

$$-y = x$$

$$\underline{T_N = \sum_{n=0}^N \frac{f^{(n)}(a)}{n!} (x-a)^n = \frac{f^{(0)}(a)}{0!} (x-a)^0 + \frac{f^{(1)}(a)}{1!} (x-a)^1 + \dots + \frac{f^{(N)}(a)}{N!} (x-a)^N}$$

$$\textcircled{2} T_5 = \sum_{n=0}^5 \frac{(-1)^n x^{2n+1}}{(2n+1)!} = \frac{(-1)^0 x^{2(0)+1}}{(2(0)+1)!} + \frac{(-1)^1 x^{2(1)+1}}{(2(1)+1)!} + \dots + \frac{(-1)^5 x^{2(5)+1}}{(2(5)+1)!}$$

$$= x - \frac{x^3}{3!} + \frac{x^5}{5!}$$

[#W3sidHlwZSI6MCwiZXEiOiJzaW4oeCkiLCJjb2xvcil6IiMwMDAwMDAifSx7InR5cGUlOjAsImVxIjoieClsmNvbG9yLjoilzAwMDAwMCI9LHsidHlwZSI6MCwiZXEiOiIlLCJjb2xvcil6IiMwMDAwMDAifSx7InR5cGUlOjEwMDAsIndpbmRvdyl6WyltMy4yNSIsIjMuMiUicltMilSljiXX1d](#)

Sec 8.8 Approximating Functions with Taylor polynomials

How can we estimate how far we are off from the actual function?

9 Taylor's Inequality If $|f^{(n+1)}(x)| \leq M$ for $|x - a| \leq d$, then the remainder $R_n(x)$ of the Taylor series satisfies the inequality

$|R_n(x)| \leq \frac{M}{(n+1)!} |x - a|^{n+1}$

for $|x - a| \leq d$

$\sum_{n=0}^{\infty} x^{2n+1}$

Ex Estimate the error of the 3rd degree Taylor polynomial for $\sin(x)$ centered at 0 on the interval defined by $|x| < 1$ What about $|x| < 20$?

$T_3 = x - \frac{x^3}{3!}$

$\sin(x)$

$|x - 0| < 1$

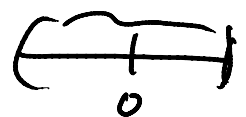
$f^{(n+1)}(x) = f^{(2)}(x) = -\sin(x)$

$|f^{(n+1)}(x)| \leq 1 \leftarrow M$

$|R_n(x)| \leq \frac{1}{(1+1)!} |x-0|^{1+1} \leq \frac{1}{2!} |x|^2 \leq \frac{|x|^2}{2}$

$\frac{|\frac{1}{2}|^2}{2} = \frac{\frac{1}{4}}{2} = \frac{1}{8}$

$|R_n(x)| \leq \frac{1}{2!} |x|^2 \leq \frac{1}{2!} 1^2 \leq \frac{1}{2}$

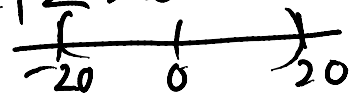


$|x| < 20$

$|R_n(x)| < \frac{1}{2!} |x|^2$

$< \frac{1}{2} |20|^2 = < \frac{1}{2} 400 < 200$

$|x| < 20$



Sec 8.8 Approximating Functions with Taylor polynomials

Alternating Series Estimation Theorem If $s = \sum (-1)^{n-1} b_n$ is the sum of an alternating series that satisfies

(i) $b_{n+1} \leq b_n$ and (ii) $\lim_{n \rightarrow \infty} b_n = 0$

then $|R_n| = |s - s_n| \leq b_{n+1}$

How far off is the third degree taylor polynomial centered at 0 for $\sin(x)$ at $x=1/2$ what about $x=20$?

$x - \frac{x^3}{3!} + \frac{x^5}{5!}$

$|R_3| = |s - s_n| \leq \frac{(1/2)^5}{5!} = \frac{\frac{1}{32}}{120} = \frac{1}{3840}$

$\approx \frac{1}{3840}$

$|R_3| = |s - s_n| \leq \frac{(20)^5}{5!} = \frac{3200000}{120} = 26666.67$

Sec 8.8 Approximating Functions with Taylor polynomials

HW Example Approximate the Taylor polynomial for $f(x)=x\sin(x)$ at $x=0$ with $n=4$ Estimate the accuracy when $-1 < x < 1$

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!}$$

$$x \sin(x) = x \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} \right)$$

$$x \sin(x) = x^2 - \frac{x^4}{3!} + \frac{x^6}{5!} - \frac{x^8}{7!} + \frac{x^{10}}{9!}$$

$$|f^{(n+1)}(x)| < M \quad |R_n(x)| \leq \frac{M}{(n+1)!} |x-a|^{n+1}$$

$$|f^{(5)}(x)| = |\cos(x)| \leq 1 \quad M=1$$

$$|R_4(x)| \leq \frac{1}{(4+1)!} |x-0|^{4+1} = \frac{1}{5!} (|x|^5) < 1$$

$$|R_4(x)| \leq \frac{1}{5!} |x|^5 \leq \frac{1}{5!} (1) = \frac{1}{120} \quad \text{since } |x| < 1$$