

Sec 9.6 Functions of two variables.



Two variables ... HA.. HA..HA...

Definition A function f of two variables is a rule that assigns to each ordered pair of real numbers (x, y) in a set D a unique real number denoted by $f(x, y)$. The set D is the **domain** of f and its **range** is the set of values that f takes on, that is, $\{f(x, y) \mid (x, y) \in D\}$.

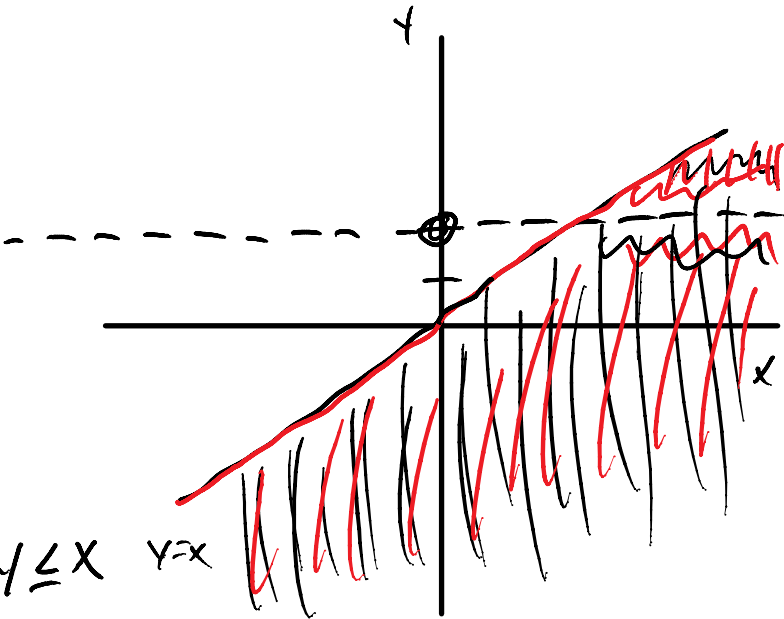
$\{f(x, y) \mid (x, y) \in D\}$
↑ ↑ ↑ ↑
z values such ordered pair belongs to
 that in

z

Domain: The domain of a function of two variables is the set of all ordered pairs (x,y) such that $(x, y, f(x,y))$ can be plotted in R^3 and $f(x,y)$ is calculable.

Ex. Find and sketch the domain of

$f(x,y) = \frac{\sqrt{x-y}}{y-2}$

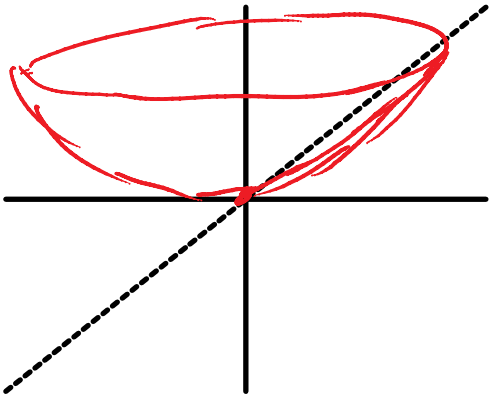


① $y-2 \neq 0$
 $y \neq 2$
② $\sqrt{x-y}$
 $x-y \geq 0$
 $x \geq y$

$y \leq x$ $y=x$

Range: The range of a function of two variables are all of the values $z=f(x,y)$ which come from all of the ordered pairs (x,y) in the Domain of f .

