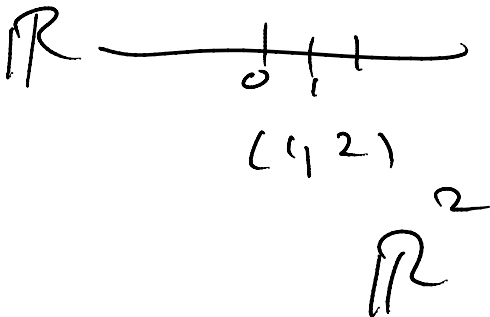
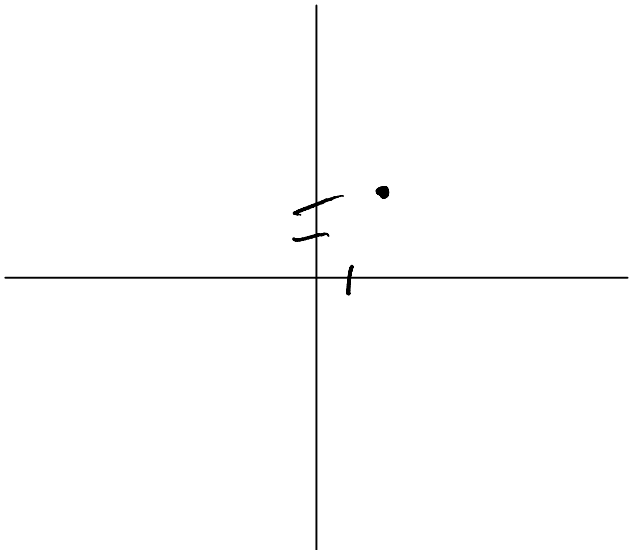
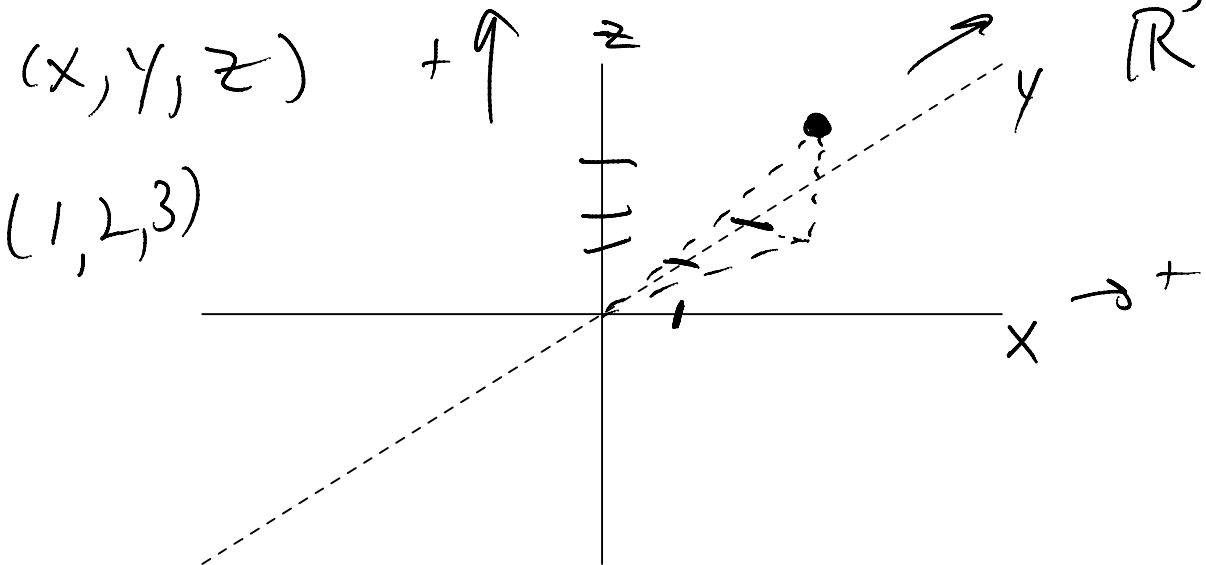


