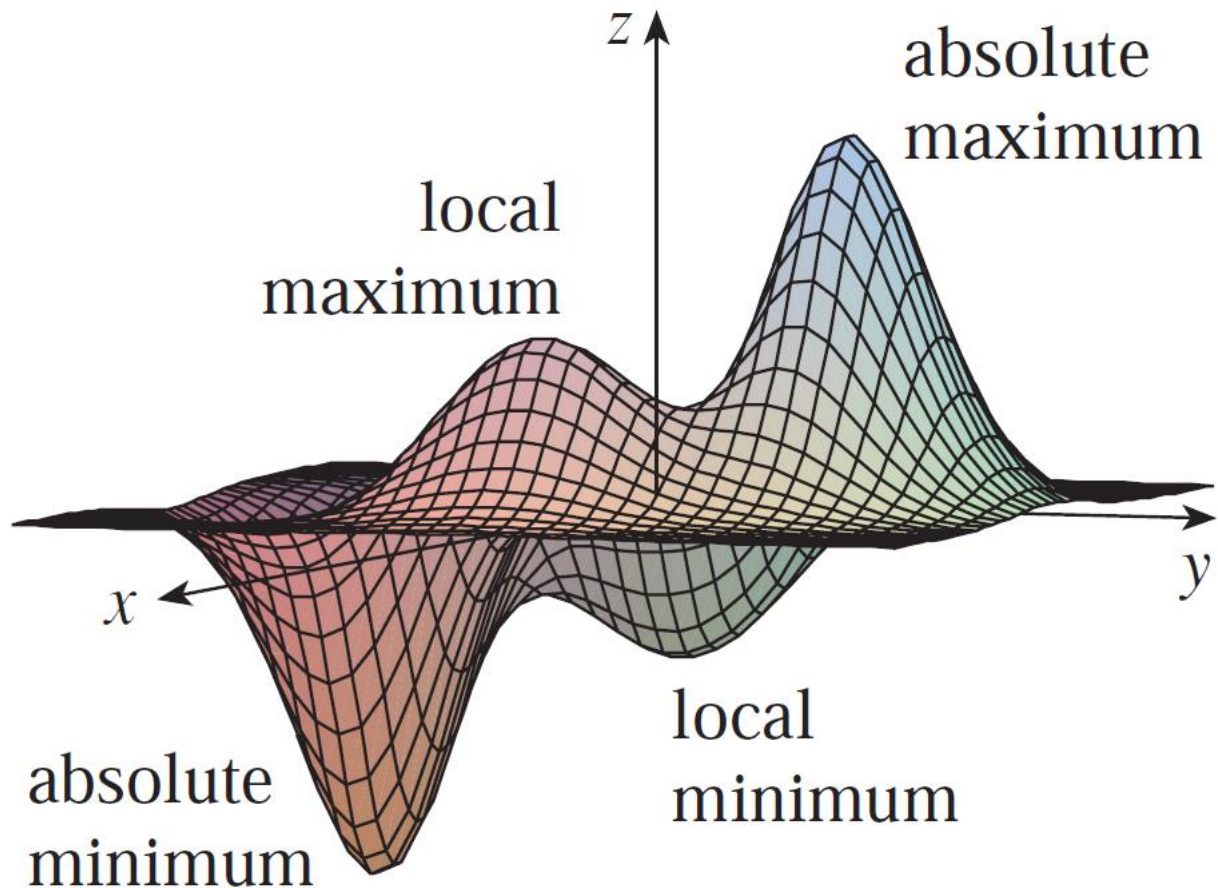


Sec 11.7 Maximum and Minimum Values.

You have found maximum and minimum values for functions of one variable but what about functions of more than one variable?