Ex Find the range of $f(x,y) = x^2 + y^2$



$x^2 + y^2 \geq 0$

$z \geq 0 \leftarrow \text{Range}$

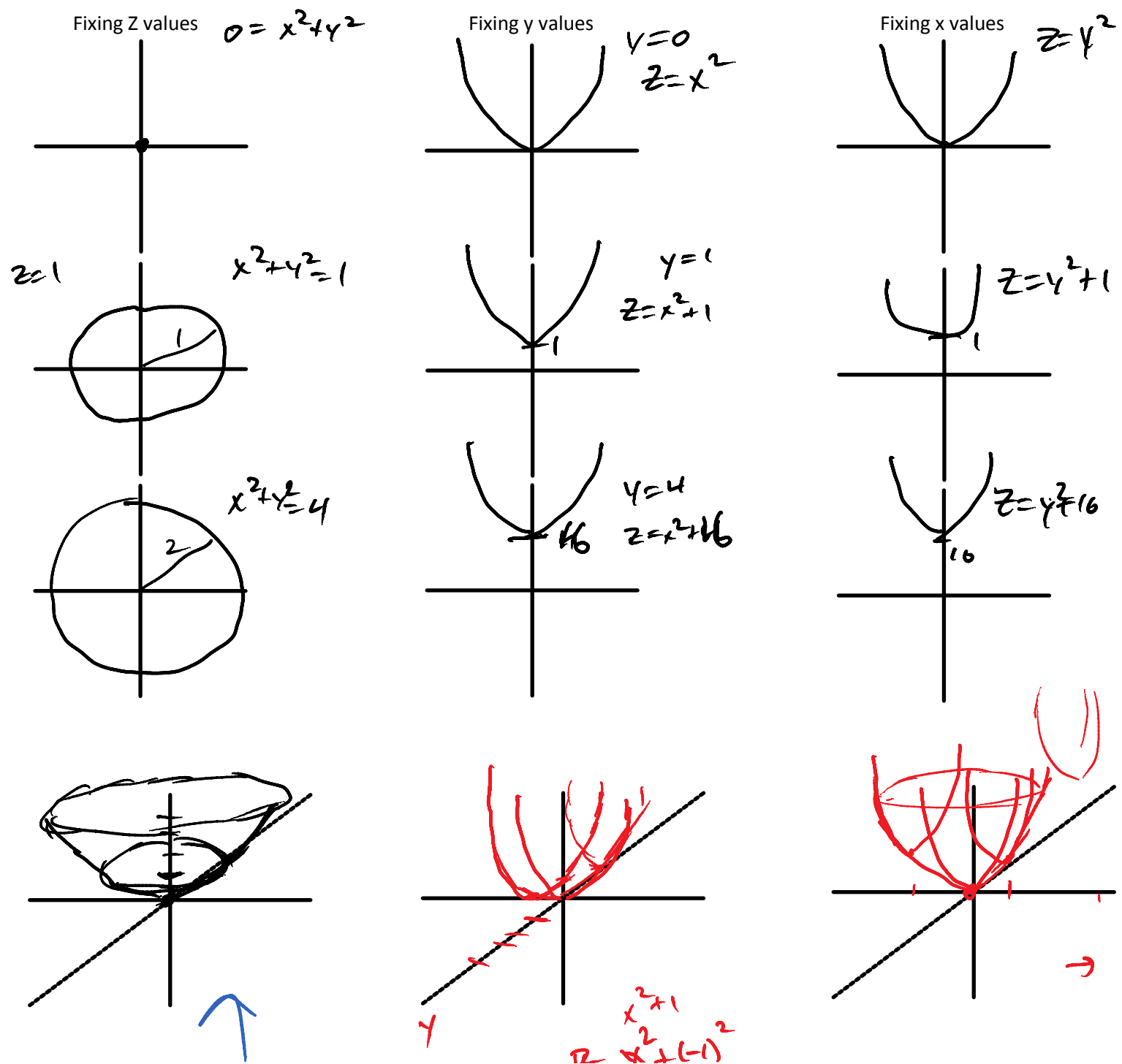
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How do we graph functions of two variables? We can do this using Level Sets or Traces as Stewart calls them.

A Level Set (Trace): For a function of two variables we may fix either an x , y , or z value in our equation which puts us in a specific plane. We can then study the intersection of the plane and our surface in that plane. This intersection is the level set or trace..