Sec 9.1 3-d coordinate system

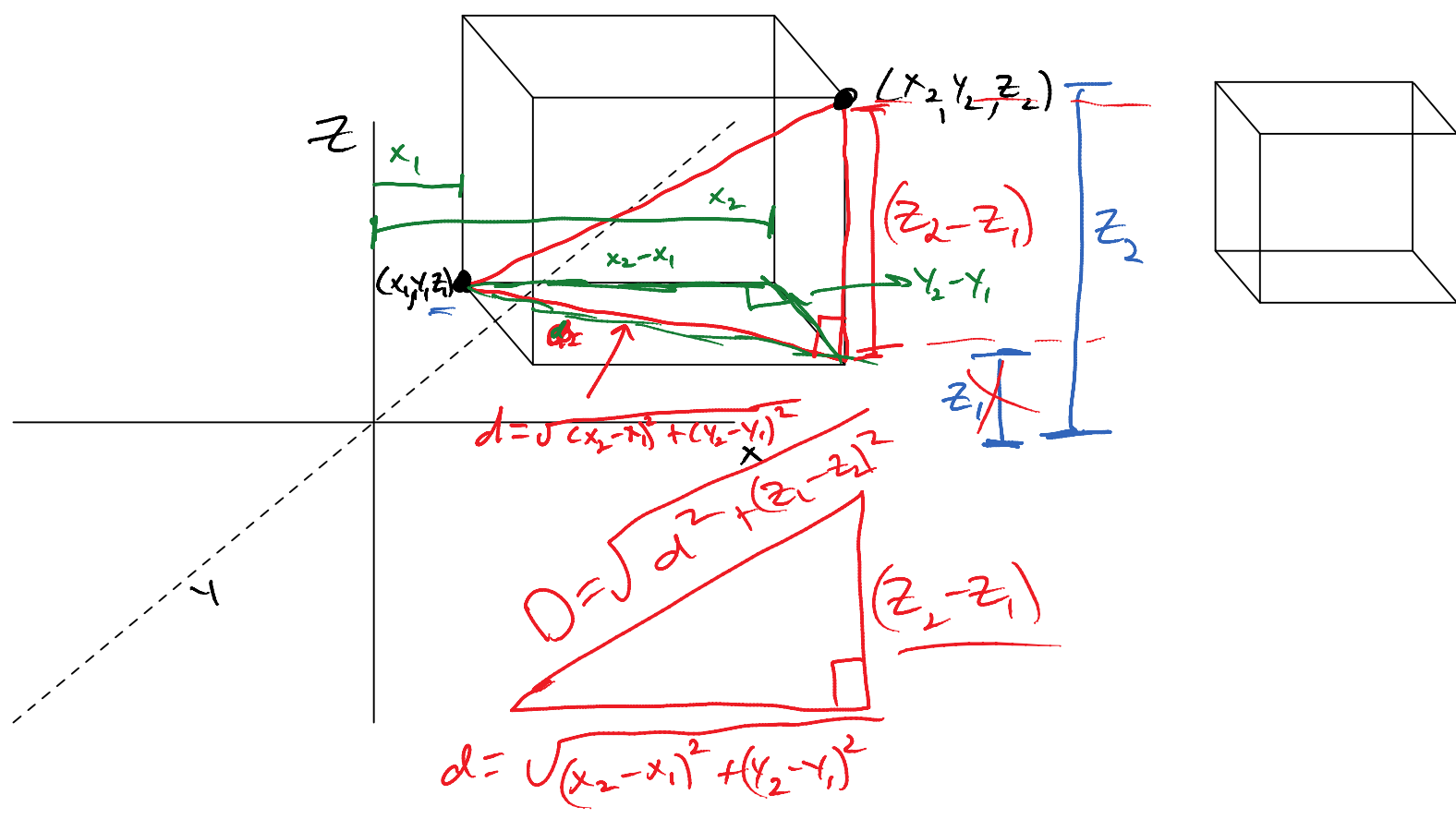
Recall how we define points in the 2d system



If we want to move to 3-d we just need one more  
Coordinate!



The distance formula in 3d.



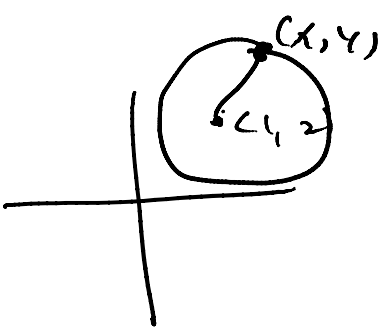
$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2$$
$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Sec 9.1 Math in 3-d!!