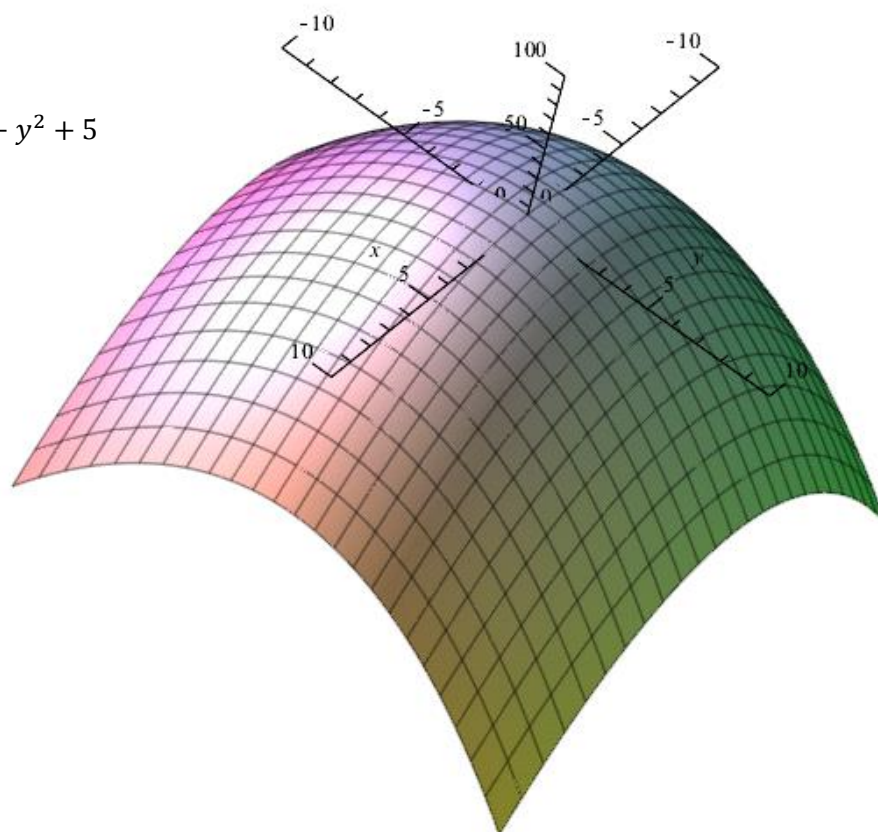
1 Definition A function of two variables has a **local maximum** at (a, b) if $f(x, y) \leq f(a, b)$ when (x, y) is near (a, b) . [This means that $f(x, y) \leq f(a, b)$ for all points (x, y) in some disk with center (a, b) .] The number $f(a, b)$ is called a **local maximum value**. If $f(x, y) \geq f(a, b)$ when (x, y) is near (a, b) , then f has a **local minimum** at (a, b) and $f(a, b)$ is a **local minimum value**.

Sec 11.7 Maximum and Minimum Values.

How can we find the maximum and minimums of functions of two variables.

2 Fermat's Theorem for Functions of Two Variables If f has a local maximum or minimum at (a, b) and the first-order partial derivatives of f exist there, then $f_x(a, b) = 0$ and $f_y(a, b) = 0$.

$$f(x, y) = -x^2 - y^2 + 5$$



Sec 11.7 Maximum and Minimum Values.

Some of the cases:

A function with an absolute minimum:

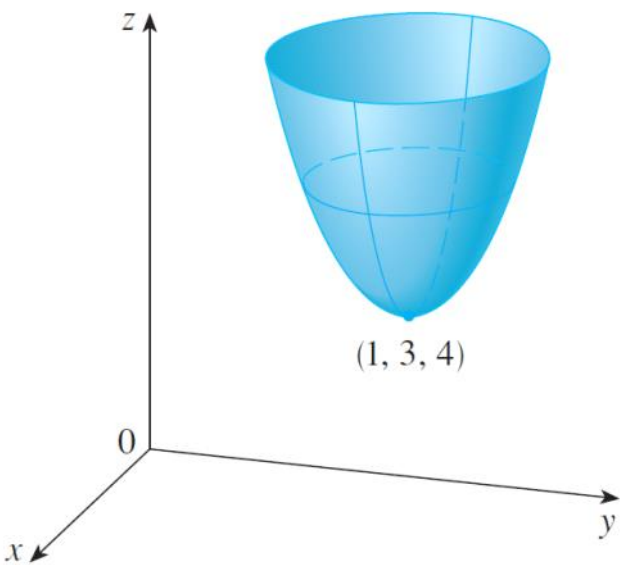


FIGURE 2

$$z = x^2 + y^2 - 2x - 6y + 14$$

A function with no maximum or minimum

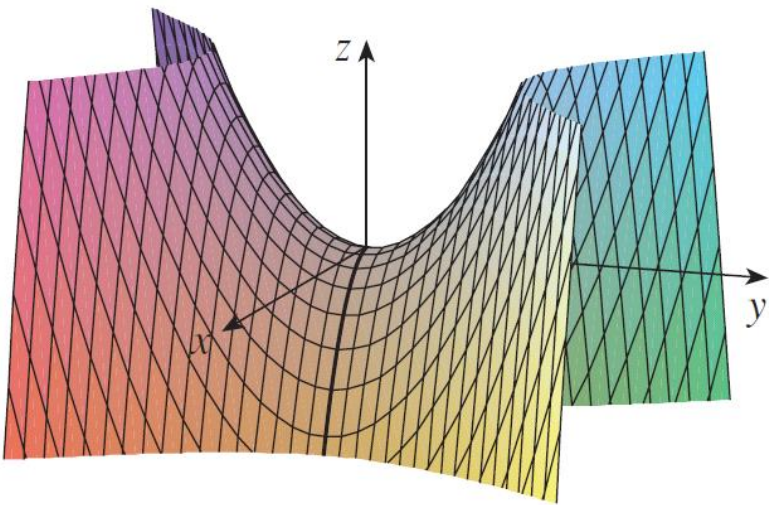


FIGURE 3

$$z = y^2 - x^2$$

$$D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = f_{xx} f_{yy} - (f_{xy})^2$$

A test to distinguish critical points.

3 Second Derivatives Test Suppose the second partial derivatives of f are continuous on a disk with center (a, b) , and suppose that $f_x(a, b) = 0$ and $f_y(a, b) = 0$ [that is, (a, b) is a critical point of f]. Let

$$D = D(a, b) = f_{xx}(a, b) f_{yy}(a, b) - [f_{xy}(a, b)]^2$$

- (a) If $D > 0$ and $f_{xx}(a, b) > 0$, then $f(a, b)$ is a local minimum.
- (b) If $D > 0$ and $f_{xx}(a, b) < 0$, then $f(a, b)$ is a local maximum.
- (c) If $D < 0$, then $f(a, b)$ is not a local maximum or minimum.

Hw example

1. Suppose $(1, 1)$ is a critical point of a function f with continuous second derivatives. In each case, what can you say about f ?
- (a) $f_{xx}(1, 1) = 4, \quad f_{xy}(1, 1) = 1, \quad f_{yy}(1, 1) = 2$

Sec 11.7 Maximum and Minimum Values.

5–16 Find the local maximum and minimum values and saddle point(s) of the function. If you have three-dimensional graphing software, graph the function with a domain and viewpoint that reveal all the important aspects of the function.

5. $f(x, y) = x^2 + xy + y^2 + y$

3 Second Derivatives Test Suppose the second partial derivatives of f are continuous on a disk with center (a, b) , and suppose that $f_x(a, b) = 0$ and $f_y(a, b) = 0$ [that is, (a, b) is a critical point of f]. Let

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- (c) If $D < 0$, then $f(a, b)$ is not a local maximum or minimum.

Sec 11.7 Maximum and Minimum Values.

11. $f(x, y) = e^x \cos y$

3 Second Derivatives Test Suppose the second partial derivatives of f are continuous on a disk with center (a, b) , and suppose that $f'_x(a, b) = 0$ and $f'_y(a, b) = 0$ [that is, (a, b) is a critical point of f]. Let

$$D = D(a, b) = f_{xx}(a, b)f_{yy}(a, b) - [f_{xy}(a, b)]^2$$

- (a) If $D > 0$ and $f_{xx}(a, b) > 0$, then $f(a, b)$ is a local minimum.
- (b) If $D > 0$ and $f_{xx}(a, b) < 0$, then $f(a, b)$ is a local maximum.
- (c) If $D < 0$, then $f(a, b)$ is not a local maximum or minimum.