Example: Consider the two variable function $f(x,y) = x^2 + y^2 \Rightarrow z = x^2 + y^2$

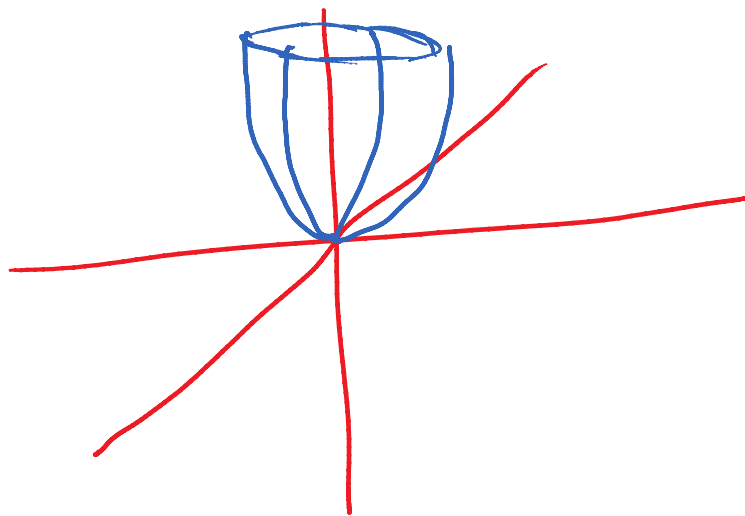
Calculate and graph Level sets for $z=0, 1, 4$ Do the same for $y=0, 1, 4$ and $x=0, 1, 4$ note $z=f(x,y)$





y

$$z = x^2 + y^2$$



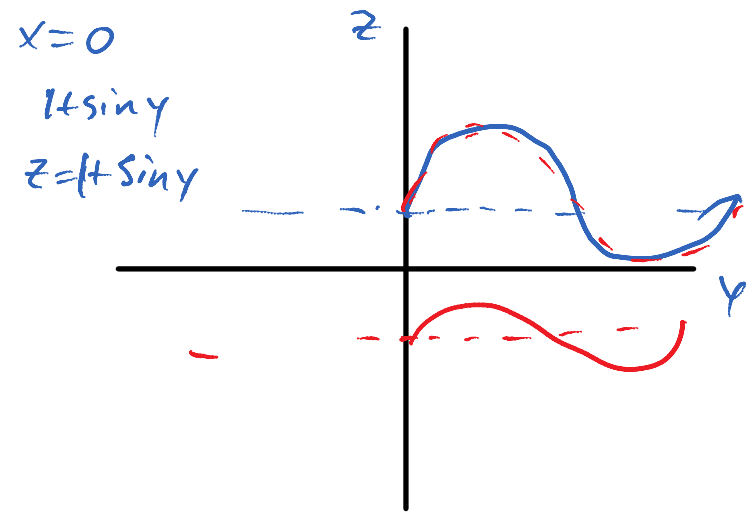
$$z = x^2$$

$$z = y^2$$

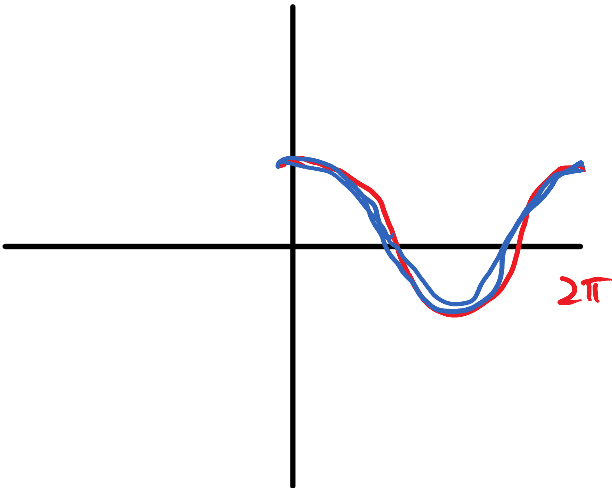
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Example Use Traces to sketch the graph of the following function on the set $\{(x,y) | 0 < x < 2\pi, 0 < y < 2\pi\}$