$$(x-h)^2 + (y-k)^2 = r^2$$

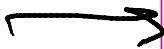
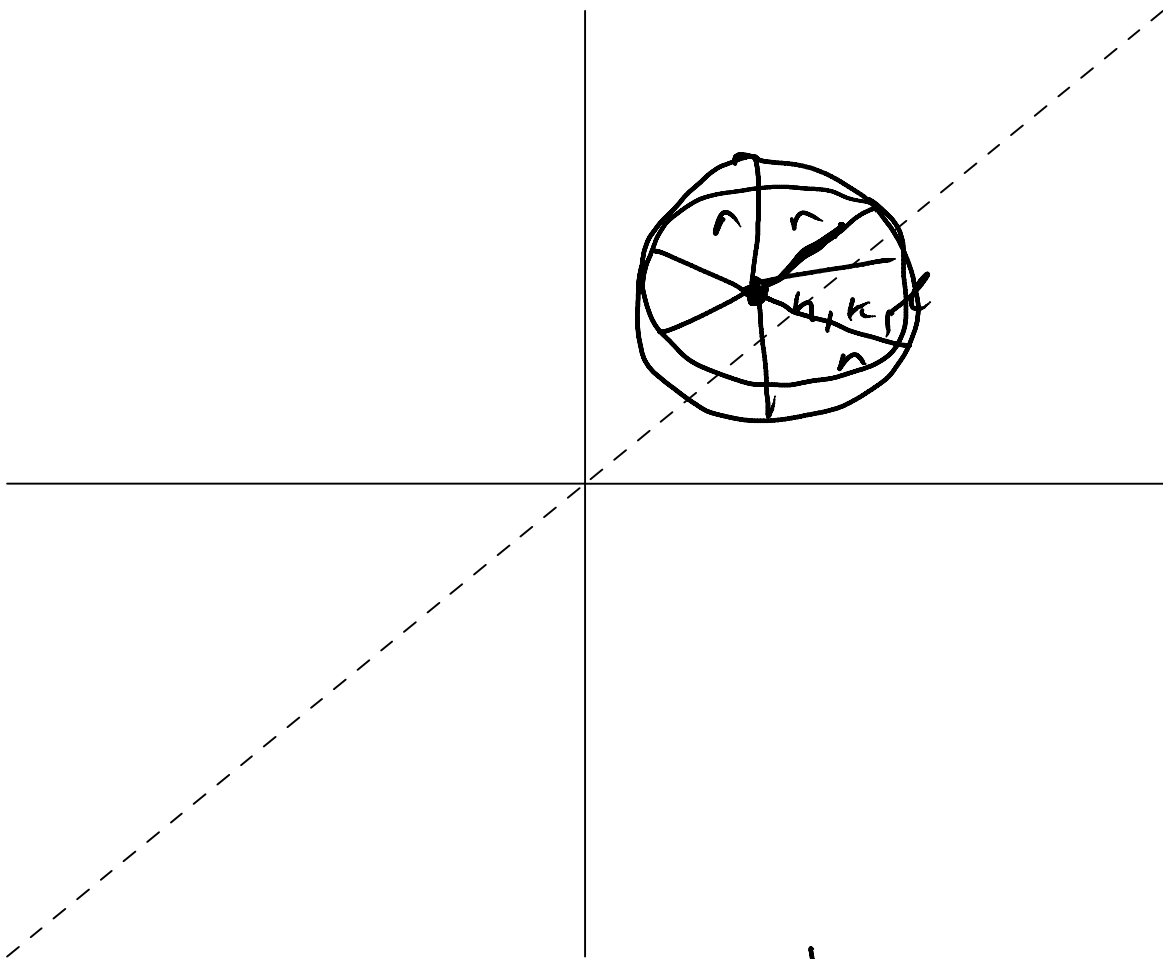
The equation of a sphere in 3d. Just like with a circle we define the equation of a sphere to be the set of all points equidistant from that of the center.

$$(h, k, l) \quad (x, y, z)$$



$$r = \sqrt{(x-h)^2 + (y-k)^2 + (z-l)^2} \quad r^2 = (x-h)^2 + (y-k)^2 + (z-l)^2$$

$$\underline{r^2 = (x-h)^2 + (y-k)^2 + (z-l)^2}$$



**Equation of a Sphere** An equation of a sphere with center  $C(h, k, l)$  and radius  $r$  is

$$(x-h)^2 + (y-k)^2 + (z-l)^2 = r^2$$

In particular, if the center is the origin  $O$ , then an equation of the sphere is