Sec 11.7 Maximum and Minimum Values.

In what case do we know we have an absolute maximum or minimum

8 Extreme Value Theorem for Functions of Two Variables If f is continuous on a closed, bounded set D in $\mathbb{R}^2$, then f attains an absolute maximum value $f(x_1, y_1)$ and an absolute minimum value $f(x_2, y_2)$ at some points (x_1, y_1) and (x_2, y_2) in D .

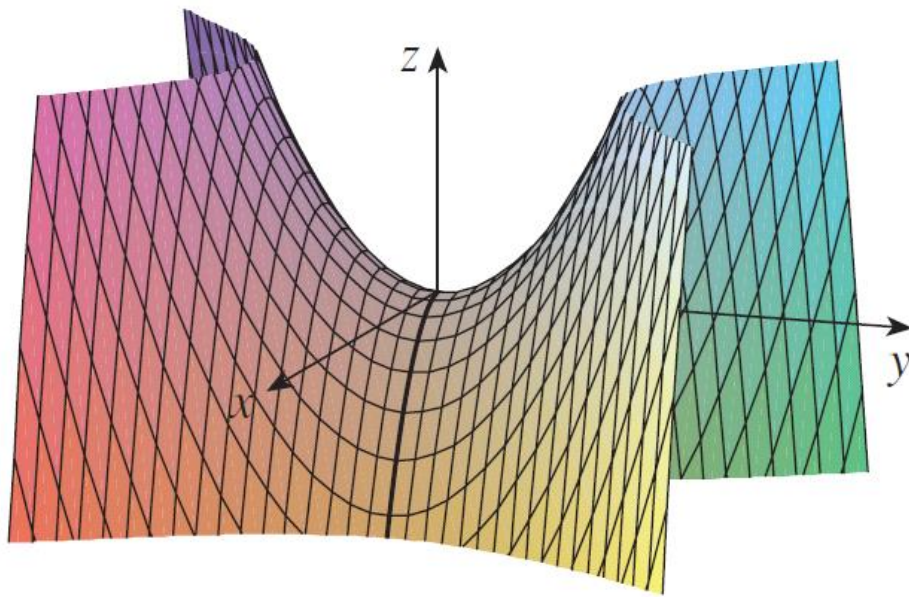


FIGURE 3

$$z = y^2 - x^2$$

Sec 11.7 Maximum and Minimum Values.

9 To find the absolute maximum and minimum values of a continuous function f on a closed, bounded set D :

1. Find the values of f at the critical points of f in D .
2. Find the extreme values of f on the boundary of D .
3. The largest of the values from steps 1 and 2 is the absolute maximum value; the smallest of these values is the absolute minimum value.

HW Example

27–32 Find the absolute maximum and minimum values of f on the set D .

- 27.** $f(x, y) = 1 + 4x - 5y$, D is the closed triangular region with vertices $(0, 0)$, $(2, 0)$, and $(0, 3)$

3 Second Derivatives Test Suppose the second partial derivatives of f are continuous on a disk with center (a, b) , and suppose that $f_x(a, b) = 0$ and $f_y(a, b) = 0$ [that is, (a, b) is a critical point of f]. Let

$$D = D(a, b) = f_{xx}(a, b)f_{yy}(a, b) - [f_{xy}(a, b)]^2$$

- (a) If $D > 0$ and $f_{xx}(a, b) > 0$, then $f(a, b)$ is a local minimum.
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- (c) If $D < 0$, then $f(a, b)$ is not a local maximum or minimum.

Sec 11.7 Maximum and Minimum Values.

47. A cardboard box without a lid is to have a volume of $32,000 \text{ cm}^3$. Find the dimensions that minimize the amount of cardboard used.