$f(x,y) = \cos(x) + \sin(y)$ Try $x = 0, \pi, 2\pi$ and $y = 0, \pi, 2\pi$

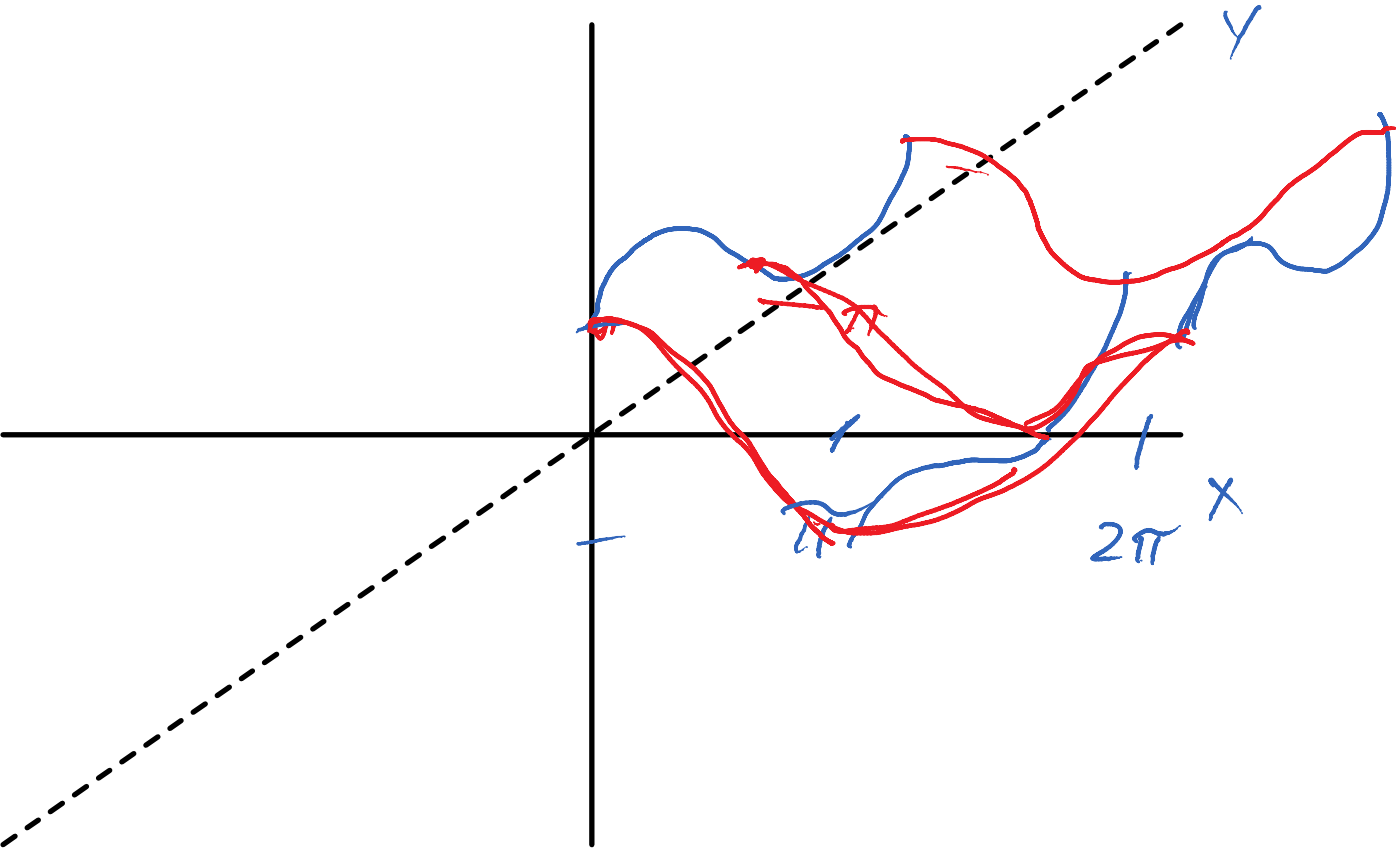