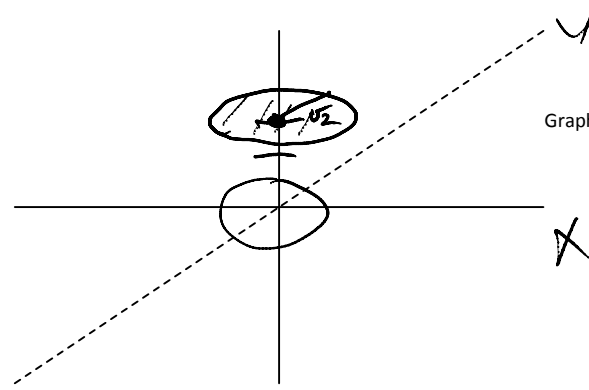
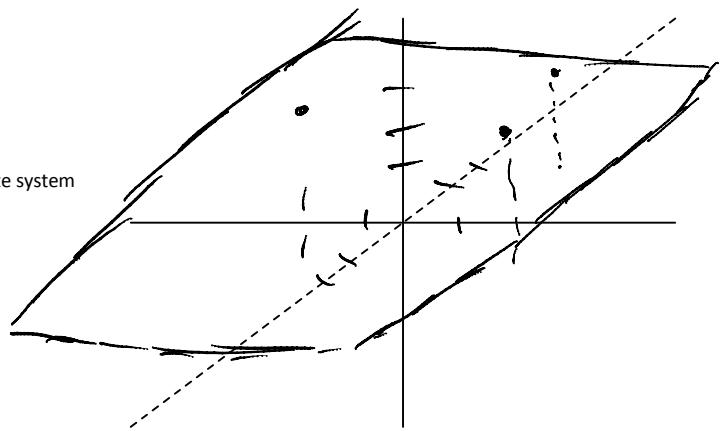
$$x^2 + y^2 + z^2 = r^2 \quad \hookrightarrow (0, 0, 0)$$

Sec 9.1 Math in 3-d!!

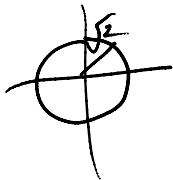
Other equations in 3-d

Ex. Graph the equation  $z=3$  in the 3d coordinate system

$(1, 2, 3)$   
 $(-1, -2, 3)$

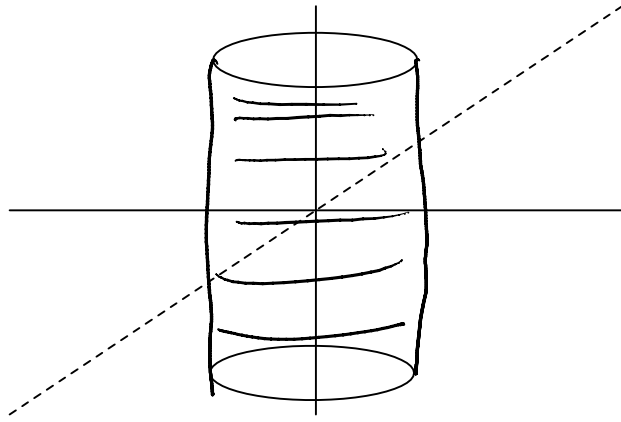


Graph the equation defined by  $x^2 + y^2 = 2, z = 2$



Graph the surface defined by  $x^2 + y^2 = 4$

$(0, 2, 1)$   
 $0^2 + 2^2 = 4$   
 $(0, 2), (0)$



Sec 9.1 Math in 3-d!!