X-level sets



Y level sets

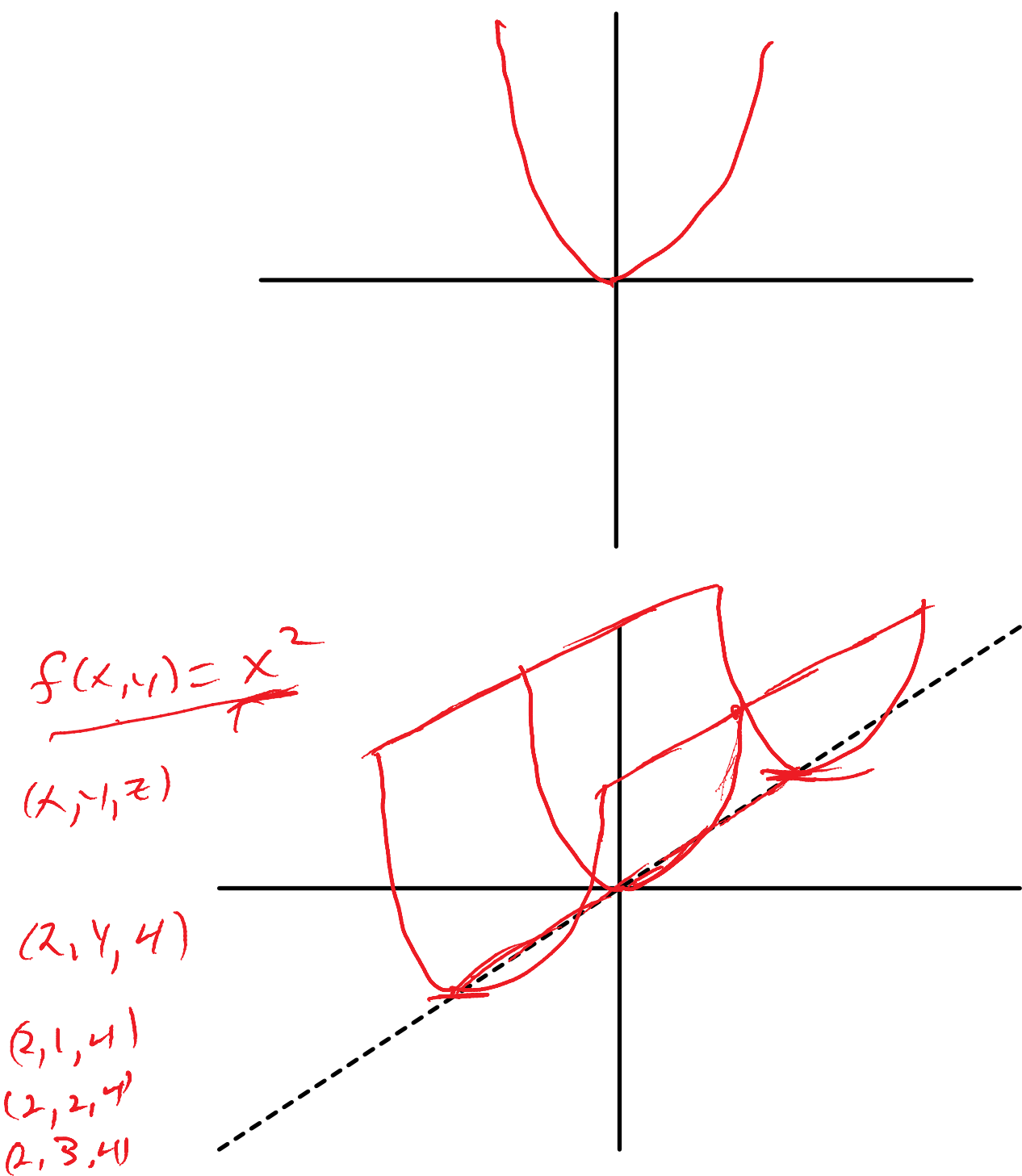
Put it all together...



<http://www.math.uri.edu/~bkaskosz/flashmo/graph3d/>

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The difference between $f(x) = x^2$ and $f(x,y) = x^2$

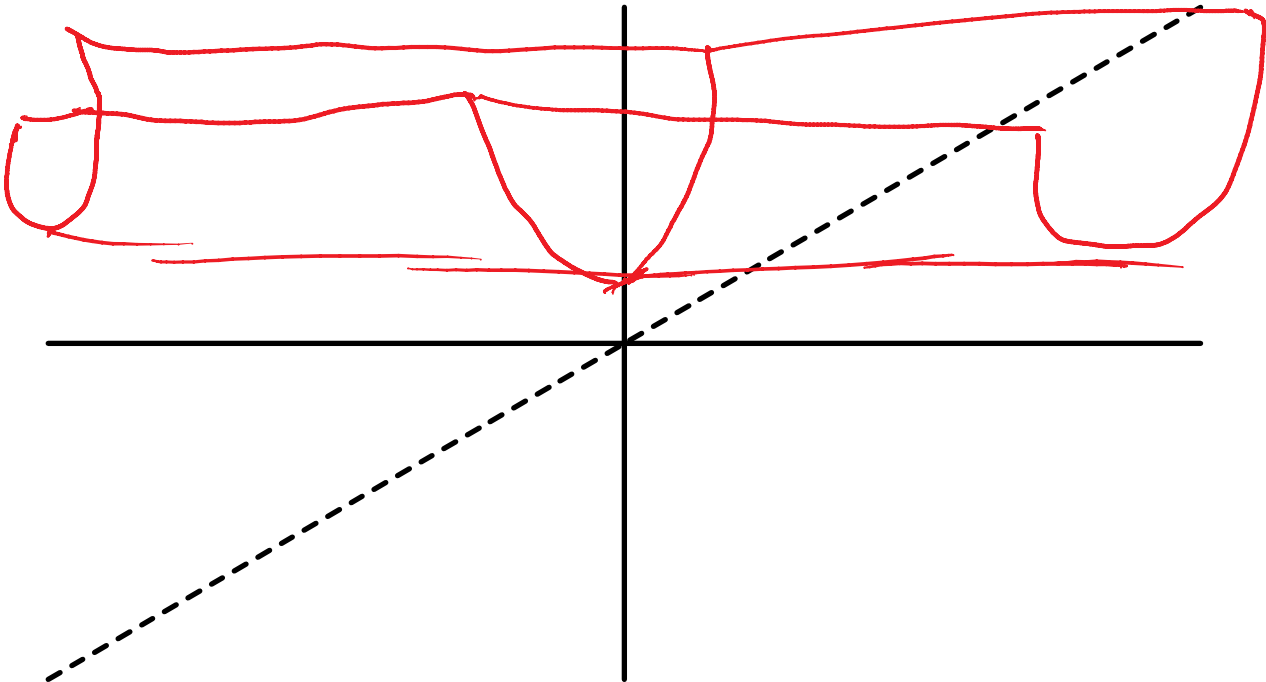
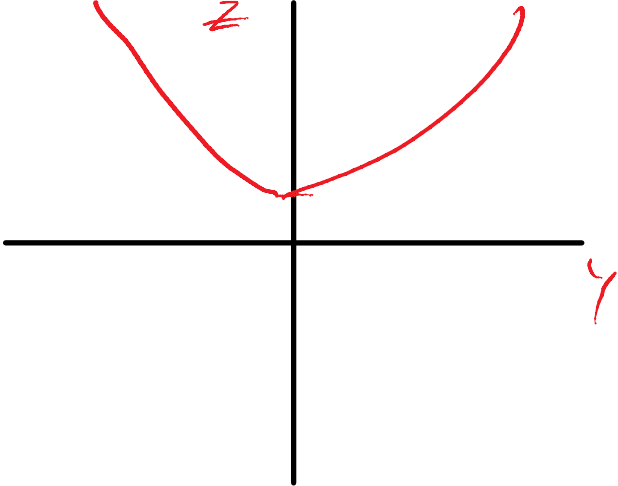


Since the graph of $f(x,y) = x^2$ consists of points (x,y,z) and there is no dependence on y , the y values may be whatever we like and it will not affect the z values.... In this way the parabola is just stretched in the y -direction for ever. This is called a parabolic cylinder

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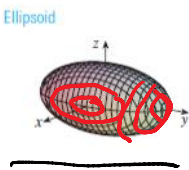
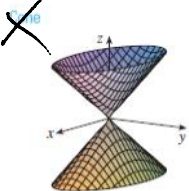
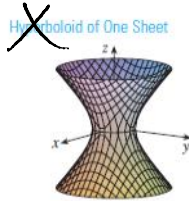
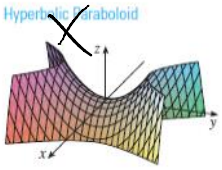
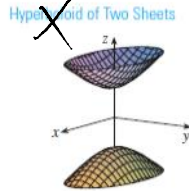
HW example Sketch the graph.