Lets try to graph  $z = x^2 + y^2$

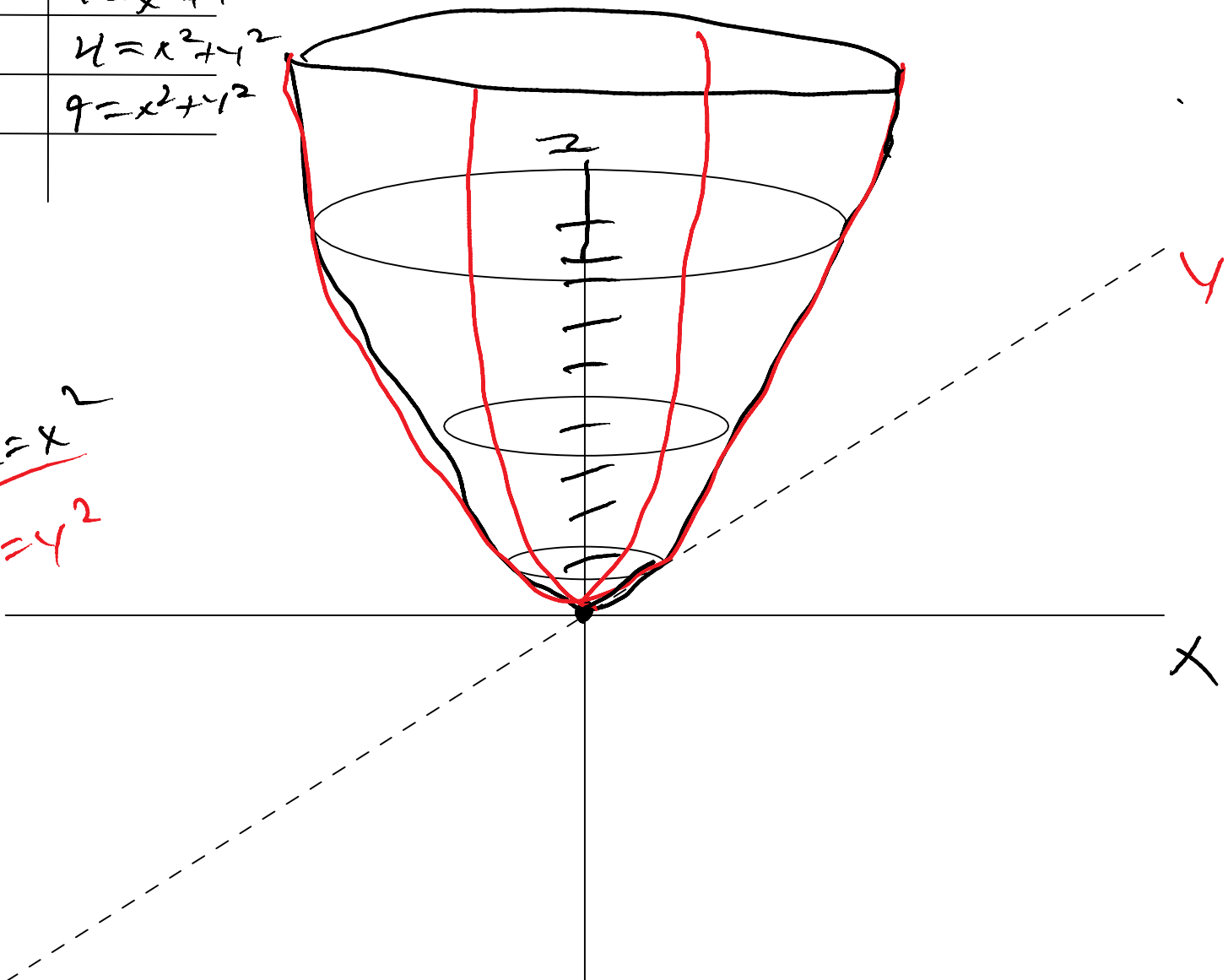
$x=2 \quad y=-2$   
 $x=1 \quad y=2 \quad 1^2 + 2^2 = 5$   
 $(1, 2, 5)$

This would be easy if  $z$  was a fixed number, one way we can accomplish our goal is to pick fixed values for  $z$  and see what the graph looks like for each one. We call these level sets.....

| z | Surface         |
|---|-----------------|
| 0 | $0 = x^2 + y^2$ |
| 1 | $1 = x^2 + y^2$ |
| 4 | $4 = x^2 + y^2$ |
| 9 | $9 = x^2 + y^2$ |

$x^2 = -y^2 \quad 0 = 0 \quad -1 = x^2 + y^2$

$z = x^2$   
 $z = y^2$



$$(x-h)^2 + (y-k)^2 + (z-l)^2 = r^2$$

Ex Hw #13 Graph the surface  $x^2 + y^2 + z^2 - 6x + 4y - 2z = 11$

$$(x+a)^2 = x^2 + \underline{2ax} + \underline{a^2}$$

$$= x^2 - 6x + \underline{y^2 + 4y} + \underline{z^2 - 2z} = 11$$

$$x^2 - 6x \quad -\frac{6}{2} = -3 \quad (-3)^2 = 9$$

$$\underline{x^2 - 6x + 9} - 9$$

$$(x-3)^2 - 9$$

$$y^2 + 4y \quad +\frac{4}{2} = 2 \quad \underline{2^2 = 4}$$

$$\underline{y^2 + 4y + 4} - 4 = (y+2)^2 - 4$$

$$z^2 - 2z \quad -\frac{2}{2} = -1 \quad (-1)^2 = 1$$

$$\underline{z^2 - 2z + 1} - 1$$

$$(z-1)^2 - 1$$

$$(x-3)^2 + (y+2)^2 + (z-1)^2 = 25$$

center (3, -2, 1)

$$r = 5$$

$$x^2 - 6x + y^2 + 4y + z^2 - 2z = 11$$

$$\begin{matrix} \downarrow & & \downarrow & & \downarrow \\ (x-3)^2 - 9 & + & (y+2)^2 - 4 & + & (z-1)^2 - 1 = 11 \\ +9 & & +4 & & +1 \end{matrix}$$

$$(x-3)^2 + (y+2)^2 + (z-1)^2 = 25$$