13. $f(x, y) = y^2 + 1$



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Quadratic Surfaces...

Surface	Equation	Surface	Equation
 Ellipsoid	$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ All traces are ellipses. If $a = b = c$, the ellipsoid is a sphere.	 ~~One Sheet Hyperboloid~~	$\frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ Horizontal traces are ellipses. Vertical traces in the planes $x = k$ and $y = k$ are hyperbolas if $k \neq 0$ but are pairs of lines if $k = 0$.
 ~~Elliptic Paraboloid~~	$\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ Horizontal traces are ellipses. Vertical traces are parabolas. The variable raised to the first power indicates the axis of the paraboloid.	 ~~Hyperboloid of One Sheet~~	$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ Horizontal traces are ellipses. Vertical traces are hyperbolas. The axis of symmetry corresponds to the variable whose coefficient is negative.
 ~~Hyperbolic Paraboloid~~	$\frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2}$ Horizontal traces are hyperbolas. Vertical traces are parabolas. The case where $c < 0$ is illustrated.	 ~~Hyperboloid of Two Sheets~~	$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ Horizontal traces in $z = k$ are ellipses if $k > c$ or $k < -c$. Vertical traces are hyperbolas. The two minus signs indicate two sheets.

HW Example Which one is it?

21. $4x^2 + y^2 + 4z^2 - 4y - 24z + 36 = 0$

$4x^2 + y^2 - 4y + 4z^2 - 24z + 36 = 0$

$4x^2 + y^2 - 4y + 4(z^2 - 6z) + 36 = 0$

$4x^2 + y^2 - 4y + 4 - 4 + 4(z^2 - 6z + 9 - 9) + 36 = 0$

$4x^2 + (y - 2)^2 - 4 + 4((z - 3)^2 - 9) + 36 = 0$

$4x^2 + (y - 2)^2 - 4 + 4(z - 3)^2 - 36 + 36 = 0$

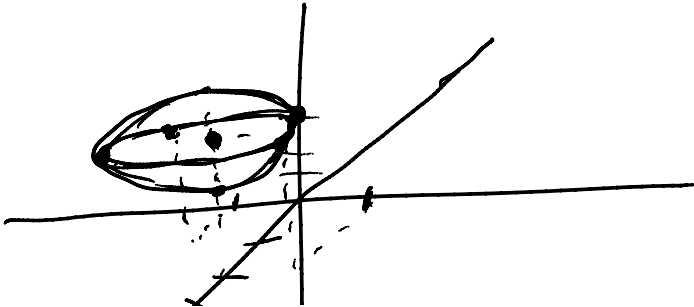
$\frac{4}{4}x^2 + \frac{(y - 2)^2}{4} + \frac{4(z - 3)^2}{4} = \frac{4}{4} \Rightarrow x^2 + \frac{(y - 2)^2}{4} + (z - 3)^2 = 1$

$(0, 2, 3)$

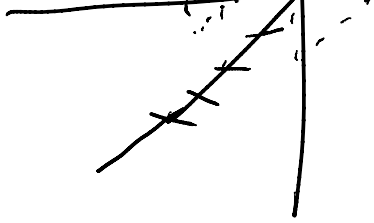
$4/2 = 2$

$2^2 = 4$

$6/2 = 3, 3^2 = 9$



$\frac{x^2}{1^2} + \frac{y^2}{2^2} + \frac{z^2}{1^2} = 1$



7 7 7

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Graphing a plane.....

Sketch a graph of the plane $2x+3y+6z=4$

$$x=0, y=0 \quad 6z=4 \quad z=\frac{2}{3}$$

$$x=0, z=0 \quad 3y=4 \quad y=\frac{4}{3}$$

$$x=0, z=0 \quad 2x=4 \quad x=2$$

